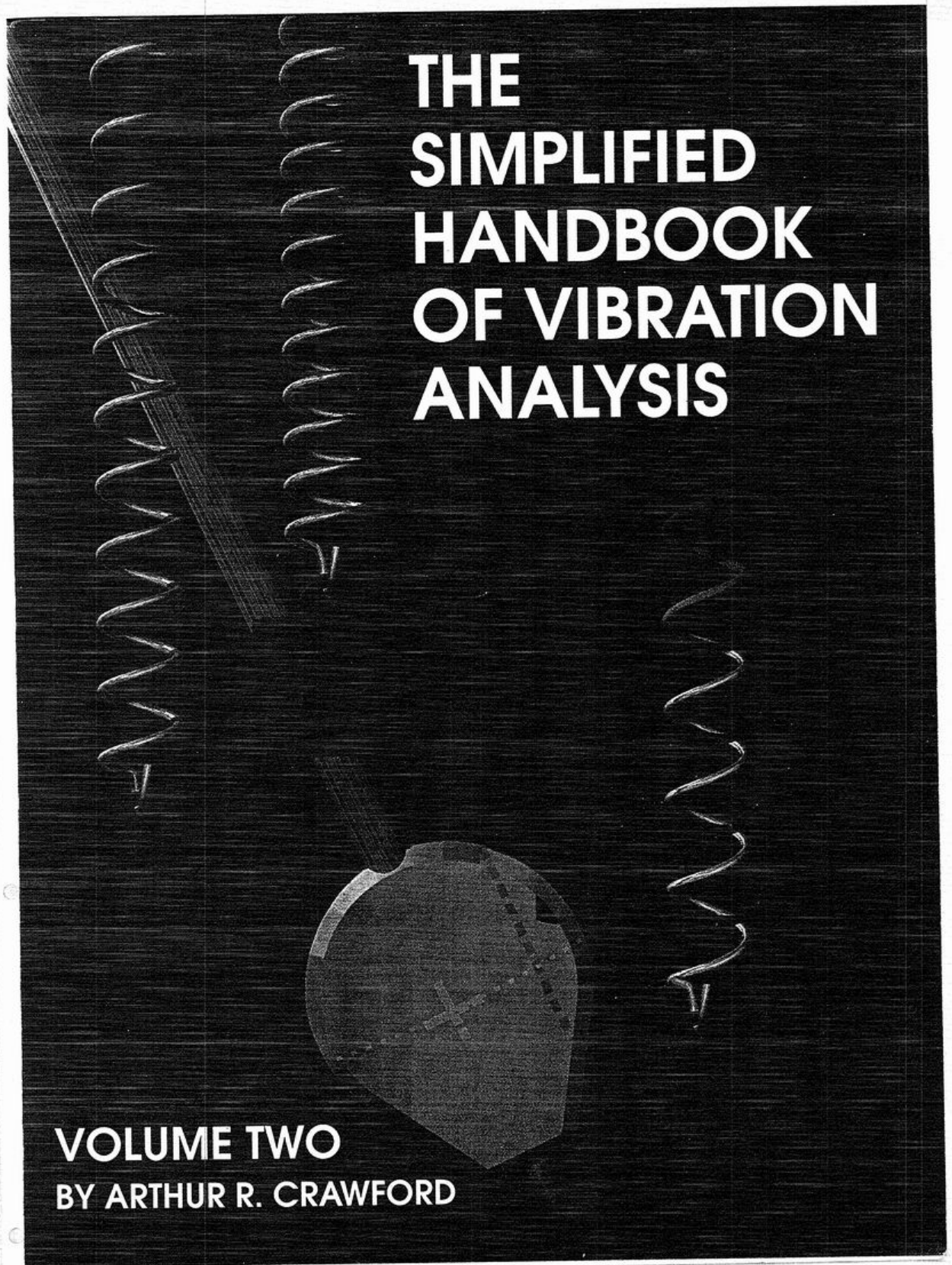
THE SIMPLIFIED HANDBOOK OF VIBRATION ANALYSIS



VOLUME TWO

BY ARTHUR R. CRAWFORD

THE SIMPLIFIED HANDBOOK OF VIBRATION ANALYSIS

Volume 2: Applied Vibration Analysis

THE SIMPLIFIED HANDBOOK OF VIBRATION ANALYSIS

Volume 2: Applied Vibration Analysis

Arthur R. Crawford
President, ARC Associates
Senior Vibration Analyst, Computational Systems Incorporated

and

Steve Crawford
President, Crawford Communications

Library of Congress Catalog Card number 92-72682

The Simplified Handbook of Vibration Analysis
Volume 2: Applied Vibration Analysis

Copyright 1992 by Computational Systems, Incorporated (CSI). All rights reserved. Printed in the United States of America. No part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

Send all inquiries to:
Computational Systems, Inc.
835 Innovation Drive
Knoxville, TN 37932

Table of Contents

MACHINE-SPECIFIC ANALYSIS: FANS 1
1.0 PRINCIPLES OF FAN OPERATION 2
1.1 Fan Curves 3
2.0 TYPES OF FANS 4
2.1 Centrifugal Fans 4
2.2 Axial Flow Fans 8
3.0 STRUCTURAL ASPECTS AFFECTING VIBRATION 9
3.1 Mounting 9
3.2 Anti-Friction Bearings 11
3.3 Axial Fans 13
4.0 OPERATIONAL SOURCES OF VIBRATION 14
4.1 Rotating Stall 14
5.0 VIBRATION ANALYSIS OF FANS 15
5.1 Determine the Problem 16
5.2 Inspect the System 16
5.3 Take Static Data 18
5.4 Take Dynamic Data 21
Time Domain Data 22
Frequency Domain Data 22
5.5 Determine the Forcing Function(s) 23
Imbalance 24
Misalignment: Coupling, Belt, and Bearing 24
Resonance and Disk Skew 24
Cavitation 24
Mechanical Non-linearities 25
Oil whirl 25
High Blade Pass Frequency 25
Variable Pitch Axial Flow Fans 26
6.0 FAN DIAGNOSIS 26
SUMMARY 29
REFERENCES 30

MACHINE-SPECIFIC ANALYSIS: PUMPS 31
1.0 PRINCIPLES OF PUMP OPERATION 32
1.1 Types of Head 33
1.2 Overview of Fluid Dynamics 35
1.3 Bernoulli's Equation 35
1.4 Viscosity and Fluid Flow 36
1.5 The Reynolds Number 37
2.0 TYPES OF PUMPS 38
2.1 Centrifugal Pumps 38

Type of Impeller 42
Orientation of Shaft 43
Type of Casing Split 43
2.2 Axial Pumps 43
3.0 OPERATIONAL SOURCES OF VIBRATION 44
3.1 Cavitation 45
3.2 Recirculation 45
3.3 Hydraulic Imbalance 46
3.4 Interaction with Volute/Diffuser 46
3.5 Misalignment 46
3.6 Imbalanced Impeller 47
3.7 Impeller Instability 47
3.8 Bent Shaft 47
3.9 Pipe Stresses 47
4.0 STRUCTURAL ASPECTS AFFECTING VIBRATION 47
4.1 Anti-Friction Bearings 48
4.2 Sleeve Bearings 49
4.3 Resonance 50
4.4 Looseness 53
4.5 Misalignment 53
5.0 ANALYSIS 53
SUMMARY 55
REFERENCES 56

MACHINE-SPECIFIC ANALYSIS: MOTORS 57
1.0 MAGNETS AND ELECTRICITY 58
1.1 Magnetic Fields 59
1.2 Electricity and Magnetism 60
1.3 Electromagnets 60
1.4 Induced Current 61
2.0 PRINCIPLES OF AC MOTORS 62
2.1 AC Motor Design and Operation 62
2.2 Synchronous Motors 64
2.3 Induction Motors 65
3.0 PRINCIPLES OF DC MOTORS 66
3.1 A Simple DC Motor 67
3.2 Commutation 68
3.3 Torque Variation 69
3.4 Design of DC Motors 70
3.5 Brushes 71
3.6 Armature reaction and brush sparking 72
3.7 Interpoles 74
3.8 Types of DC Motors 74
4.0 ELECTRICAL SOURCES OF VIBRATION 76

4.1 Electrical Sources of Vibration in AC Motors 76
4.2 Electrical Sources of Vibration in DC Motors 78
4.3 External Sources of Vibration 79
5.0 MECHANICAL SOURCES OF VIBRATION 79
Rotor Loose on Shaft 80
Thermal Growth 81
Uneven Air Gap 81
Out of Round Rotor 81
Stator Looseness 81
Rotor Bars 82
Shorted Lamination 86
Commutation 86
6.0 ANALYSIS 87
SUMMARY 89
REFERENCES 90

MACHINE-SPECIFIC ANALYSIS: COMPRESSORS 93
1.0 PRINCIPLES OF COMPRESSOR OPERATION 93
2.0 TYPES OF COMPRESSORS 94
2.1 Reciprocating Compressors 95
2.2 Rotating Compressors 95
2.2.1 Screw compressors 96
2.3 Centrifugal Compressors 97
2.4 Axial Compressors 98
3.0 STRUCTURAL ASPECTS AFFECTING VIBRATION 98
3.1 Thermal Growth and Misalignment 98
3.2 Anti-Friction Bearings 99
3.3 Resonances 100
3.4 Looseness 100
4.0 OPERATIONAL SOURCES OF VIBRATION 100
4.1 Pulsations 100
4.2 Flow Characteristics 101
4.3 Surge 101
4.4 Rotary Screw Compressors 101
5.0 VIBRATION ANALYSIS OF COMPRESSORS 102
5.1 Determine the Problem 103
5.2 Inspect the System 103
5.3 Take Static Data 104
5.3.1 Resonant Frequencies 104
5.3.2 Blade Pass Frequency 105
5.3.3 Rotational and Reciprocating Forces 105
5.3.4 Mechanical Data 105
5.3.5 Background Vibration 105
5.4 Take Dynamic Data 105

5.4.1 Startup Data 106
5.4.2 Normal Load Data 106
Time domain data 106
Frequency Domain Data 106
5.4.3 Coastdown Data 107
5.5 Determine the Forcing Function(s) 107
Imbalance 108
Misalignment: Coupling and Bearing 108
Resonance 108
Cavitation 108
Mechanical Non-linearities 108
Oil whirl 109
High Blade Pass Frequency 109
Shaft Orbits 109
SUMMARY 109
REFERENCES 110

MACHINE-SPECIFIC ANALYSIS: ENGINES 111

1.0 ENGINES 111
1.1 Pressure Volume (PV) Curves 113
Ideal Otto Cycle 113
Ideal Diesel Cycle 114
1.2 Compression Ratio 115
2.0 VIBRATORY FORCES IN RECIPROCATING ENGINES 115
2.1 The Slider Crank Mechanism 115
Rotating and Reciprocating Forces 116
2.2 Knock and Preignition 122
2.3 Gas Pressure Forces 122
2.4 Torsional Vibration Forces 123
2.5 Crankshaft Balance 123
3.0 ANALYZING ENGINE VIBRATION 124
3.1 Reciprocating Loads on Bearings 124
3.2 Vibrations from Engine Accessories 124
SUMMARY 125
REFERENCES 125

MACHINE-SPECIFIC ANALYSIS: TURBINES 127

1.0 PRINCIPLES OF TURBINE OPERATION 127
2.0 TURBINE VIBRATION 129
2.1 Imbalance 129
2.2 Rotor Bow 130
2.3 Misalignment 130
2.4 Thermal Growth and Distortion 131
2.5 Resonance 132
3.0 BALANCING STEAM TURBINES 132

3.1 Acceptable Methods 133
3.2 Determining Which Method to Use 134
Making the Compromise 135
4.0 THE STATIC COUPLE METHOD 135
4.1 Performing Static Couple Balancing 136
Locate the Heavy Spot 138
Determine Where to Place Your Weight 139
Take Vibration Readings 139
Solve for Correction 139
Locate the Heavy Spot 140
Place Your Weight 140
Take Vibration Readings 140
Solve for Correction 140
5.0 THE INTERMEDIATE SOLUTION METHOD 141
5.1 Performing Intermediate Solution Balancing 141
Identify and Catalog the Correction Planes 141
Record the As-Is Readings 141
Place Trial Weight in First Plane 142
Calculate Correction 142
6.0 GENERATOR ROTORS 142
SUMMARY 143
REFERENCES 143

GEARS 145

INTRODUCTION 145
1.0 GEAR THEORY 145
1.1 Spur Gears 148
1.2 Helical Gears 149
1.3 Crossed Helical 149
1.4 Worm Gears 150
1.5 Bevel Gears 150
2.0 GEAR MESHING AND VIBRATION 151
2.1 Load Pulses 151
2.2 Transmission Error 151
2.3 Deflections 152
3.0 VIBRATIONS IN GEARING 153
SUMMARY 154
REFERENCES 155

POWER TRANSMISSION COMPONENTS 157

1.0 BELTS AND BELT DRIVES 157
1.1 Types of Belts 157
1.2 V-Belts 158
2.0 BASIC THEORY OF BELT DRIVES 161
2.1 A Belt in a Belt Drive 161

2.2 Belt Slip and Belt Creep 162
2.3 Drive Ratios 163
2.4 Belt Tension and Alignment 164
3.0 NOISE IN BELT DRIVES 165
4.0 BELT AND SHEAVE WEAR 165
4.1 Sheave Wear 165
5.0 BELT DRIVE VIBRATION 166
5.1 Sheave Alignment 166
5.2 Belt Tension 166
5.3 Primary Belt Frequency 166
5.4 Belt Strand Frequency 168
5.5 Belt Types and Vibration Severity 170
6.0 CHAINS AND CHAIN DRIVES 171
6.1 Types of Chains 171
7.0 PRINCIPLES OF CHAIN DRIVES 173
7.1 Drive Ratio 174
7.2 Number of Sprocket Teeth 174
7.3 Center-to-Center Distance 174
7.4 Chain Slack 175
7.5 Chain Wear and Tension 175
7.6 Chordal Action 176
7.7 Determining Chain Length 176
8.0 CHAIN DRIVE VIBRATION 178
8.1 Tooth-Induced Vibration 178
8.2 Other Sources 178
9.0 UNIVERSAL JOINTS 178
9.1 Non-Uniform Joints 179
9.2 Uniform Joints 179
9.3 Other Classifications 179
10.0 CARDAN UNIVERSAL JOINTS 180
10.1 Non-Uniform Motion 180
10.2 Achieving Constant Velocity 182
11.0 BALL AND TRUNNION UNIVERSAL JOINT 183
12.0 ROLLER AND TRUNNION UNIVERSAL JOINT 184
13.0 SLIDING BLOCK AND TRUNNION UNIVERSAL JOINT 184
14.0 RUBBER COUPLINGS 185
15.0 VIBRATION IN UNIVERSAL JOINTS 185
15.1 Torsional Vibration 185
15.2 Inertial Vibration 186
15.3 Secondary Couple Vibration 186
15.4 Other Vibrations 186
16.0 FLEXIBLE COUPLINGS 187
16.1 Types of Couplings 187
16.2 Vibration Considerations 187

SUMMARY 189
BIBLIOGRAPHY 191

ANALOG INSTRUMENTS 193
SUMMARY 196

DIGITAL ANALYSIS 199
1.0 DIGITAL ANALYZERS 199
1.1 Frequency Analysis with Analog Analyzers 199
1.2 The Fourier Transform 200
1.3 Time and Frequency Domain Displays 201
1.4 Aliasing 202
2.0 SIGNAL ANALYSIS 204
2.1 Line Spectra 205
2.2 Continuous Spectra 206
2.3 Time vs. Frequency Displays 207
2.4 Leakage 211
3.0 INSTANTANEOUS SPECTRA 212
4.0 TRIGGERING 213
5.0 AVERAGING 213
5.1 Linear Averaging 213
5.2 Exponential Averaging 213
5.3 Peak Hold 214
5.4 Synchronous Averaging 214
5.5 Synchronous Time Averaging 214
6.0 CROSS-CHANNEL CAPABILITIES 215
6.1 Coherence 215
6.2 Transfer Function 217
6.3 Cross Channel Phase 222
SUMMARY 222

MEASUREMENT AND COLLECTION TECHNIQUES 223
1.0 MACHINERY MAINTENANCE 223
1.1 Catastrophic (Hysterical) Maintenance 223
1.2 Preventive (Historical) Maintenance 224
1.3 Predictive (Histogram) Maintenance 224
2.0 VIBRATION DATA COLLECTION 225
2.1 Collect Broadband or Spectral Data? 226
2.2 What Frequency Range Should Be Covered? 226
2.3 What Units Are Used for Machine Health Baseline? 227
2.4 How Much Vibration is Too Much Vibration? 227
2.5 What Type of Transducer Should Be Used? 228
2.6 How Will the Transducers Be Mounted? 233
2.7 How Many Measuring Locations Are Required? 234
2.8 How Often Will Data Be Collected? 234

2.9 Will Machines be Loaded or Unloaded During Collection? 234
2.10 Who Will Collect and Analyze the Data? 235
SUMMARY 235

EXPERIMENTAL MODAL ANALYSIS 237

1.0 EXPERIMENTAL MODAL ANALYSIS 237
1.1 Disadvantages of Experimental Modal Analysis 238
1.2 A Comparison with Finite Element Analysis 238
2.0 EXPERIMENTAL MODAL ANALYSIS THEORY 239
2.1 A Single Degree of Freedom System 239
2.1.2 Imaginary Frequency Response Curve 242
2.2 A Multiple Degree of Freedom System 243
3.0 FREQUENCY RESPONSE FUNCTIONS 246
3.1 Basic Frequency Response Functions 246
4.0 EXPERIMENTAL MODAL ANALYSIS PROCEDURES 247
4.1 Develop a Test Procedure 247
4.2 Field Test 249
4.3 Modal Parameter Identification 252
4.4 Model Verification 254
5.0 STRUCTURAL MODIFICATION METHODS 255
6.0 CASE HISTORY #1 256
6.1 Statement of Problem 256
6.2 Experimental Modal Analysis 256
6.3 Finite Element Model 258
6.4 Structural Modifications 260
7.0 CASE HISTORY #2 260
7.1 Statement of Problem 261
7.2 Experimental Modal Analysis 261
8.0 OPERATING DEFLECTION SHAPE ANALYSIS 263
8.1 Test Procedure 264
8.2 Curve Fitting 265
8.3 Single-Channel Operating Deflection Shape 266
8.4 Case History 266

FINITE ELEMENT ANALYSIS 269

2.0 PRINCIPLES OF FINITE ELEMENT ANALYSIS 270
3.0 STATIC ANALYSIS 272
4.0 DYNAMIC ANALYSIS 274
4.1 The Modal Superposition Method 274
5.0 MESH REFINEMENT 276
6.0 HIGHER ORDER DISPLACEMENT FUNCTIONS 278
6.1 Shape Functions 281
7.0 ASPECT RATIO AND SKEWNESS 284
8.0 ELEMENT TYPES 285
8.1 Spar Elements 285

8.2 Beam Elements 286
8.3 Thin Shell Elements 287
8.4 Two Dimensional Solid Element 288
8.5 Three Dimensional Solid Element 288
8.6 Mass Elements 289
8.7 Spring Elements 290
9.0 ANALYSIS PROCEDURES 290
9.1 Nodes 290
9.2 Elements 291
9.3 Boundary Conditions 291
9.4 Dynamic Loading 292
9.5 Matrix Condensation 292
10.0 CASE HISTORY #1 293
10.1 Problem Description 293
10.2 Finite Element Solution 295
11.0 CASE HISTORY #2 297
11.1 Problem Description 297
11.2 Finite Element Analysis 298
12.0 CASE HISTORY #3 299
12.1 Problem Description 300
12.2 Finite Element Analysis 301
13.0 CONCLUSION 303
14.0 REFERENCES 303

APPENDIX B - EXAMPLE ANALYSES 305

ANALYSIS #1 - MULTIPLE RESONANT FREQUENCIES 305
ANALYSIS #2 - BEAT FREQUENCY BETWEEN SPINDLES 306
ANALYSIS #3 - HARMONIC GROUPS OF BELT FREQUENCY 307
ANALYSIS #4 - RUBBING GIBS 308
ANALYSIS #5 - USING COHERENCE 309
ANALYSIS #6 - BEARING FAULT 310
ANALYSIS #7 - PRESSURE PULSATIONS 311

APPENDIX C - RESONANT FREQUENCY CALCULATIONS 313

Definition of Terms 313
Assumptions 314
1.0 SHAFT CASE ONE 315
2.0 SHAFT CASE TWO 316
3.0 SHAFT CASE THREE 316
4.0 SHAFT CASE FOUR 318
5.0 SHAFT CASE FIVE 319
6.0 BEAM CASE ONE 320
7.0 BEAM CASE TWO 321
8.0 BEAM CASE THREE 322
8.1 Resonant Frequency for Mass *m1* 323

8.2 Resonant Frequency for Mass m_2 324
8.3 The Combined Natural Frequency 324
9.0 BEAM CASE FOUR 325
10.0 BEAM CASE FIVE 326
11.0 ADDING SHAFT OR BEAM MASS 327
11.1 Adding Shaft Mass 327
11.2 Adding Beam Mass 328

Glossary of Terms **329**
Index **337**

List of Figures

Figure 1 - fan curves 3
Figure 2 - a centrifugal fan 4
Figure 3 - center hung and cantilevered centrifugal fans 5
Figure 4 - backward bladed centrifugal fan 5
Figure 5 - airfoil blade 6
Figure 6 - radially tipped fan 6
Figure 7 - forward curved blade 7
Figure 8 - axial flow fan 8
Figure 9 - vibration mounts 10
Figure 10 - floating and fixed bearings 11
Figure 11 - rotating stall 14
Figure 12 - the belt strand 19
Figure 13 - belt strand resonance 19
Figure 14 - head and pressure 32
Figure 15 - measurements of static head 33
Figure 16 - a Venturi tube 36
Figure 17 - viscous fluid flow 37
Figure 18 - a basic centrifugal pump 39
Figure 19 - uniform pressure on the impeller 40
Figure 20 - unequal pressure on the impeller 40
Figure 21 - a twin volute pump 41
Figure 22 - diffusion vanes 41
Figure 23 - incline of axial impeller vanes 44
Figure 24 - floating and fixed bearings 48
Figure 25 - oil whirl 49
Figure 26 - effect of dynamic absorber on system response 52
Figure 27 - absorber mass and resonance spread 52
Figure 28 - magnetic attraction and repulsion 59
Figure 29 - a magnetic field 59

Figure 30 - effect of electricity on a compass needle 60
Figure 31 - a basic electromagnet 61
Figure 32 - induced current 61
Figure 33 - a typical motor stator 62
Figure 34 - three phase stator 63
Figure 35 - a rotor with a uniform air gap 64
Figure 36 - an induction motor rotor 65
Figure 37 - circular current flow in the rotor 66
Figure 38 - motor action 67
Figure 39 - a simple DC motor 67
Figure 40 - torque through one-half revolution 68
Figure 41 - a two-segment commutator 68
Figure 42 - torque vs. position plot for a two-segment commutator 69
Figure 43 - a four-segment commutator 69
Figure 44 - a DC motor armature 70
Figure 45 - a typical commutator 71
Figure 46 - brush assembly on a DC motor 72
Figure 47 - main magnetic field 73
Figure 48 - magnetic field around the armature coil 73
Figure 49 - the load neutral plane 74
Figure 50 - one slip cycle 77
Figure 51 - half wave rectification 78
Figure 52 - full wave rectification 79
Figure 53 - head and pressure 94
Figure 54 - a reciprocating compressor 95
Figure 55 - a sliding vane compressor 96
Figure 56 - a two-lobed screw compressor 97
Figure 57 - a centrifugal compressor 97
Figure 58 - an axial compressor 98
Figure 59 - fixed and floating bearings 99
Figure 60 - drive train of a rotary screw compressor 101
Figure 61 - the PV curve for an ideal otto cycle 113
Figure 62 - the PV curve for an ideal diesel cycle 114
Figure 63 - the slider crank mechanism 116
Figure 64 - rotating and reciprocating mass 118
Figure 65 - reciprocating piston with no moment 119
Figure 66 - an unbalanced counterbalance force 119
Figure 67 - the origin of side-to-side motion 120
Figure 68 - a turbine/generator set with high vibration 133
Figure 69 - pure static imbalance 134
Figure 70 - pure couple imbalance 134
Figure 71 - a turbine rotor with static imbalance 136
Figure 72 - two gear teeth in contact 145
Figure 73 - conjugacy 146

Figure 74 - the Pitch Circle 146
Figure 75 - tracing an involute curve 147
Figure 76 - spur gears 148
Figure 77 - a helical gear 149
Figure 78 - worm gear and wormwheel 150
Figure 79 - a straight bevel gear 150
Figure 80 - how a V-belt wedges against the sheave 158
Figure 81 - typical V-belt cross section 159
Figure 82 - various belt types 161
Figure 83 - a sample belt drive 162
Figure 84 - belt drive ratio 163
Figure 85 - drive ratio and output speed 164
Figure 86 - determining belt length 167
Figure 87 - the belt strand 168
Figure 88 - belt strand resonance 169
Figure 89 - time domain plot of support resonance excited by belt strand 170
Figure 90 - common chain types 172
Figure 91 - a basic chain drive 173
Figure 92 - a hunting tooth sprocket 174
Figure 93 - allowing for chain slack 175
Figure 94 - chordal action 176
Figure 95 - determining chain length 177
Figure 96 - the Cardan joint 180
Figure 97 - motion of a point at zero joint angle 181
Figure 98 - motion of a point at an angle 181
Figure 99 - speed variations during one revolution 182
Figure 100 - polar plot of speed variations 182
Figure 101 - joint phasing 183
Figure 102 - the Ball and Trunnion joint 183
Figure 103 - maximum angle of ball and trunnion joint 184
Figure 104 - a gear coupling 188
Figure 105 - sampling at the input frequency 201
Figure 106 - sampling at twice the input frequency 202
Figure 107 - 128 Hz signal sampled at 1,024 Hz 202
Figure 108 - 1,152 Hz signal sampled at 1,024 Hz 203
Figure 109 - locating a precise frequency 206
Figure 110 - time domain waveform for Case 1 207
Figure 111 - frequency spectra of Case 1 207
Figure 112 - time domain waveform of Case 2 208
Figure 113 - frequency spectra of Case 2 208
Figure 114 - time domain waveform of Case 3 209
Figure 115 - frequency spectra of Case 3 209
Figure 116 - time domain waveform of Case 4 210
Figure 117 - frequency spectra of Case 4 210

Figure 118 - coherence plot of cracked machine spindle 216
Figure 119 - coherence plot of a new machine spindle 217
Figure 120 - Inertance plot of turbine generator 218
Figure 121 - apparent stiffness calculated from inertance 219
Figure 122 - imaginary portion of Figure 120 220
Figure 123 - impact locations on turbine generator 220
Figure 124 - rocking mode of turbine generator 221
Figure 125 - integration and double integration 231
Figure 126 - differentiation and double differentiation 232
Figure 127 - a single degree of freedom vibrating system 239
Figure 128 - magnitude of force varies with time 240
Figure 129 - real and imaginary components of total displacement 241
Figure 130 - the magnification factor 242
Figure 131 - a typical imaginary frequency response function 243
Figure 132 - a multiple degree of freedom system 243
Figure 133 - mode shape obtained from one data point 248
Figure 134 - mode shapes obtained from multiple data points 248
Figure 135 - erroneous mode shape display resulting from aliasing 249
Figure 136 - impact and response plots 250
Figure 137 - effect of tip hardness on excited frequency band 250
Figure 138 - a force-exponential window 251
Figure 139 - coherence before and after repairs 252
Figure 140 - light and heavy modal coupling 253
Figure 141 - frequency response curve for Case 1 257
Figure 142 - first and second mode shapes for Case 1 257
Figure 143 - finite element model for Case 1 258
Figure 144 - first mode shape of Case 1 259
Figure 145 - side and top views of second mode shape of Case 1 259
Figure 146 - coherence and frequency response plots for Case 2 261
Figure 147 - animated mode shape of Case 2 first mode 262
Figure 148 - animated mode shape of Case 2 second mode 263
Figure 149 - setup for operating deflection shape analysis 264
Figure 150 - pseudo FRF for an 1,800 RPM fan 265
Figure 151 - coherence and frequency response plots for ODS Case 1 266
Figure 152 - result of ODS analysis 267
Figure 153 - approximating beam deflection 271
Figure 154 - two uniaxial truss elements coupled in series 272
Figure 155 - meshes for a steel cantilever slab 276
Figure 156 - plotted mode shapes from Table 11 277
Figure 157 - higher order displacement functions 278
Figure 158 - a simple uniaxial truss element 281
Figure 159 - solid elements obtained from a parent element 284
Figure 160 - a spar element 285
Figure 161 - a beam element 286

Figure 162 - a thin shell element 287
Figure 163 - a two-dimensional solid element 288
Figure 164 - a three-dimensional solid element 289
Figure 165 - node generation 290
Figure 166 - impact data from ductwork 294
Figure 167 - frequency spectra at fan bearing 294
Figure 168 - finite element model for existing ductwork 295
Figure 169 - finite element model of new duct design 295
Figure 170 - first two predicted mode shapes of new ductwork design 296
Figure 171 - 12th mode shape of new ductwork design 296
Figure 172 - finite element model of machine foundation 297
Figure 173 - first two predicted mode shapes of machine foundation 298
Figure 174 - pressure-time variation for an air burst 299
Figure 175 - finite element model of the control house 301
Figure 176 - first mode shape of control house 301
Figure 177 - second mode shape of control house 302
Figure 178 - predicted response of control house front wall 302
Figure 179 - time and frequency domain plots of air handler vibration 306
Figure 180 - beat frequency between two boring spindles 306
Figure 181 - beat frequency plot after balance 307
Figure 182 - frequency domain spectra of fan vibration 307
Figure 183 - time domain plot of Station 2 cutter 308
Figure 184 - vibration at Station 2 only 309
Figure 185 - time domain plot after readjusting gibs 309
Figure 186 - coherence plot obtained from impacting spindle 309
Figure 187 - coherence after repairing spindle casing 310
Figure 188 - time domain plot of boring machine vibration 310
Figure 189 - time domain plot after replacing spindle bearings 311
Figure 190 - vibration reading at 296 Hz 311
Figure 191 - vibration on plant floor, adjacent to pad 311
Figure 192 - vibration on machine with lube pump off 312
Figure 193 - vibration after inserting tee in lube pump line 312
Figure 194 - Shaft Case One: single wheel 315
Figure 195 - Shaft Case Two: outboard mass 316
Figure 196 - Shaft Case Three: two outboard masses 317
Figure 197 - Shaft Case Four: one mass inboard, one mass outboard 318
Figure 198 - Shaft Case Five: two outboard wheels, one inboard wheel 319
Figure 199 - Beam Case One: one mass at end of beam 320
Figure 200 - Beam Case Two: mass not at end of beam 322
Figure 201 - Beam Case Three: two masses on one beam 323
Figure 202 - Beam Case Four: one mass supported by two beams 325
Figure 203 - Beam Case Five: skid resonance 326

List of Tables

Table 1 - engine types and reciprocating forces 121
Table 2 - crankshafts requiring bobweights during balancing 123
Table 3 - standard belt sizes 159
Table 4 - vibration severity for various belt types 171
Table 5 - the effect of resolution on the time window 204
Table 6 - coherence as a function of the number of averages 216
Table 7 - how resonances are displayed in rectangular transfer function plots 219
Table 8 - categorizing levels of vibration 228
Table 9 - comparing FEM and EMA predictions 260
Table 10 - natural frequency and damping estimates for Case 2 262
Table 11 - calculated mode shapes using models from Figure 155 277
Table 12 - mode shapes calculated using a quadratic displacement function 279
Table 13 - linear vs. quadratic displacement functions 279
Table 14 - solution of the cantilever slab problem using several models 280
Table 15 - calculating first stretching mode using the three displacement functions 280

PREFACE

We have tried to remain true to our original intention: to write a Simplified Handbook of Vibration Analysis in this second volume. We recognize that interest in vibration and vibration analysis is either highly technical or intensely practical. Therefore we have attempted, in both volumes, to deal with the technical aspects of vibration and vibration analysis in terms that operating personnel can not only understand, but put to practical use.

To what extent we have been successful rests in the minds of our readers. We are certain that we have fallen far short of what we wanted to accomplish, and trust that others will add to this effort and make the value of vibration analysis more apparent to our country's industrial base.

It will be apparent to readers of this second volume that, first of all, we have not dealt in as much detail as we could have in each of the machine-specific chapters. Secondly, some of the material (like the chapters on Experimental Modal Analysis and Finite Element Analysis) is hard-pressed to meet the classification "Simplified." We did the best we could.

If possible, we would like to expand this series with several detailed volumes, each of which dealing with an individual aspect of this valuable and interesting field. For example, Resonance, Imbalance, Alignment, Rolling Element Bearings, Journal Bearings, and Spectrum Analysis are some of the topics we could cover.

Again I want to take this opportunity to express my thanks to everyone who has helped in the endeavor, especially my wife of over 50 years who, in her words, has been a "vibration widow." THANKS!

Art Crawford

Prefaces are scary. It never occurred to me how dangerous they are until I set out to write one.

Those of you who read prefaces have noticed how authors usually do three things: they thank their spouses, they thank a stream of people you know nothing about, and they apologize for errors and

omissions. I am going to thank my wife and several other people, but the last one bothers me. If the author is so sure that there is an error in the book, then why didn't they fix it?

The answer, of course, is that they haven't found it. But they are so sure it's there that they take pains to warn you (albeit in the preface, where most of you won't see it). I could cover myself as well, and tell you that "at least one statement in this book is wrong."

Well, if we assume that this preface is true, then the book contains at least one error. But what if we did our job better than I realize, and the rest of the book is accurate? Then the error in the book is in the preface, and that makes my disclaimer incorrect. But if the disclaimer is incorrect, then that makes the preface true. And if the preface is true, then there is at least one error in this book...

Maybe it would be best if you didn't read this preface. For if you do, you'll get so bound up in its inherent paradox that you won't be able to move on to the rest of the book.

Even though you may not know any of them, there are several people whom I must thank. If you meet any of them later in life, be sure and tell them you saw them here first.

Bob Sayer has contributed an incredible amount of time and effort to this book. Besides writing the Finite Element Analysis chapter, and co-authoring the Experimental Modal Analysis chapter, Bob has contributed drawings, explanatory text, references, data, and a thorough review of several other chapters. This second volume could not have been completed without Bob's generous help. Both my father and I extend our deepest gratitude.

Nelson Baxter has also spent a great deal of time helping us. He answered many questions we both had about turbines, and provided a detailed, 22-page outline of important vibration considerations (albeit on a Sunday afternoon, when I thought the book was done!).

Dr. John Starkey, a professor in the Mechanical Engineering department of Purdue University, spent a great deal of time and research deriving the resonant frequency formulas found in Appendix C. My father and I deeply appreciate the effort.

Lamont Gee and Bill Roper of Kiemle Hankins Service Co. provided a great deal of useful information concerning AC and DC motor vibrations. Rowland Hill of Magnetek (and my neighbor) has been very helpful in explaining interpoles and armature reaction. And Jenny Hecker did her usual thorough and excellent job of making English out of what we have written, and keeping our layout and style consistent. If anyone has kept this book simplified, Jenny has.

I must also thank my friends Tony Machosky, Mike Schoenborn, and Glenn Emelko. Tony, the writer I will never be, offered guidance on the scope of the project and writing style as I kept him filled with pasta. Mike helped me squeeze the last bit of performance from my system, and was a good ear when things went wrong. Glenn provided a great deal of insight into digital signal processing, and exhaustively verified formula derivation. He also found time to marry Liz, a wonderful woman (but he didn't invite any of us to the wedding!).

Two other people provided tremendous assistance, and I must thank them. The irony is that I haven't met either of them! Keith Campbell of American Small Business Computers, Inc., provided me with their excellent software, DesignCAD 3D, so that I could complete several of the drawings in the Motors and Power Transmission Components chapters.

Emily Beck Considine of Zurn Industries Inc. provided the universal joint photo in the Power Transmission Components chapter. She was considerate and helpful, willing to send product photos to someone she had never met.

And, of course, I must thank the staff of Computational Systems Incorporated for sponsoring this project. Most of the time and frequency domain plots found in this volume were taken with CSI's Wavepak and MasterTrend software packages. Dan Nower, Dave Drief, and Arby Dickert took time from their busy schedules to review the contents of this book. Ron Moore has been patient and

generous with a project that seemed to have no end. And Ken Piety, the father of the project, has been helpful and supportive beyond description. It has been a pleasure to work with you all.

I must also thank my family. I am so thankful that I have been able to spend these last two years working with my father. No matter how busy he has been, he has always taken the time to answer stupid questions from his youngest kid. He is a wonderful man. Thanks, Dad! My mother has endured the hectic schedules with her typical charm and composure, many times giving up her few private times with her husband to me and this book. Jerry Pruden, my brother-in-law, is a mountain of practical advice, hands-on knowledge, and patience (not to mention always willing to get a sub at Tommy's). On more than one occasion Jerry has clarified obscure information, and thrown together a demonstration of a concept I was writing about. Thanks, Jerry!

Finally, and most importantly, I want to thank my wonderful wife Ann and son Adam. For eating dinner without me while I worked in the evenings, for going to the zoo without me while I worked on the weekends, and for putting up me when I was distracted, exhausted, and grumpy. Ann may not be a vibration widow, but she has been a WordPerfect widow. Adam has taken it all in stride, sitting comfortably on my lap while I write, even throwing dirt into the floppy drive of my laptop to help his dad. You are both dear and wonderful, and I thank you.

Steve Crawford

ABOUT THE AUTHORS

Art Crawford has been an active consultant in the vibration industry for the last 45 years. He is the Founder and President of ARC Associates, a vibration consulting firm with customers worldwide, and is a senior vibration analyst for Computational Systems, Incorporated (where he also teaches courses in advanced vibration analysis).

In 1969 Crawford founded Production Measurements Corporation (PMC), a leading manufacturer of electronic equipment for engine analysis, vibration analysis, and vibration monitoring. Prior to 1969 he was director of electronic research for ABEX Corporation, where he was retained as a consultant on the design and development of engine vibration analysis and dynamic balance equipment for the automotive industry.

Art was the Founder and President of IRD Mechanalysis in 1952. IRD, profitable every year under his direction, remains a leader in the field of vibration analysis and dynamic balance equipment. He holds a B.S. and M.S. in Physics from The Ohio State University.

Steve Crawford is the Founder and President of Crawford Communications, a technical publication consulting firm in Columbus, Ohio. He holds a Bachelor of Arts from The Ohio State University, much to the chagrin of his father.

8 MACHINE-SPECIFIC ANALYSIS: FANS

A one hundred horsepower belt-driven center-hung centrifugal fan at a nuclear power plant exhibited excessive vibration on the outboard fan bearing at motor frequency. The fan was classed as CAT 1 (Category 1), meaning that it was critical for plant operations and could not be shut down.

The analyst assumed that the fan bearing pedestal was resonant at the frequency of the motor and was being excited by imbalance in the motor. He requested permission to remove the belts and balance the motor in place. Since this was a CAT 1 fan, he was not allowed to remove the belts. Therefore he elected to balance the motor with the belts on. Vibration data taken on the motor was unstable, making the balance operation almost impossible.

After completion of the in-place balance operation, data was again taken on the fan bearings. The high vibration at motor frequency on the belt side fan bearing was the same as the original data.

The analyst then assumed that the bearing pedestal resonance was a high Q (lightly damped) system. He recommended that the pedestal be redesigned in order to move the resonance well above motor frequency.

An engineering order was written and the necessary steps were taken to remove the unit from service and modify the fan bearing pedestal. In a nuclear facility, this is not a simple procedure.

After the modifications were completed and the unit was returned to service, the vibration level on the outboard fan bearing at motor frequency was still above that permitted for a CAT 1 fan.

The authors were then retained to examine the unit and make recommendations to reduce the vibration on the fan bearings.

Vibration data was taken in-line and at 90 degrees to the belt direction, and these rectangular coordinate vectors were summed in order to determine the direction of maximum amplitude. The phase of the maximum amplitude at the fan bearing was in-line with the dynamic belt tension, indicating pitch line runout on the motor sheave.

Once again an engineering order was written to check the motor pulley. When this work was done, the pitch line runout on the motor pulley was measured as 0.011 inches. The pulley was replaced and the fan bearing vibration was reduced well below the acceptable level.

Incomplete data collection and faulty analysis can result in very expensive yet unnecessary modifications to the system.

INTRODUCTION

This chapter begins a section on machine-specific analysis. For the next six chapters we will be focusing on vibration problems specific to equipment that is common in industry: fans, pumps, motors, compressors, engines, and turbines.

When discussing machine-specific problems it becomes difficult to adequately define the vibration analyst's role in correcting these problems. The design, operation, and maintenance of industrial equipment is a science in itself, and years of study are required to specialize in just one area.

The vibration analyst, therefore, may use data collection and analysis techniques to describe *how* a machine is shaking, but may not be able to say *why* the machine is shaking the way it is. In these cases, a specialist will use the vibration analyst's data to suggest an appropriate solution.

This chapter will focus on vibration problems that are common in fans. Discussion of the fluid mechanics principles governing fan design is kept to a minimum, except when needed to define terminology and explain the source of vibratory forcing functions.

1.0 PRINCIPLES OF FAN OPERATION

Fans are designed to move air against moderate pressures (the total air resistance in a duct system). The pressure, temperature, density, viscosity, and humidity of the air all affect the design and performance of a fan.

Fluid dynamics, the study of fluids in motion, is a branch of science that figures prominently in fan design, since moving air behaves like a fluid. Some of the terms used to describe fan operation are listed below:

- **pressure**: force per unit area, such as pounds per square inch or Newtons per square meter. Fan pressure is typically measured in Pa (for Pascal); 1 Pa = 1 N/m^2. Fan pressure is expressed in the following three terms:

  1. **Fan total pressure** - the difference between the pressure at the fan inlet and the fan outlet. Fan total pressure is equal to the sum of the fan velocity pressure and the fan static pressure.

  2. **Fan velocity pressure** - the pressure resulting from the average air velocity at the fan outlet. Any moving fluid (whether a liquid or a gas) exerts some force per unit area by virtue of its motion. If you hold your hand at the fan outlet you will feel the moving air pushing against you. The force of this push, due to the velocity of the moving air, is the result of the fan velocity pressure.

3. **Fan static pressure** - the fan total pressure minus the fan velocity pressure. Once the velocity pressure has been removed, all that remains is the pressure of the non-moving (i.e. static) air. Fan static pressure is what most people think of when referring to fan pressure.

- **head**: the height of a column of fluid above a specified plane, typically expressed as inches of water. Head is another measure of pressure, since pressure is what pushes the water up the tube.
- **air power**: the work done by a fan in moving air against a constant pressure. Fan air power is found by the formula

$$P = pQ$$

where

P = air power
p = pressure difference between the fan inlet and outlet
Q = volume of air moved in unit time

- **fan efficiency**: equals air power/fan input power * 100%. The fan input power may be the power absorbed by the rotor only, or may include power absorbed by bearings and other drive components.

Quite often the terms fan, blower, and compressor are used interchangeably. This is not correct, but the distinction between the three is not precise. Generally a fan develops rather low pressure, usually a maximum of a 7 percent increase in air density between the fan inlet and the fan outlet. A blower develops somewhat higher pressure than a fan, while a compressor develops very high air pressure.

1.1 Fan Curves

A fan's pressure, power requirements, and efficiency are all related to the volume of air flow through the fan. If each of these components is plotted on a single graph relative to air flow, a set of curves called **fan curves** are created, as shown in Figure 1.

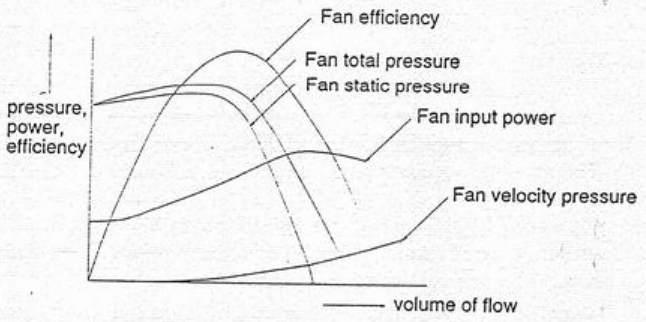


Figure 1 - *fan curves*

The shape of these curves varies with the type of fan. This graph, called a fan characteristic, shows the relationship of pressure, power, and efficiency to air flow and to each other. For example, the fan curves in Figure 1 show that, at low volume flow, a small increase in flow requires a large increase in power (since the fan power curve rises sharply at low volume flow).

This increase in power also improves the fan's efficiency, since increasing volume flow puts the fan at a higher point on its efficiency curve. At higher volumes beyond the peak efficiency, however, increases in air flow require smaller increases in power and result in decreased fan efficiency.

2.0 TYPES OF FANS

There are two primary types of fans:

1. Centrifugal
2. Axial Flow

2.1 Centrifugal Fans

Centrifugal fans (also called "radial flow" fans) consist of an impeller running in a spirally-shaped casing. Usually the air enters the casing from the impeller's axial direction and is discharged at the periphery of the impeller. A typical centrifugal fan is illustrated in Figure 2.

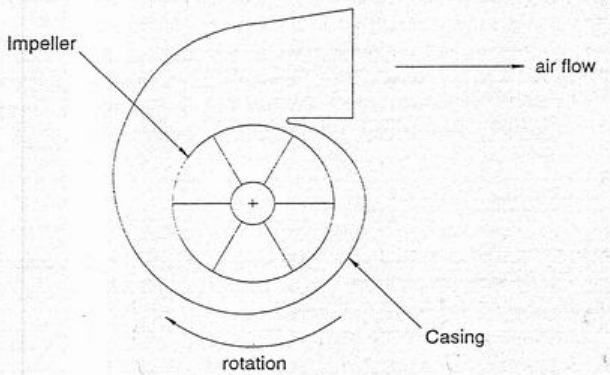


Figure 2 - *a centrifugal fan*

Sometimes the blades on the impeller are shrouded; that is, they have plates welded at one or both ends of the blade. This gives the impeller strength and reduces pressure loss due to air leaking between the impeller and the fan casing.

The work done by a centrifugal fan (pressure) depends on the **blade angle**: the angle of the blade tip with respect to the direction of rotation. The fan pressure increases as the blade angle increases.

Centrifugal fans may be either **center hung** or **cantilevered**:

- **center hung**: The impeller is hung between two bearings and is supported on either side as shown in Figure 3(a).

- **cantilevered**: Also called **overhung**, the fan is mounted so that the mass of the rotor is outboard of both bearings and is supported at one end only. Figure 3(b) illustrates a cantilevered centrifugal fan.

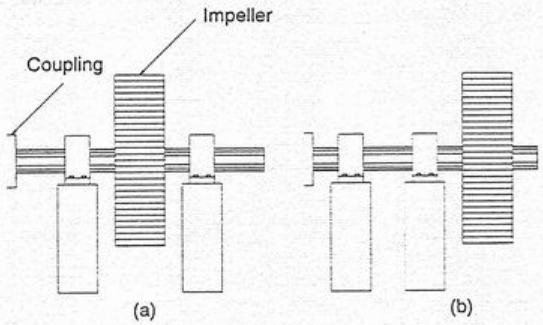


Figure 3 - *center hung and cantilevered centrifugal fans*

Centrifugal fans may be driven in a variety of ways, including

- coupling the fan directly to a motor
- driving the fan with a belt
- using a fluid coupling (although this is rare)

There are three types of centrifugal fan blades:

1. backward curved blade
2. radial blade
3. forward curved blade

2.1.1 Backward Curved Blade

A backward curved blade inclines away from the direction of rotation at an angle greater than 90 degrees. Figure 4 illustrates a backward curved blade. Also shown is the fan's **radial depth**: the radial distance between the inner blade tip and the circumference of the impeller.

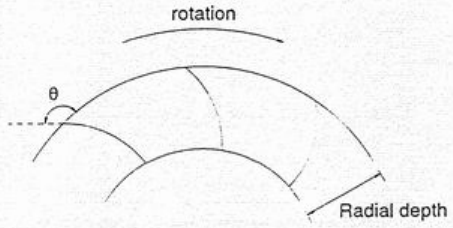


Figure 4 - *backward bladed centrifugal fan*

The efficiency of a backward bladed centrifugal fan is around 80 percent. For higher efficiencies, a variation on the backward blade, called the **airfoil blade**, is used. An airfoil blade is shown in Figure 5. This blade is so named because it is an airfoil in cross section, much like an airplane wing. Due to its aerodynamic design an airfoil blade has efficiencies around 90 percent: the highest efficiency of any centrifugal fan blade type. Since their efficiency is so high, backward bladed (and airfoil blade) fans are used to ventilate large installations such as coal mines and tunnels.

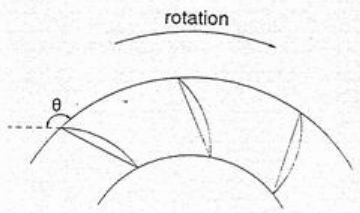


Figure 5 - *airfoil blade*

2.1.2 Radial Tip

The blade tips are radial on a radial tip fan: they meet the direction of rotation at exactly 90 degrees (just like the radius of the impeller circumference). The efficiency of a radial tip blade rarely exceeds 75 percent. Figure 6 illustrates a centrifugal fan with a radial tip blade.

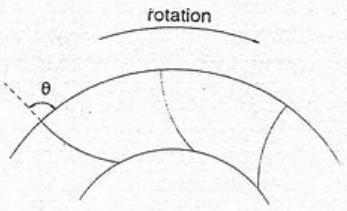


Figure 6 - *radially tipped fan*

A variation on the radial tip is the **radial blade**, where the entire blade is radial and meets the wheel outer diameter at 90 degrees. These fans are sometimes called "paddle blade" fans since they resemble the flat-bladed paddlewheels on old steamboats.

Radial blade impellers have thick, sturdy blades and are usually unshrouded. They are used primarily to move air that is laden with solids, since the fan stays free of blockage and can generally withstand considerable wear before the fan fails.

2.1.3 Forward Curved Blade

A forward curved blade inclines into the direction of rotation at an angle less than 90 degrees, as shown in Figure 7.

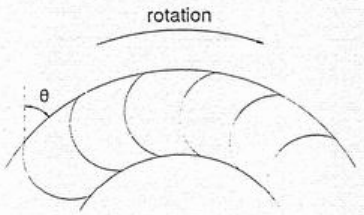


Figure 7 - *forward curved blade*

Because of the small radial depth of forward curved blades, an impeller with forward curved blades can have a larger inlet opening than a backward curved or radial tip impeller of the same diameter. Therefore centrifugal fans with forward curved blades generally develop higher pressure for a given size and speed.

Forward curved blade fans are commonly used to ventilate commercial buildings, since a compact fan can produce a substantial air movement. However, the efficiency of forward curved blades rarely exceeds 75 percent.

Depending on their location on the fan curve, forward curved and radial blade fans may require a large increase in power for a small increase in airflow. Increasing airflow without regard to this non-linear relationship may overload the drive motor.

Increasing the air flow in a backward curved fan, however, has little effect on the fan's power requirements. This is called a **non-overloading** fan characteristic.

2.1.4 Other Types of Centrifugal Fans

Other types of centrifugal fans include:

- **dual inlet**: The air enters the impeller axially from both ends.
- **in-line**: The air enters the impeller radially and flows in-line with the direction of rotor. This design is rare, and is seen most often in furnace blowers.

2.2 Axial Flow Fans

The flow of air is parallel to the impeller axis in an axial flow fan. Axial fan blades are usually an airfoil in cross section, and run at a very close clearance to the side of the fan casing. Figure 8 illustrates an axial flow fan.

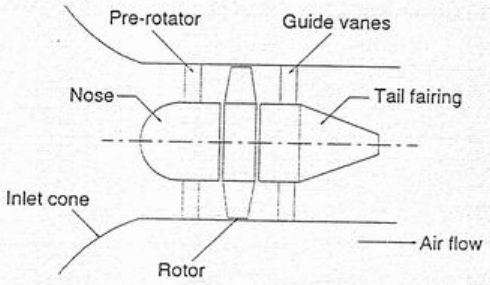


Figure 8 - *axial flow fan*

Air enters the fan in an axial direction. Unlike centrifugal fans, however, the air leaving an axial flow fan has both an axial and a rotational component. Some of the rotational energy of the blades is imparted to the air, causing it to rotate as it moves down the duct. Consequently, the total pressure developed by the impeller is not all available as useful fan total pressure.

Most axial fans employ some method of removing the rotational component in order to recapture the rotational velocity pressure. The most common methods of eliminating rotating air are:

- guide vanes: radial fins mounted downstream of the impeller that block the rotating air. Guide vanes are normally mounted behind the impeller; however, some axial fans have the guide vanes in front.

  A "relatively prime" relationship must exist between the number of guide vanes and the number of fan blades. That is, the number of fan blades and the number of guide vanes can not have any common factors; pressure pulses will be created if these two numbers have a common factor. For example, an axial fan with 12 blades might have 7, 11, or 13 guide vanes, but would not have 6, 8, or 9 guide vanes.

- vanes installed upstream of the impeller to pre-rotate the incoming air in the opposite direction. The rotation introduced by the impeller cancels out this pre-rotation and the air leaves axially.

- a second impeller downstream of the first rotating in the opposite direction. This is called a **contra-rotating** fan.

Axial flow fans can be driven in a variety of ways, including:

- direct drive
- belt driven

An axial flow fan does not generate the same pressure as a centrifugal flow fan of the same diameter and speed, and is primarily used to move air that is not laden with solids such as ash and dust. Fans without guide vanes have an efficiency of around 75%, while efficiencies of as high as 87% have been recorded for fans with guide vanes.

Axial fans also have characteristic fan curves, and can usually operate further from their best efficiency point than can centrifugal fans.

2.2.1 Other Types of Axial Flow Fans

Other types of axial flow fans include:

- **propeller fans**: The blades on the impeller are made of flat stock such as sheet metal, like a desk or a ceiling fan. Since these fans run in an orifice with a large clearance, the flow of air is not completely axial.

  Cooling tower fans are a special type of propeller fan. These fans are large (possibly 30 feet in diameter), gear box driven, and operate at very low RPM (anywhere from 40 to 250 RPM). A motor is connected to a gear box by a long, horizontal connecting shaft (around 20 feet long). The gear box has a vertical shaft to which a large propeller fan is attached.

- **cross-flow fans**: Also called **tangential** fans, these have blades shaped like forward curved blades from a centrifugal fan. Both ends of the impeller are sealed, and the air enters at the periphery on one side and exits from the periphery on the other side. The air flow is along a curved path with the inlet and outlet ports nearly at right angles to each other.

- **fixed pitch**: Pitch is the angle between the fan blade and an imaginary line perpendicular to the center line of the shaft. For example, a fan blade with zero pitch would be parallel to a line that is perpendicular to the shaft center line. A fan blade with a pitch of 10 degrees would be 80 degrees off of the shaft center line, but only 10 degrees off of the line perpendicular to the shaft center line. The pitch of propeller blades is fixed, and can only be changed by loosening the mounting bolts when the fan is shut down.

- **variable pitch**: mechanical, hydraulic, or air-driven mechanisms are used to change the pitch of each blade while the fan is operating. Variable pitch fans are used to accommodate varying demands for air, and can be more difficult to diagnose.

3.0 STRUCTURAL ASPECTS AFFECTING VIBRATION

This section discusses structural aspects of both centrifugal and axial fans that can affect vibration. Most of the structural aspects covered here apply to both types of fan (mounting, bearing considerations, belts, alignment, and so on); considerations that are specific to one type of fan are discussed under the appropriate sub-heading.

A troubleshooting guide that includes the considerations from this section is at the end of this chapter.

3.1 Mounting

The fan assembly may be solidly mounted to the supporting structure or be suspended on vibration isolators. The mechanical integrity of both mountings is an important consideration to the vibration analyst.

Vibration mounts have many different designs. One popular design employs two U-brackets as shown in Figure 9(a). The two U-brackets are of different widths; the wider U-bracket is bolted to the floor, while the narrower U-bracket fits upside down inside the wider bracket. A spring connects the two brackets, and the adjusting nut is used to vary the tightness of the spring. The sides of both brackets are cupped and are filled with a neoprene compound for damping.

Figure 9(b) illustrates another type of vibration mount, consisting of a large spring mounted between the floor and the base of the fan. This simple spring mount is the most common vibration mount for fans. A third type of vibration mount is shown in Figure 9(c), where the fan rests on hard rubber or felt.

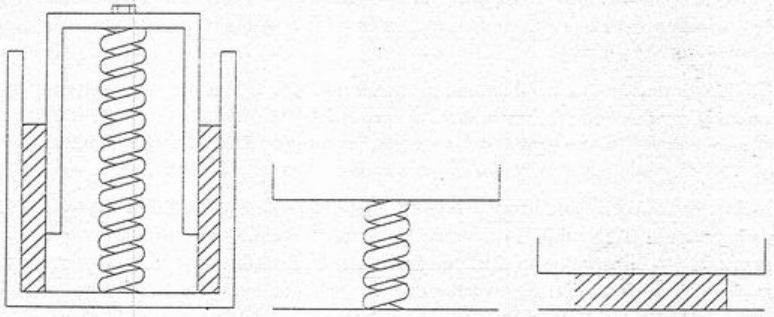


Figure 9 - *vibration mounts*

To work properly, vibration mounts must not be cocked or shorted out. If the vibration mount shown in Figure 9(a) were cocked (turned at an angle), the inner U-bracket might come into direct contact with the outer bracket. This direct contact bypasses any damping from the rubber and causes the vibration mount to behave like a solid mount.

A vibration mount can "short out" if the damping material compresses or breaks down, or if the mount is so tightened that direct contact occurs. For example, the adjusting nut in Figure 9(a) might be tightened so much that the inner U-bracket bottoms out against the wider bracket. The weight of the fan might compress the spring in Figure 9(b) so that the coils are all pressed against each other.

If a spring mount is properly designed, the center of gravity of the system should be below the top of the springs. Otherwise, if the center of gravity is above the spring, the system will be unstable. The smallest motion of the center of gravity results in a very large deflection of the springs.

Unfortunately, it is common for fans to be improperly mounted, so that their center of gravity is above the springs. Therefore, there are many fan mountings that are inherently unstable and cause vibration problems.

One solution to this problem is an **inertia base**, which is a heavy slab of steel-reinforced concrete. The fan is mounted to the inertia base, and the entire system is then mounted on springs that extend down from angle iron bolted to the top of the inertia base. This causes the major portion of the mass to be below the top of the springs. Although they are very effective in eliminating instability, inertia bases are expensive and are not often used.

Weight, age, and airborne chemicals can cause the damping material in Figure 9(c) to break down and allow the fan base to settle against the floor. In all these cases, shorting out causes the vibration mount to behave exactly like a solid mount with looseness. The loss of damping that results can contribute to excessive fan vibration.

Vibration mounts, like any other mechanical system, have their own natural frequency. The natural frequency (or harmonic of a natural frequency) of a vibration mount should be significantly below any running speed (or harmonic) within the system.

If vibration mounts are used, there should also be a flexible connection between the duct work and the fan. Flexible bellows made of canvas or neoprene are used to connect the fan outlet to the duct. The bellows help isolate any vibrations in the fan from the duct work.

In most cases the bellows are held on with clamp rings. On some high-pressure fans, however, the bellows may be held in place by large bolts that span the gap between the fan outlet and the duct work. Typically these bolts are mounted on rubber. If the bolts are tightened too much, they can short out, causing a direct coupling between the fan and the duct work. Any vibration in the fan is then transmitted to the duct.

Solid mounts must be tight and properly grouted. Looseness may cause several harmonics of the first-order vibration; the second- or third-orders may be higher than the first order. If the problem is pure looseness, then the harmonics will mostly be odd-order (3*RPM, 5*RPM, and so on).

3.2 Anti-Friction Bearings

Errors in mounting anti-friction bearings can cause excessive vibration. For example, anti-friction bearings might not be mounted square to the shaft or in the same plane (one bearing is higher or lower than the other). This usually results in elevated bearing temperature and can cause second, third, and fourth order vibrations in addition to first order.

Anti-friction bearings are mounted to a shaft as shown in Figure 10. The inner race is locked to the shaft, while the outer race is held against the bearing housing. The outer race is preloaded so that it does not rotate.

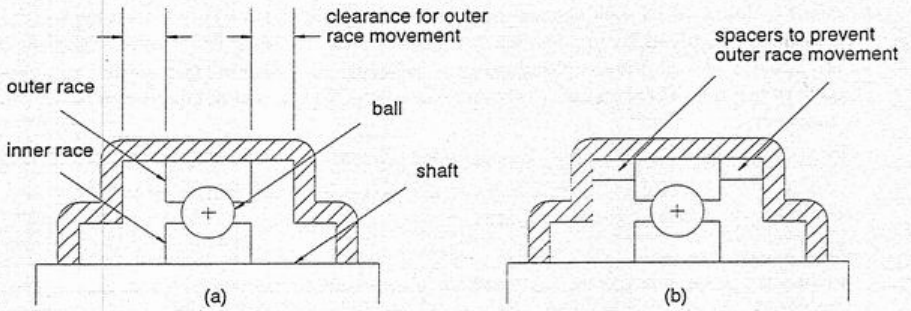


Figure 10 - *floating and fixed bearings*

During thermal growth, the inner race moves with the shaft since the two are locked together. The preload prevents the outer race from rotating, but not from moving with the shaft and the inner race.

Make sure that, during thermal growth of the shaft, the outer race is not restricted from growth movement by either side of the bearing housing.

A bearing whose outer race is free to move with the shaft is called a **floating bearing**, while a bearing whose outer race is prevented from moving is called a **fixed bearing**. Spacers are inserted on either side of the outer race of the fixed bearing to hold it in place. Figure 10(a) illustrates a floating bearing, and a fixed bearing is shown in Figure 10(b).

Only one bearing should be fixed; since some thermal growth is inevitable, one of the bearings must be floating. Normally the fixed bearing is the one closest to the coupling or the pulley that is driving the fan; the floating bearing is the one on the outboard side. This allows the fan shaft to grow into the floating bearing.

If the reverse were true, and a floating bearing was next to a coupling, the shaft could grow into the coupling. Excessive growth could use up the coupling clearance and result in a locked coupling. The fan would run within vibration limits when started cold but the first- (and maybe the second- and third-) order vibration would increase as the fan came up to operating temperature.

All the cautions that apply to anti-friction bearings should be observed for fan bearings:

- **Bearing lubrication**: too much lubricant seals in the heat and causes early bearing failure. Too little lubricant does not provide the lubrication necessary to minimize wear.

- **Bearing preload**: The preload on the journal and in the bearing housing clearance must be set to maintain the proper preload on the bearings.

- **Bearing clearance**: Clearance on journal bearings should be between 1 and 1.5 mils per inch of shaft diameter, with a minimum of 2 mils. Insufficient clearance increases bearing temperature as well as the possibility of wiping. Too much clearance decreases the damping and increases the possibility of shaft instability.

- **Maximum frequency**: The operating frequency of an anti-friction bearing fan should be within the bearing tolerance (7,200/shaft diameter for greased lubricant, 9,600/shaft diameter for oil lubrication). For example, if the fan has a 5 inch shaft with grease-lubricated bearings, then the shaft frequency should not be higher than 1,440 RPM.

- **Belt drives**: Both the static and the dynamic load of the belts on the bearings (at least on the bearing closest to the belt sheave) must not increase the radial bearing load beyond the maximum rated load for that bearing. Angular and parallel shaft and sheave alignment is important in all belt drives and especially so in fan drives. In some cases parallel misalignment of less than 1/8 of an inch has resulted in a two-fold increase in vibration at fan or belt frequencies.

- **Shaft alignment**: Alignment is a concern if the fan is direct coupled to the drive motor. One of the first effects of shaft center line misalignment is an increased radial load on the bearings, which can lead to early bearing failure.

- **Disk skew**: Disk skew occurs when the entire disk assembly is not mounted perpendicular to the shaft center line. As the fan comes up to speed, the centrifugal forces try to straighten out the disk (since centrifugal force goes up by the square of the shaft frequency). This force (a couple) causes a bow in the shaft which can reduce the shaft critical frequency so that the fan is running closer to critical speed than the design intended.

  In addition to decreasing shaft critical frequency, disk skew usually results in an increase in vibration amplitude after the fan passes through the first mode resonance (the expected result

would be a reduction in amplitude). On overhung fans, the skew causes rapid phase shifts at the outboard bearing above critical speed.

- **Resonances**: Fan shaft and bearing pedestal resonances that are close to the fan's operating frequency result in high first-order vibration levels and make field balancing of the fan difficult, if not impossible.

  The fan wheel itself has a fundamental plate resonance as an assembly. In addition, each blade has its own resonant frequency. The operating frequency and the blade pass frequency forcing functions must not be able to excite any of these resonant frequencies.

  Fan drive components produce their own forcing functions which must not excite any resonances in the fan. If the fan is direct-coupled to the driver, then the operating frequency of the driver is the same as the fan, and any 1*RPM forcing function in the motor (imbalance, angular misalignment, bent shaft, for example) is transmitted to the fan.

  If the fan is belt-driven, then the motor is likely operating at a different frequency than the fan. The motor's operating frequency, the belt frequency, and misalignment between the motor and fan sheaves are all forcing functions that can cause vibration in the fan.

  The duct work may also have resonances excited by the fan forcing functions, especially in stall or surge conditions.

- **Looseness**: Loose bearing or hold-down bolts can change the system stiffness and result in high vibration levels. A fan wheel loose on the shaft (excessive clearance between the wheel bore and the shaft diameter) allows the fan wheel to shift on the shaft between runs so that acceptable balance may not be possible.

- **Cap loading**: Centrifugal fans are often cantilevered. If the fan is mounted in sleeve bearings and the wheel is not properly located (moved outboard slightly), the bearing farthest from the rotor can be cap loaded. This could result in hydraulic instability and early bearing failure if the bearing is not designed for cap loading.

3.3 Axial Fans

Many axial fans are direct driven: the propeller is mounted on the motor itself or on a shaft directly coupled to the motor. Some axial fans may consist of a double-shafted motor with a propeller on each end of the motor. There are also belt-driven axial fans, some having two propellers rotating in opposite directions.

The axial fan rotor should be within .001 inch of the center of the fan housing. Tests have shown that high first-order vibrations (especially at blade pass frequency) can result if the axial fan is not precisely centered in the fan housing. If the rotor is not accurately centered within the housing, then 1*RPM and *Number of blades**RPM vibrations may be high.

It is very common for one or more of the blades not to be pitched accurately. This means that one blade is either doing more (or less) work than the others, and results in a high axial vibration.

Another source of axial vibration is **track error**. The track of a fan blade is a measure of how it is placed ahead or behind ("upstream" or "downstream") of the other blades.

If a reference mark is made on the housing exactly in line with the tip of one fan blade, the tip of all the other blades should also sweep by the mark exactly in line. A track error means that a blade tip passes by the mark slightly off to one side. Track error attempts to speed up and slow down the mass of air flowing through the fan during one revolution, which causes an axial vibration. Axial

vibration on an axial fan can translate into horizontal and vertical vibration by causing a rocking mode in the fan.

4.0 OPERATIONAL SOURCES OF VIBRATION

Both centrifugal and axial fans should be operating close to their best efficiency point on the fan curve. Also, flow characteristics at the inlet to the fan should be as smooth as possible. Analysis of fan efficiency and flow involves the determination of the average static and velocity pressures in the duct and is covered in several of the references at the end of this chapter.

Every attempt should be made to have laminar (non-turbulent) flow for a distance of at least 6 fan wheel diameters at the input. It may also be necessary to use an inlet cone to reduce turbulence.

The fan outlet should have some reasonable run (at least 4 to 6 wheel diameters) of laminar flow before a transition (a change in the shape of the air duct, a turn, an elbow, for example). If a transition is close to the fan outlet, then steering vanes should be added to the duct to reduce air turbulence.

A blockage on the outlet of a fan can cause **surge**. Surge occurs when the pressure developed by the fan is not equal to or greater than the downstream pressure in the duct. This causes air to surge back across the clearances in the fan. Surge results in a high noise level and high pulsations in the ductwork, and these pulsations normally occur between 30 to 45% of fan speed, with higher harmonics and wide skirt noise.

If the velocity of the air flow through the fan is not equal to the flow required by the angle of attack on the fan blades, then the flow pattern will break down across the fan blade. This condition is known as **stall**. Stall can often cause a very significant vibration at approximately 1/3 to 1/2 the operating frequency of the fan.

4.1 Rotating Stall

Rotating stall is often caused by an air incident mismatch at the fan blades. That is, the air does not meet the fan blades at the correct angle. This is often caused by improper preswirl. The development of a rotating stall condition depends on several paramters such as inlet/shroud clearance, inflow velocity profile, and inlet damper settings.

Figure 11 illustrates a rotating stall condition in a centrifugal fan.

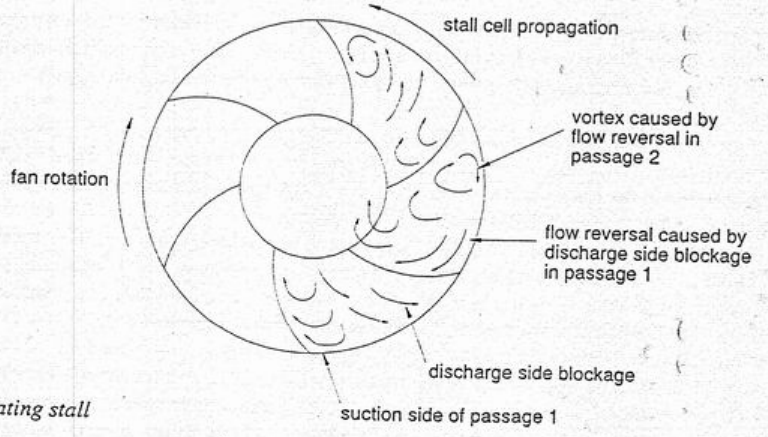


Figure 11 - *rotating stall*

First, the boundary layer on the suction side of the fan blade in passage 1 (a **passage** is the space between two fan blades) separates as air flow to the fan is reduced. Reduced air flow can be caused by increased system resistance, for example, or by unfavorable inlet conditions. This air flow reduction results in a backflow: the air begins to flow backwards along the suction side of the blade.

This backflow of air blocks off the discharge side of the passage, and causes the air flow in passage 2 to reverse. The reversed flow pattern in passage 2 creates a relative vortex in the fan, causing another change in air flow direction in the third passage. This cycle repeats for other passages, working its way around the entire circumference of the fan, opposite to the direction of the fan's rotation.

Pressure pulses caused by rotating stall are usually low in magnitude. However, these pressure pulsations can cause fatigue cracks in flexible ductwork, where the dynamic force produced by the small pulsations becomes very large due to the large surface area of the ducts.

You can detect a rotating stall condition by spectral analysis. Rotating stall usually produces pressure pulsations at a frequency around 2/3*RPM. The frequency spectrum for a rotating stall condition can also contain multiples of the stall frequency, such as 2/3*RPM, 4/3*RPM, 2*RPM (that is, 6/3*RPM), 8/3*RPM, and so on. The magnitude of vibration at each of these frequencies depends on whether a system resonance is present at (or near) that frequency.

When you take field data to investigate the possibility of rotating stall, make sure you place the reference transducer at a point that is sensitive to the low-pressure pulsations associated with the stall. An accelerometer installed on the ductwork usually provides more signal than one mounted on a fan bearing. A pressure transducer located in the air stream usually provides another good signal of stall frequencies.

Rotating stall behaves much the same in axial fans as it does in centrifugal fans.

5.0 VIBRATION ANALYSIS OF FANS

Vibration analysis and the solution of vibration problems is a logical process, where the analyst follows a procedure to gather the information necessary to pinpoint the vibratory forcing function. The logical approach to vibration problem solving can be broken into five steps:

1. **Determine the problem** - Ask *what, where, and how:* What is the problem? Where is the problem? How was the problem identified? Always solve one problem at a time; if there is more than one, solve the most severe problem first.

2. **Inspect the system** - Do a complete walk-around to determine the condition of the system; look for wear, cracks, looseness, and other problems. Identify the number and type of components (belts, couplings, bearings, motors, and so on). Take physical data on the system, such as spacing and clearances, diameters, RPM(s), and type of mounting.

3. **Take static data** - Calculate resonances within the system (shaft, bearing pedestal, housing, base, belt strand, blade, for example). Calculate other possible forcing functions, such as bearing fault frequencies, blade/vane pass frequency, and belt frequency. Section 1.3 of the General Analysis Techniques chapter provides information on calculating possible forcing functions. Check alignment within the system. Record the background vibration.

4. **Take dynamic data** - Record vibration data during startup, at normal load, and during coastdown in both the time and frequency domains.

5. **Determine the forcing function(s)** - Examine the data for excessive amplitude at any of the calculated frequencies from Step 3. Check for synchronous, sub-synchronous, and

non-synchronous vibrations. Look for impacts, non-periodic events, and non-symmetric waveforms.

Now, let's apply this procedure to the analysis of fan vibration.

5.1 Determine the Problem

The first step in diagnosing fan vibration is to determine the problem by asking "what, where, and how:"

- **What is the problem?**
  - The fan doesn't "feel good."
  - Someone feels the fan vibrates excessively.
  - There are high levels of vibration in the ductwork.
  - The fan makes unusual or excessive noises. It is difficult to say absolutely that a noisy fan is the responsibility of a vibration specialist. The noise study of a fan requires analysis by a fan expert: the air flow is calculated to determine if it is too fast for that size of duct, the duct work is analyzed to determine if it is properly designed and supported, and so on.
  - The fan noise is transmitted through the building.

- **Where is the problem?** Or, at least, where does someone *feel* the problem is located: the fan, the motor, the drive, the bearings, the ductwork, the floor?

- **How was the problem identified?** If there is a vibration data collection program in effect, you may be able to study historical data on this unit.

5.2 Inspect the System

Then make a general walk-around inspection, carefully noting the following:

- The condition of the unit:
  - Does the fan appear to be well maintained or neglected?
  - Is the grouting in good shape or is it crumbling and cracked?
  - Are the hold-downs tight?
  - Are there shim packs that appear to be solid? Not many thin ones, but the fewest number of shims in order to obtain the proper height and location of the unit.
  - Are the shim packs in good condition?

- The type of fan:
  - Centrifugal
  - Axial
  - Forward curved

- Backward curved
- Airfoil

• Fan design data:

- Fixed pitch or variable pitch?
- The number of blades
- The operating frequency
- The shaft diameter and shaft material
- The distance between bearing centers
- The mounting of the fan (spring mounted, on inertia base, rubber mounted, bolted to the floor, etc.)

• The type of motor:

- AC, DC, variable speed, synchronous?
- The motor's operating frequency
- The type of bearings in the motor
- Whether the actual bearing type is available, so that fault frequencies can be calculated and checked

• The type of drive:

- Direct-driven or belt-driven?

If the fan is direct-driven, note:

- The coupling type
- The condition of the coupling
- How the coupling is aligned, and how often that alignment is checked

If the fan is belt-driven, note:

- The number of belts
- The size of the belts
- The belt length: Section 5.3.1 of the Power Transmission Components chapter explains how to calculate the belt length if the belt length is unavailable
- Whether the sheave shows excessive wear; many times the wear pattern in the sheave sets up resonant vibration in the belt drive
- The condition of the belts
- How the belts are aligned and tensioned, and how often

- The type of bearings:
  - Anti-friction or sleeve
  - How the bearings are lubricated, and how often
  - Whether the bearing clearances have been checked recently, and what the clearances are?
  - What bearing fault frequencies are, if possible

If the fan is not running, then static measurements can be made first. Otherwise, running checks can be made first, and static measurements can be made once the fan is shut down.

5.3 Take Static Data

Static data (that is, data taken when the fan is not moving) is used to calculate system resonances and possible forcing functions. Dynamic vibration data taken when the fan is running is then analyzed to determine if any of these calculated frequencies are present.

Remember, the major vibration within the fan may be a harmonic of one of these forcing functions. System resonances can amplify one of the harmonics of the forcing function, causing the amplitude of that harmonic to exceed the amplitude at the forcing function frequency.

So when the dynamic vibration data is analyzed, look for excessive amplitudes at *harmonics* of the calculated forcing function frequency as well as *at* the forcing function frequency.

5.3.1 Resonant Frequencies

Impact tests should be performed to determine the resonant frequency of various fan components such as bearing pedestals, fan shafts, the fan wheel assembly, the foundation, and the air duct. Bearing pedestals and motor supports should be tested for resonant frequencies in the horizontal, vertical, and axial direction.

Very large fan systems, with drive motors in the hundreds of horsepower, are often mounted on heavy concrete bases. Their mass makes it nearly impossible to determine their resonant frequency from an impact test: it is simply too difficult to hit hard enough with a force hammer to make the system vibrate. You can often determine the resonant frequency of these systems by analyzing the vibration data during startup. Section 5.4.1 of this chapter describes this technique in detail.

5.3.2 Blade Pass Frequency

For both centrifugal and axial fans, count the number of fan blades so that you can determine the blade pass frequency. Blade pass frequency is equal to the number of blades times the RPM of the fan. High blade pass frequencies occur when a fan wheel is not centered in the duct (especially so for axial fans), or when a cutoff plate is installed too close to the fan wheel on a centrifugal fan. Both centrifugal and axial fans can exhibit high blade pass frequencies when the fan is operated well below or above its best efficiency point.

5.3.3 Belt Frequency

The belt frequency of a belt-driven fan is the number of times per minute that the belt makes one complete circuit. Vibrations caused by worn or faulty belts can occur at 2 * belt frequency and up to 2 * # belts * belt frequency.

Several harmonics of belt frequency will be present in the spectra. It is very rare for the first-order belt frequency to have the highest amplitude in the harmonic family. Usually, the order that is equal to the number of pulleys has the highest amplitude: if there are two pulleys, the second order will be highest; if there are three pulleys the third order will be highest.

5.3.4 Belt Strand Frequency

It is very important to check the belt strand frequency on belt-driven fans. The **belt strand** is that portion of the belt that is not in direct contact with one of the sheaves, as shown in Figure 12.

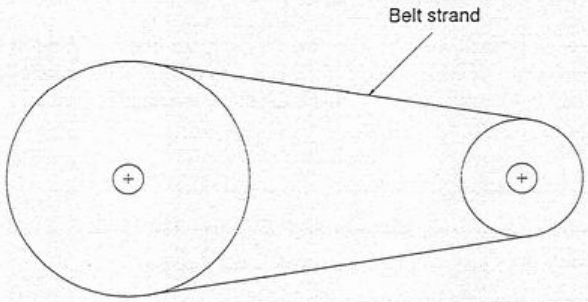


Figure 12 - *the belt strand*

A belt strand has a resonant frequency for each of its mode shapes; usually the first mode shape (the belt strand's first degree of freedom, as shown in Figure 13) is the one of greatest concern.

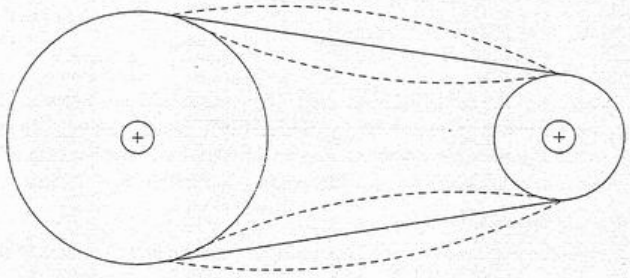


Figure 13 - *belt strand resonance*

Mount an accelerometer on the strand of the belt; an easy method is to set the accelerometer on the top of the belt strand and wrap several layers of electrical tape around the transducer and the belt. Tap the belt strand lightly, in order to determine the first mode resonance of the belt strand. When the motor is not running, both belt strands have the same tension. Therefore, the resonant frequency found on one belt strand is the same on the other.

However, when the motor is started up, the belt strand that is pulled by the drive wheel is in tension; the other belt strand is loose, and actually sags a little bit. When this happens, the resonant frequency of the belt strand under tension increases above the static belt strand frequency. Likewise, the resonant frequency of the slack belt strand decreases below the static belt strand frequency.

When examining dynamic vibration data for belt strand resonance, look for a spread of 2-3 Hz above and below the static belt strand frequency. A vibration at belt strand frequency may have a broad skirt (a wide base in a the frequency spectra) since the two belt strand resonances are so close together.

5.3.5 Track, Pitch, and Blade Spacing

Verify that the blade track and pitch are identical for each fan blade. Errors in blade track and pitch will result in high axial vibration. Blade pitch is not as critical a measurement in centrifugal fans as it is in axial fans. For centrifugal fans, check the blade spacing if possible.

5.3.6 Mechanical Data

For both centrifugal and axial fans:

- Check fan bearing clearances, and determine if the bearings are square with the shaft.
- Identify which is the fixed bearing, and which is the floating.
- Check for proper lubrication and preload on each fan bearing.
- Check for any looseness (bearing bolted to the pedestal, the bearing cap, pedestal to the foundation, and so on).
- Check shaft alignment if the fan is direct driven, or belt alignment if the fan is belt driven.
- Check the coupling clearance or the belt tension.
- Check the condition of the belt or coupling.
- Inspect both the motor and fan sheaves for wear.
- Check the motor hold-downs for looseness.

Additional checks for axial fans:

- Check the centering of the fan in the inlet cone. If the fan is not located precisely on the inlet cone center line, there is a great possibility of high blade pass frequency. In certain critical applications (such as operating rooms and clean rooms), the authors have found that the inlet cone must be round and that the fan must be centered to within ± .001 inch.

Additional checks for centrifugal fans:

- Check for proper spacing between the fan cutoff and the outside diameter of the fan wheel.
- Verify that the fan shaft is level.
- Make sure that the fan wheel is tight on the shaft.
- Check that inlet and outlet dampers, if present, are installed correctly.

5.3.7 Background Vibration

Measure the background vibration in both the time and frequency domains. Check the time domain for impacting and other non-periodic components that would not appear in the frequency domain

spectra. There are many cases where excessive vibration has been attributed to a fan when it is actually due to the background vibration.

5.4 Take Dynamic Data

Before any dynamic data can be recorded, transducers must be mounted at the appropriate locations. This is usually not too difficult for centrifugal fans, but axial flow fans present special problems.

One of the major difficulties in diagnosing an axial flow fan is collecting vibration data off the motor when it is inside the air duct. If baffles, filters, grids, or heating elements are present in the air duct behind the motor, then access to the motor is even more difficult.

The back of the motor is virtually inaccessible: you can't run a cable through the fan, since the blades will cut the cable when the fan is started. Running a cable between the fan and the housing is not possible either; the clearance between the blade tips and the housing is kept as small as possible to reduce surge and leakage back across the fan. On some fans there is even evidence that the blades have rubbed the inside of the housing.

Many times there are pitot tube holes (a **pitot tube** is a tube pointing into the air flow with a pressure gauge attached to the downstream end; used to measure fan pressure) or wire access holes in the duct work which can be used to run cables to transducers on the front and rear of the motor.

On most fans the guide vanes are behind the propeller, and the motor is mounted on a plate that is welded to the guide vanes. The normal procedure is to look for that weld on the outside of the air duct and mount a transducer on the weld. Since that guide vane goes directly to the motor mounting plate, the guide vane provides good transmission of vibration from the front of the motor.

Obtaining vibration readings from the back of the motor is usually more difficult. On smaller axial flow fans the motor just hangs in the air duct. If the fan is 25 hp or greater, there are usually 2-4 supporting rods that are threaded into the back of the motor. These rods come through the air duct and are held in place by two nuts, one inside and one outside the duct. Transducers are often mounted on these nuts to obtain radial vibration readings from the back of the motor.

The problem with these methods is that the air duct is very thin and flimsy, and may be mounted on springs, vibration mounts of rubber or cork, or on the floor. The flexibility of the duct and the mounts prevents good mechanical impedance between the transducer and the motor.

The mechanical impedance becomes worse as the fan frequency increases, making it more and more difficult to see high frequency vibrations (such as bearing fault frequencies). Low frequencies, such as resonances in the duct and the mounting, however, are easier to see.

If an inspection hatch is present, you can crawl inside and mount transducers solidly to the front and back of the motor in the horizontal and vertical directions, and take axial readings at the rear of the motor on the top, bottom, right, and left sides. If there is a serious vibration problem on the fan, an inspection hatch *must* be cut in the duct so that transducers can be mounted on the motor.

Dynamic vibration data is recorded during fan startup, normal load operation, and coastdown.

5.4.1 Startup Data

Take startup data in the time domain. Sometimes the startup torque shock can act as a high-energy impact, and a "ringdown" from this shock can establish a first mode resonance in the motor or the fan mounting. This resonance can be clearly seen in time domain data taken during startup. Examining this ringdown for the first-mode resonance is useful on very large fan systems, whose mass makes it nearly impossible to calculate the resonance from an impact test.

If a multi-channel data recorder is available, record a tach pulse and startup vibration from as many locations as possible (horizontally, vertically, and axially at each bearing). If a multi-channel data recorder is not available, check phase and amplitude at running frequency on the drive side bearing in the horizontal direction.

5.4.2 Normal Load Data

Data taken during normal load operation of the fan is taken in both the time and frequency domains. Use time domain data to check for impacts, non-periodic components, and non-symmetrical waveforms that can indicate looseness, rubs, electrical problems, or surge. Use frequency domain data to check the amplitudes of the various resonant and fault frequencies calculated from the fan static data.

Time Domain Data

Once the fan is running under normal load, use an accelerometer to examine the time domain for impacts, non-periodic components, and non-symmetrical waveforms. If there are impacts in the time domain, look for

- looseness
- bearing fault frequencies
- a cracked shaft
- the fan wheel loose on the shaft

If there are non-periodic components in the time domain, look for

- foreign material in the fan (if the fan is airfoil-bladed or a hollow-shaft fan)
- an outside source of vibration causing non-periodic events, like bad SCR timing in the electrical drive
- surge across the fan

If there are asymmetries in the time domain, look for

- rubs or restrictions to motion in the fan assembly.

Frequency Domain Data

First, take low-resolution broadband spectra in the horizontal, vertical, and axial directions on all bearings, the bearing pedestals, the fan housing, the duct work, the motor, and the motor foundation.

You can take these broadband low-resolution spectra very quickly, making it easy to determine where the vibration energy lies in the frequency domain. Then make a more detailed investigation at the frequency location of those energy levels.

Many times an analyst fails in attempting to solve an air handling vibration problem simply because they did not take enough high-frequency data to provide the information necessary for solving the problem. Low resolution, high baseband spectra allow you to identify very quickly where the energy levels are. Then you can examine each of these energy areas in more detail.

Then take high-resolution data in those energy areas. Take readings in each direction at each bearing, the bearing pedestal, the motor, and the motor base. Take phase and amplitude readings with a tach pulse in the horizontal, vertical, and axial directions.

Also, take readings in the axial direction at the top, bottom, left, and right sides of each bearing to determine how the bearing is moving in space (whether it is cocking or rocking, for example). If

possible, take these phase and amplitude readings at first-, second-, and third-order to check for mechanical non-linearities in the system.

Phase readings at operating frequency are very important in identifying vibrations due to resonance and misalignment. To identify resonance, check the phase from horizontal to vertical on each bearing. You can identify misalignment by checking phase from horizontal to horizontal and vertical to vertical between bearings and across the coupling.

A useful test you can perform with a fan (or with any rotor) is to obtain 1, 2, 3, and 4 pulses per revolution. One way is to use a tracking adaptor and 4 equally-spaced pieces of reflecting tape. Many times a coupling has 4-8 bolts around it that can be sensed using a proximity pickup. With a tracking adapter, you can divide the data down into 1 pulse per revolution, 2 pulses per revolution, etc.

Once you have this tach data, you can use the vibration analyzer to look at the vibration component at each order. It is not always necessary to use a tracking adapter; some analyzers allow you to look at multiple orders from a single tach pulse.

5.4.3 Coastdown Data

Take coastdown data as the fan coasts to a stop after shutdown. During coastdown, take phase and amplitude on the worst bearing at first-, second-, and third-order if possible. If a multi-channel data recorder is available, use that to record data from as many locations as possible.

5.5 Determine the Forcing Function(s)

If all of the preceding data is taken, it should be possible to pinpoint vibration problems in any air-handling situation. Sections 5.5.1 and 5.5.2 discuss forcing functions that are common to both centrifugal and axial flow fans. Section 5.5.3 discusses additional diagnostic steps specific to centrifugal fans, and Section 5.5.4 discusses additional axial flow fan diagnostics.

5.5.1 Examination of Time Domain Data

Examine the time domain carefully for impacts, non-periodic events, and for non-symmetrical waveforms. Impacting in the time domain is usually a good indication of some mechanical non-linearity in the system. When examining impacts, it is important to determine that they are, in fact, periodic. For example, in some cases the phase firing of the triacs in a DC drive are not timed properly. These non-timed firings can appear to be impacts in the time domain, but they are not periodic. This non-periodicity indicates that these events will not have a high energy level in the frequency domain.

Non-periodic information in the time domain coupled with noise and vibration in the duct can indicate surge, which indicates that the fan is operating into a pressure which is higher than that developed by the fan.

Also, the fan may stall (the airflow on the blade breaks down because of an angle of attack which is greater than the airflow); this results in high levels of vibration somewhere between 30 to 45% of operating frequency, with higher harmonics.

If the vibration is erratic (it comes and goes, and the amplitude varies dramatically), and there is also the presence of some second- and maybe half-order vibration, then it is possible that the sleeve bearing is loose.

If the coupling locks up, there can be high second-, third-, or fourth-order vibrations. However, a locked coupling is usually a load-dependent problem, and there should be a sudden drop out of the high vibration when the unit is shut down.

5.5.2 Examination of the Frequency Domain

Imbalance

If the primary vibratory energy is at operating frequency, and there is not great disparity between horizontal and vertical readings (2 to 1 or less), then the fan wheel is not in balance. Imbalance causes the phase angles to move approximately 70 to 90 degrees when you go from horizontal to vertical on a bearing (if there is no resonance amplification).

If many harmonics of running frequency are present, then there is some kind of non-linearity in the system (usually some kind of looseness or excessive bearing clearance). Some typical examples of non-linearities include the fan wheel being loose on the shaft, a cracked fan shaft, excessive coupling clearance, or the fan rocking on its foundation.

Misalignment: Coupling, Belt, and Bearing

The presence of second-, third-, and fourth-order components can indicate some kind of alignment problem for direct coupled machines.

The misalignment may also be bearing misalignment (the bearings are not at the same level or are not normal to the shaft). If both bearings are fixed, leaving no possibility for thermal growth, it is possible to get high axial vibration.

If the belts are not properly aligned, it is possible to have high components both at belt frequency (or one of its harmonics) and at fan frequency (or one of its harmonics). The authors have encountered cases where sheaves out of parallel alignment by as little as 1/8 inch have caused vibration levels 2 to 3 times what one would normally expect. There are installations where the alignment of the belts is extremely critical.

Bearings that are cocked on the shaft result in high axial vibration, and many times high second- and third-order vibration as well.

Resonance and Disk Skew

Look at the relationships between horizontal, vertical, and axial amplitudes. For example, if the horizontal amplitude is much higher than the vertical, then probably the system is resonant in the horizontal direction. If so, the system will probably not be resonant in the horizontal direction at 2, 3, or 4 times operating frequency.

Check to see if disk skew is present. Measure the blades on the rotor, then slowly rotate the rotor to see if the blades are perpendicular to the center of rotation. If disk skew is present, the centrifugal force that increases with shaft frequency may bend the shaft. As a result, axial phase on the bearings slowly shifts as the disk skew takes over. The gyroscopic effects of disk skew dramatically change the fan shaft resonance.

There are many cases where the calculated fan shaft resonance is well above the operating frequency, but the actual resonance is much closer to operating frequency as a result of disk skew.

Cavitation

Cavitation is usually accompanied by orders of vibration harmonically related to running speed with higher frequency noise and relatively wide skirts.

Mechanical Non-linearities

Another way to check the fan for mechanical non-linearities is by adding a known weight as an imbalance weight. This weight is a first-order disturbance: it is a rotating vector at fan frequency, so it should not affect any of the higher orders. The weight should be calculated to produce an imbalance of one to two mils. Refer to the Field Balancing chapter in Volume I for more information on calculating trial weights.

Checking the phase and amplitude at first-, second-, third-, and maybe fourth-order with and without the weight indicates whether one of the higher orders of vibration is being affected by a first-order disturbance. A first-order disturbance such as imbalance should not affect the higher orders. However, if the system has a mechanical non-linearity, the higher orders are affected.

Oil whirl

Oil whirl causes vibration at less than half-order in sleeve bearings. If shaft orbits are available, the oil whirl appears as an inside loop on the orbit.

High Blade Pass Frequency

If a very high blade pass frequency is present, you should determine where the fan is running on the fan curve. An increase in blade pass frequency accompanies fan operation well below or above the best efficiency point. Pitot tubes are used to measure air flow and calculate fan efficiency, and may require the assistance of a fan expert.

Once the fan efficiency has been determined, you should look for the following:

- Is there a cutoff plate that directs air out the fan outlet? Sometimes the fan casing is designed to serve as a cutoff plate. If the cutoff plate is too close to the blade, a dramatic difference in blade pass frequency and noise can result. Cutoff clearance is more critical on a forward curved fan than a backward curved fan.
- Is there a resonance close to blade pass frequency that is amplifying the blade pass frequency? Look for the resonance in the air duct, the foundation, the bearing pedestals, and the fan shaft itself. Impact tests should be performed on the shaft in both the vertical and axial directions. Mount transducers on the bearings and impact the fan wheel in the vertical, horizontal, and axial directions. The fan should be off when performing impact tests on ducting.

5.5.3 Additional Centrifugal Fan Diagnostics

Shaft orbits can be very helpful in diagnosing centrifugal fans in sleeve bearings. Many units have eddy current probes already installed in the bearings; however, not all such installations include two probes mounted at 90 degrees to each other in each bearing.

If the weight of the rotor is equal to at least 50% of the total weight of the unit, it is rarely necessary to use non-contact probes in sleeve bearings. If the rotor weight is small compared to the total weight, it becomes more important to examine the movement of the shaft in the bearing clearance as well as the bearing casing vibration.

With probes mounted at 90 degrees to each other in the bearing, the orbit of the shaft movement in the bearing can be examined. Use a tach signal to trigger the orbit. The information provided by analyzing shaft orbits includes the following:

- The direction of the maximum movement of the shaft.

- The movement of the shaft centerline from start to running condition. This also indicates the remaining clearance between the shaft and the bearing.
- The stability of the phase of the orbit.
- The presence of contact areas or rubs.
- The presence of misalignment, indicated by banana shaped or figure eight-shaped orbits.
- Secondary loops indicate shaft resonances.
- Unstable orbits indicating hydraulic instability such as oil whirl.

5.5.4 Additional Axial Fan Diagnostics

Most axial flow fans consist of a motor with a propeller mounted on the front, so the air flows over the motor (the air never flows over the motor on a centrifugal fan). Very few axial fans are belt-driven, where the motor sits outside and the belt comes through the fan duct.

If the air blows right into the motor, dust may be forced into the motor bearings and cause the bearings to fail. Most fans have a spinner in the center that directs the air over the outside of the motor and fan hub but protects the endbell from high pressure air.

Variable Pitch Axial Flow Fans

Variable pitch fans have a mechanical, hydraulic, or air-driven mechanism that moves the pitch of the fan blades while the fan is running. This mechanism is subject to wear and high centrifugal forces, and many times will not pitch the blades evenly from zero pitch to full pitch. A common problem with variable pitch axial flow fans is that the fan can be balanced at zero pitch, half pitch, or full pitch, but cannot be balanced across the entire pitch range. This is almost always due to one of three things:

1. The pitch does not change uniformly as the mechanism varies the pitch.
2. A resonance excited by certain blade pitches and pulsations of air going down the duct.
3. The blade is not centered in the inlet cone.

A least squares balancing method is an effective procedure for balancing a fan in this situation. Data is taken at several different pitch settings, and the best possible compromise is calculated.

Another approach is to find out what pitch is most often used during normal operation and obtain the best balance at that pitch setting. The other pitch settings will not have the best balance, but this may be acceptable since the fan does not run at these pitch settings for much of the time. Quite often, the blades on a variable pitch fan do not track uniformly. Blade track error results in high axial vibration at blade pass frequency.

6.0 FAN DIAGNOSIS

Refer to the following tables to determine the probable cause (or causes) of fan vibration.

TIME DOMAIN ANALYSIS		
TYPE OF WAVEFORM	**PROBABLE CAUSE**	**COMMENTS**
Impacts	Looseness	
	Bearing fault frequency	
	Cracked shaft	
	Fan wheel loose on shaft	
Non-periodic components	Foreign material	
	Outside source of vibration	
	Surge	
Asymmetrical waveforms	Rubs	
	Restrictions to motion	

FREQUENCY DOMAIN ANALYSIS				
FREQUENCY	HARMONICS	AMPLITUDES	PROBABLE CAUSE	COMMENTS
1 * RPM	No	1*RPM H > .15 in/sec 1*RPM V > .12 in/sec Low axial both bearings	Imbalance	Consistent 1*RPM phase
		1*RPM H > .15 in/sec 1*RPM V > .12 in/sec 1*RPM A > .1 in/sec	Bent shaft	Look for axial phase 180° out
		1*RPM H > .18 in/sec 1*RPM V < .08 in/sec	Horizontal resonance	High amplitude may be at some other order excited by non-linear system
		1*RPM H < .08 in/sec 1*RPM V > .18 in/sec	Vertical resonance	
		Increases after fan startup	Thermal bow	
		High motor 1*RPM at fan bearing	BELT DRIVE: - Pitch line runout on motor pulley - Fan mount resonance at motor speed	
		High fan 1*RPM at motor bearing	BELT DRIVE: - Pitch line runout on fan pulley - Motor mount resonance at fan speed	
	2*RPM	1*RPM H > .18 in/sec 1*RPM V < .08 in/sec	Horizontal resonance plus vertical misalignment	Phase across direct drive coupling will either be in phase or ± 10° out of phase
		1*RPM H < .08 in/sec 1*RPM V > .18 in/sec	Vertical resonance plus horizontal misalignment	
			Disk skew	Vibration builds as in resonance but phase does not change
	2*RPM, 3*RPM	1*RPM H > .15 in/sec 1*RPM V > .12 in/sec Low axial both bearings	Imbalance plus system non-linearity (looseness, cracks, excessive clearance)	- Consistent 1*RPM phase - Can also produce ½ order harmonics
		High axial on bearing	Bearing cocked on shaft Both bearings are fixed (rare but possible)	
	Up to number of blades x RPM	H, V, or A vibration > .1 in/sec	AXIAL FAN: non-uniform blade track	
			AXIAL FAN: - Blade pitch error - Structural element too close to fan blades	Higher harmonics may also be present

FREQUENCY DOMAIN ANALYSIS				
FREQUENCY	HARMONICS	AMPLITUDES	PROBABLE CAUSE	COMMENTS
1*RPM	3*RPM, 4*RPM ½ orders		- Locked coupling - External side force	- Locked coupling is usually load-dependent - Asymmetric time signal may also be present
	2*RPM-4*RPM higher than 1*RPM		- Bearings cocked on shaft - Bearings not in same plane	Axial vibration on the bearings will be high
			Cavitation	- High frequency noise also present - Wide skirts
	Group at belt frequency		Belt misalignment	
# blades * RPM	No	H, V, A vibration > .15 in/sec	- Fan operating too far from best efficiency point - Fan cutoff too close to blades - Structural resonance AXIAL FAN: non-uniform blade pitch	
Belt frequency	# pulleys order usually highest		- loose belts - faulty belts	Harmonics can be as high as 9th or 10th order
# belts * belt frequency			Shafts not parallel	
Belt strand resonance			Resonance excited by belt strand	Wide skirt, since resonance of tight and loose strands are within 3-6 Hz of each other
Approximately 40% of 1*RPM	No		Oil whirl	Appears as inside loop on shaft orbit
33-50% of 1*RPM	No		Fan in surge	- Accompanied by pulsations in duct - High noise usually present
60-75% of 1*RPM	Yes		Rotating stall	- caused by inlet disturbance - check damper design and installation

SUMMARY

Fans are designed to move air against moderate pressures (the total air resistance in a duct system). The pressure, temperature, density, viscosity, and humidity of the air all affect the design and performance of a fan.

There are two primary types of fans: centrifugal and axial flow.

Centrifugal fans consist of an impeller running in a spirally-shaped casing. Usually the air enters the casing from the impeller's axial direction and is discharged at the periphery of the impeller.

The flow of air is parallel to the impeller axis in an axial flow fan. Axial fan blades are usually an airfoil in cross-section, and run at a very close clearance to the side of the fan casing. Air enters the fan in an axial direction. The air leaving an axial flow fan has both an axial and a rotational component.

Consequently, the total pressure developed by the impeller is not all available as useful fan total pressure.

Most axial fans employ some method of removing the rotational component in order to recapture the rotational velocity pressure.

Many aspects of the construction of a fan affect vibration. The structural aspects that most often contribute to fan vibration include

- mounting
- bearing lubrication
- bearing preload
- bearing clearance
- maximum frequency
- belt drives
- shaft alignment
- disk skew

Fans are highly sensitive to belt misalignment, much more than other belt-driven systems.

A fan should be operating at its best efficiency point in order to prevent turbulence in the duct. Stalls and surges resulting from obstructions in the duct can also cause vibration.

REFERENCES

Wallis, R. Allan, Axial flow fans: design and practice, Copyright 1961, Academic Press, New York, NY.

Osborne, William C., Fans, first ed., Copyright 1966, Pergamon Press, New York, NY.

Eck, Bruno, Fans: design and operation of centrifugal, axial-flow, and cross-flow fans, Copyright 1973, Pergamon Press, New York, NY.

Design and Specification Guidelines for Large Draft and Fan Systems, EPRI Publication CS-3431, Project 1649-8, January, 1984.

Power Plant Fans Specification Guidelines, Air Movement and Control Association, Publication 801, 1977.

9 MACHINE-SPECIFIC ANALYSIS: PUMPS

The inboard bearing of the high pressure (HP) stage of a turbine-driven high pressure core injection pump (HIPCI) at a nuclear power station had overall vibration levels as high as 1.8 inches/second. A vibration consultant was called in to determine the cause of these high vibration levels.

The vibration in the horizontal direction was more than 5 times the level in the vertical direction and was almost all at a frequency which appeared to be 2 times the operating frequency of the pump.

The analyst reported that the high amplitude in the horizontal direction was the result of misalignment between the turbine and the HP stage, which was amplified by a resonance in the bearing pedestal of the pump.

The power plant had the turbine and HP stage of the pump aligned at considerable expense in both dollars and hours, but when the unit was restarted, the vibration level on the inboard bearing of the HP stage was unchanged.

The authors were called in to run further tests on the system. The initial data taken with and without synchronous time averaging indicated that the vibration at the inboard bearing of the HP stage was not at a second order frequency of the pump shaft. However, synchronous time averaging at the frequency of the low pressure (LP) stage showed the vibration to be the fourth order of the LP.

The gear box ratio between the HP and the LP was 1.99:1 and the LP had a four-vane rotor so that 4/1.99 = 2.01. Thus it was clear that the high vibration on the inboard bearing of the HP was

at a frequency equal to 2.01 times the HP stage frequency and equal to 4.00 times the LP stage frequency.

Once it was known that the high vibration was phase locked to the LP stage, it was apparent that the resonance of the bearing pedestal of the HP stage was being excited by the acoustical resonance of the inter-stage piping which, in turn, was being excited by the pressure pulsations from the LP stage. The LP stage pump rotor was changed to one with 5 vanes instead of 4, and the problem was solved.

1.0 PRINCIPLES OF PUMP OPERATION

Pumps move fluid from one point to another by adding energy (not pressure) to the fluid. Since energy is defined as the ability to do work, adding energy to a fluid causes the fluid to do work: move through a pipe, fill a tank, or flow to a higher level.

A pump transforms mechanical energy from a rotating impeller into the kinetic and potential energy required to move the fluid. The mechanical energy that is imparted to the fluid (in foot-pounds per pound of fluid) is independent of the fluid's density and is called the **pump head**.

Pump head is measured in feet, and the head generated by a pump operating at a given speed and capacity is the same for all fluids. Therefore pump heads are expressed in feet of liquid.

The terms *head* and *pressure* are often used interchangeably, but this is not correct. By expressing head in feet you are measuring the energy produced by the pump, since a certain amount of energy is required to move the fluid a certain number of feet. For example, consider a column of fluid as shown in Figure 14.

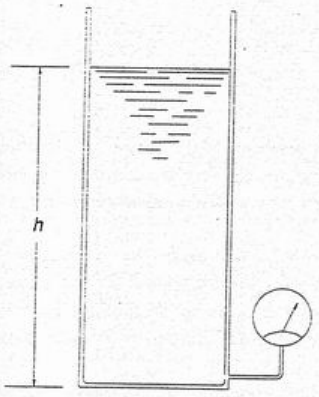


Figure 14 - *head and pressure*

The column of fluid in Figure 14 is a certain height *h* above the bottom of the column. Due to its height, the fluid exerts pressure (equal to its mass times gravity) on the bottom of the tube. Head is the height of the fluid; pressure is the force exerted by the fluid by virtue of its height.

A column of water 2.31 feet high produces a pressure of 1 psi (pound per square inch) at its base. Therefore, any pressure measured for a column of water can be converted to feet of water by

multiplying the pressure by 2.31. For fluids other than water, the height needed to produce 1 psi is found by dividing 2.31 by the specific gravity of the fluid.

For example, a column of water 231 feet high produces a pressure at its base of 100 psi. The height of a column of gasoline (with a specific gravity of 0.75) needed to produce 100 psi would be 308 feet, since 231/0.75 equals 308.

1.1 Types of Head

The general term "head" refers to the total energy imparted to the fluid at a given operating speed and capacity of the pump. There are many different measurements of head, however, and each one refers to a different aspect of the pump's operation.

The total head against which the pump must operate is called the **system head**. System head is made up of the following three components:

1. Static head
2. Friction head
3. Velocity head

1.1.1 Static Head

Static heads refer to differences in elevation between suction and discharge, and may either be measured between the fluid levels or from the pump centerline. Figure 15 illustrates the three different measurements of static head.

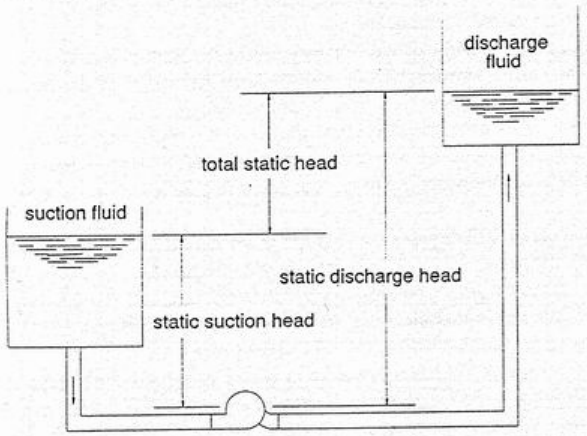


Figure 15 - *measurements of static head*

The **total static head** is the difference in elevation between the levels of the suction and discharge fluids, while the **static discharge head** is the difference between the pump centerline and the discharge fluid level. The **static suction head** is the elevation difference between the pump centerline and the suction fluid level (this may be called *submergence* if the fluid is under pressure other than atmospheric).

1.1.2 Friction Head

Any fluid pumped through a system of pipes encounters friction from the pipe and the fittings. Friction is an energy loss; therefore, in order for the fluid to move, enough energy must be imparted to the fluid to overcome the energy loss due to friction.

Friction head is the head (measured in feet) that is required to overcome the friction losses in the system. The friction head varies with the quantity of flow, the size of the pipe, components in the piping system, and the type of fluid that is being pumped.

1.1.3 Velocity Head

As a fluid is moving through a pumping system, it has a certain velocity at any given moment. This velocity (which is also a measure of the kinetic energy of the fluid) is measured in feet, and is called the **velocity head**. Velocity head is equal to the distance the mass of fluid would have to fall in order to obtain that same velocity.

When attempting to determine the total head of a system, the velocity head must be added to the gauge pressure reading. This is because the actual head is a sum of pressure and velocity heads, and a pressure gauge can only read pressure energy.

1.1.4 NPSH (Net Positive Suction Head)

All matter exists in one of three states: solid, liquid, or gas. In the design and operation of pumps, we are primarily concerned with the pumping of liquids, although gas pumps do exist. The transition from one state to another is called a **phase change**.

A phase change requires a transfer of heat energy. When a liquid goes from a liquid state to a gaseous state (a process called **vaporization**), energy must be added to the liquid. Likewise, to condense a gas into a liquid, energy must be removed from the gas.

Phase changes are usually associated with temperature (for example, liquid water turns to solid ice at a temperature of 32 degrees Fahrenheit and to gaseous steam at 212 degrees Fahrenheit). But phase changes are also dependent on pressure: the lower the pressure, the lower the temperature at which a phase change occurs.

For example, water boils at 100 degrees Centigrade at normal atmospheric pressure (14.7 lb/in^2, the pressure at sea level). If the atmospheric pressure were decreased to only 1 lb/in^2, the water would boil at 40 degrees C. This is why cooking times must be adjusted for higher altitudes: since atmospheric pressure is lower at higher altitudes, the amount of heat being put in is much less than at sea level.

At any given temperature, a liquid has a pressure at which it immediately turns to vapor if more temperature is added. This pressure is called the **vapor pressure**, and is a very important consideration in pumping liquids. The suction pressure of a pump must always exceed the liquid's vapor pressure. Otherwise the liquid will flash into vapor and the pump will cavitate, causing excessive vibration, impeller damage, and pump failure.

Because of the vapor pressure, the energy that is available to move the liquid into the pump is equal to the total suction head minus the vapor pressure. If the total suction head exactly equaled the vapor pressure, then all the energy of the suction head would be expended in keeping the fluid in a liquid state, and none would be available to move the fluid.

This available head (total suction head - vapor pressure in feet) is called the "net positive suction head," and is abbreviated **NPSH**. A distinction is made between *available* NPSH and *required* NPSH:

- Available NPSH is the difference between the absolute suction head and the vapor pressure of the fluid at its operating temperature. This is a characteristic of the pumping system.
- Required NPSH is the smallest margin necessary between the suction head and the vapor pressure. This is a characteristic of the pump design.

As the temperature increases, the vapor pressure also increases. The boiling point of a liquid is the temperature at which its vapor pressure equals the external pressure.

1.2 Overview of Fluid Dynamics

The dynamics of a fluid in motion are very complex. Experts in the fields of fluid dynamics and hydraulics begin by considering an **ideal fluid** in **steady flow**. An ideal fluid is incompressible and has no internal friction, and steady flow means that there are no vortices, eddies, or turbulent areas in the fluid.

The **equation of continuity** is used to describe the effect of the cross-section of a flow on the flow's velocity. For any given tube of flow, the product of the tube's area A at a given point times the velocity v of the fluid at that point is equal to $A*v$ at any other point along the tube.

In order for the equation of continuity to remain true, the velocity of the fluid must decrease when the tube's cross-section increases. Likewise, the velocity increases when the tube's velocity decreases. For example, consider a tube with two points. The area at point 1 is noted as $A1$, and the velocity at point 1 is noted as $v1$; the same convention is used for the area and velocity at point 2. The equation of continuity states that:

$$A1 * v1 = A2 * v2 \qquad \text{(Formula 9-1)}$$

Therefore, if area $A2$ is smaller than $A1$, then velocity $v2$ must increase proportionally so that $A2 * v2$ equals $A1 * v1$.

This relationship can be easily seen in a stream or river. An object moving in an open, unobstructed part of the river moves along at a given rate. If the object flows between two large rocks spaced closely together, the object's velocity increases. Then, once it has left the rocks and the cross-section increases, the object's velocity decreases.

1.3 Bernoulli's Equation

In 1738, a Swiss mathematician named Daniel Bernoulli (1700-1782) expanded on the equation of continuity by relating a fluid's pressure to its velocity. Bernoulli stated that the sum of a fluid's pressure, density, and kinetic energy (velocity head) are equal at any point along the flow of an ideal fluid. This equation states that if a fluid's velocity increases, then the pressure drops; likewise, the pressure increases as the velocity decreases.

Bernoulli's equation has many applications, most notable of which is the Venturi tube. A Venturi tube is a tube with a constriction (or **throat**) as shown in Figure 16. The tapers on either side of the throat are designed to eliminate turbulence.

A fluid entering from the left has a pressure $p1$ and velocity $v1$ as it enters the throat. Inside the throat, the fluid has pressure $p2$ and velocity $v2$. Bernoulli's equation (greatly simplified) states that:

$$p1 + v1^2 = p2 + v2^2 \qquad \text{(Formula 9-2)}$$

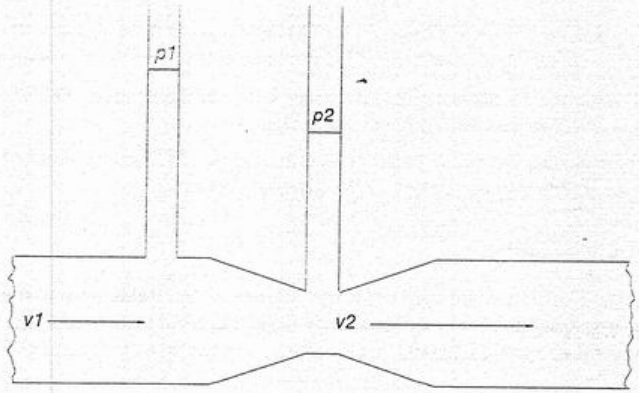


Figure 16 - *a Venturi tube*

The equation of continuity tells us that the fluid will flow faster in the narrower cross-section of the throat than in the wider cross-section of the tube. Therefore velocity $v2$ is higher than velocity $v1$. In order to maintain the equality of Bernoulli's equation, the pressure $p2$ must decrease so that $p2 + v2^2$ equals $p1 + v1^2$.

Aspirated internal combustion engines take advantage of the decreased pressure within a Venturi tube to help pull gasoline vapors into the intake manifold. The carburetor is attached to a Venturi throat at the top of the intake manifold. As the gas/air mixture flows into the intake manifold, its velocity increases at the narrow throat. The decreased pressure helps to pull the mixture on into the manifold.

Airplane wings, baseballs, and golf balls all take advantage of Bernoulli's equation. The airfoil cross-section of an airplane wing is longer across the top than across the bottom. As air flows around the wing the top section acts as a Venturi tube, increasing the air's velocity and decreasing its pressure. The air on the bottom of the wing, however, remains nearly at atmospheric pressure. This higher pressure underneath the wing creates a lifting force.

A spinning baseball creates turbulence along one side of the ball. This turbulence creates a region of high pressure and, depending on the type of spin, causes the ball to drop or curve to one side. Likewise, the slant of a golf club causes the ball to spin, so that the high pressure area is underneath the ball, giving the ball lift. The dimples on a golf ball act as miniature wings: air flowing inside the dimple behaves like air on top of a wing, so that high-pressure zones are created outside the dimple along the ball's surface.

1.4 Viscosity and Fluid Flow

Viscosity is the measure of the internal friction of a fluid (its "thickness" or resistance to flow). Gases are also viscous, but fluids have much higher viscosities than gases. Fluids flow in layers, and force is required to make these layers slide past each other.

A simple illustration of how a viscous fluid flows is shown in Figure 17. Two parallel plates are held apart by a viscous fluid. The bottom plate is stationary, while the top plate moves to the right with a constant velocity v.

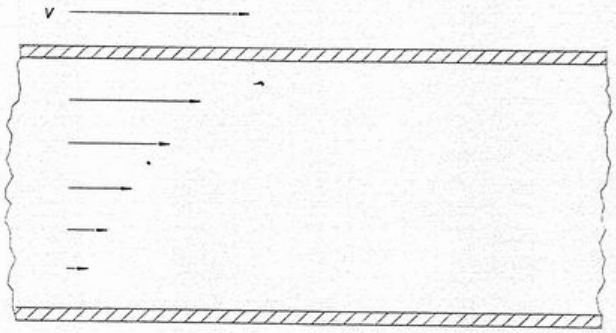


Figure 17 - *viscous fluid flow*

The fluid that is in contact with each plate has the same speed as the plate. Therefore the fluid next to the stationary plate has a velocity of 0, while the fluid in contact with the moving plate has the same velocity *v* as the plate.

The speed of the fluid layers between the two plates varies between 0 and *v*. Fluid layers close to the stationary plate have very low velocities, while layers closer to the moving plate have velocities closer to *v*.

This type of flow is called **laminar** (a *lamina* is a thin sheet); the layers of the fluid slide smoothly over each other. Laminar flow in a pipe has the same distribution of velocities as illustrated in Figure 17. Fluid closest to the pipe walls flows in an area of near zero velocity called the **boundary layer**. The flow velocity increases uniformly toward the center of the pipe; the velocity is highest at the exact center.

Because the flow velocity varies throughout the pipe diameter, fluid flow is often expressed as an average. The average velocity is defined as the uniform velocity over the entire pipe diameter that would result in the same rate of flow.

When the average flow velocity exceeds a critical value, the flow ceases to be laminar and becomes **turbulent**. Flow within the boundary layer remains laminar, but circular currents called **vortices** are formed at random locations throughout the rest of the flow. These vortices are highly resistant to flow and affect the efficiency of a pump.

1.5 The Reynolds Number

Experiments conducted by Professor Osborne Reynolds showed that four factors determine when a fluid flowing through a pipe is laminar or turbulent:

1. The density of the fluid
2. The fluid's average velocity
3. The diameter of the tube
4. The viscosity of the fluid

The combination of these four factors is known as the **Reynolds number**, and is found by the formula:

(Formula 9-3)

$$N_R = \frac{\rho v D}{\eta}$$

where:

N_R = the Reynolds number
ρ = the fluid density
v = the average velocity of the fluid
D = the tube diameter
η = the fluid's viscosity

The Reynolds number is a *dimensionless* number; that is, the number will be the same regardless of the units used to measure the four quantities.

Reynolds showed by experimentation that a fluid will be laminar with a Reynolds number around 2,000 or less. A Reynolds number higher than 3,000 or so means that the flow will be turbulent, and numbers in between characterize an unstable flow that may be either laminar or turbulent.

These numbers are not exact, since the average velocity in Formula 9-3 assumes a straight-line flow. Anything that disturbs the velocity distribution of the flow (such as an elbow or a change in pipe diameter) changes both the flow pattern and the Reynolds number. Also, variations in temperature affect the fluid's viscosity, which changes the Reynolds number.

A common example of the Reynolds number and the transition from laminar to turbulent flow can be seen in the smoke rising from a candle or the end of a cigarette. Since the smoke is less dense than air, the air pressure accelerates the smoke upward. This upward thrust acts continuously so that the smoke's velocity continues to increase.

The smoke flow near the cigarette has a low velocity and is laminar. After a certain point the smoke's velocity exceeds a critical value and the flow becomes turbulent. In still air, the smoke can be seen to rise smoothly for a foot or so before it breaks up into curls and eddy currents.

2.0 TYPES OF PUMPS

There are two main types of pumps:

1. Centrifugal pumps
2. Axial pumps

Please note that there are many different pump designs, not all of which fit neatly into these two categories. These two categories of pump design encompass most pumps, and are useful when discussing the vibration characteristics of pumps.

2.1 Centrifugal Pumps

A centrifugal pump consists of a rotating element called an **impeller** that is enclosed in a casing. The impeller is mounted to a shaft that is connected to an external power source by a coupling. The impeller rotates at high speed within the casing, spinning the fluid being pumped along with it. Centrifugal force then forces the fluid out of the pump. Figure 18 shows a cross-section of a typical centrifugal pump.

When the fluid departs the impeller periphery, it is moving at a higher velocity than when it entered the pump casing. Since Bernoulli's equation states that the fluid's pressure decreases as its velocity

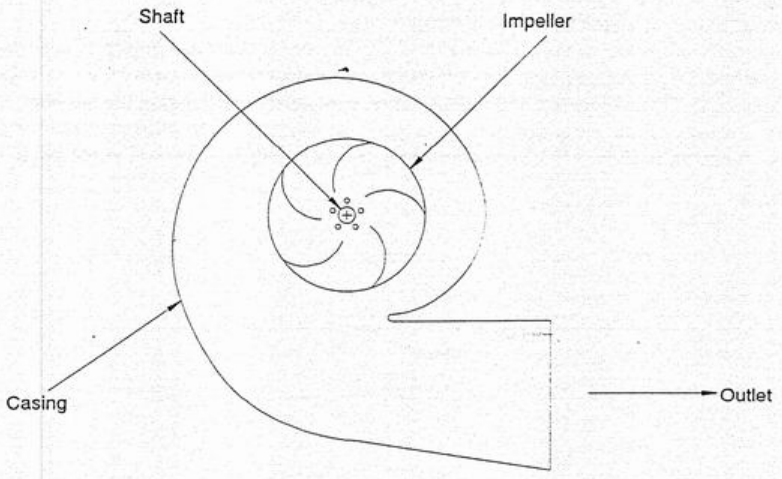


Figure 18 - *a basic centrifugal pump*

increases, the low pressure created by the impeller creates a suction at the pump inlet. This suction (the suction head) pulls more fluid into the pump through the inlet.

Many pumps use only one impeller to create the necessary head; such pumps are called **single stage pumps**. To create a very large head, however, it is often more efficient to use several impellers operating in series: the suction of one impeller is taken from the discharge of another. Such pumps are called **multi-stage pumps**.

The pump casing has two purposes:

1. To guide the fluid in toward the impeller and provide a pressure gradient
2. To convert some of the velocity energy of the fluid into pressure energy

It is obvious that, in order to develop and maintain a low-pressure area, some means of containing that area is necessary. The pump housing not only helps guide the fluid into the impeller, but provides a gradient between the low pressure created by the impeller and the higher pressure outside the pump. A component called the **stuffing box** is used to seal the interior of the pump from the outside, even around the impeller shaft.

The pump head has both pressure and velocity components. In order to get more useful work out of the pump, some of the velocity energy must be converted into pressure energy. This conversion is accomplished by one of two means:

1. A volute
2. Stationary diffusion vanes

2.1.1 Volute

A **volute** is an ever-widening spiral curve (like a snail shell). A volute casing is a spiral-shaped casing that surrounds the impeller. The area of the volute steadily increases around the impeller, until

it reaches its greatest area at the pump discharge opening. This increasing area helps to convert some of the velocity energy of the fluid into pressure energy.

Centrifugal pumps with a volute casing (such as the one illustrated in Figure 18) are often called **volute pumps**. Most volute pumps are *single volute pumps*; that is, there is only one volute within the pump casing. The volute is a 360 degree curve, completely surrounding the impeller.

When a single volute pump operates at its designed capacity, near uniform pressures act on the impeller, as shown in Figure 19. This results in a near-zero radial movement of the impeller.

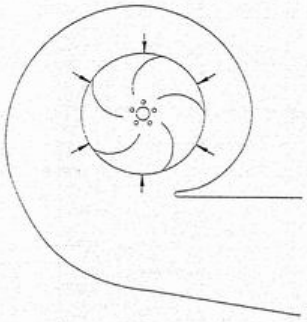


Figure 19 - *uniform pressure on the impeller*

If the pump is operating at other than its designed capacity, however, the pressures on the impeller can become unequal. These unequal forces result in a net radial movement of the impeller, as shown in Figure 20. This radial deflection can lead to increased bearing wear and excessive flow turbulence, both of which can be causes of excessive vibration.

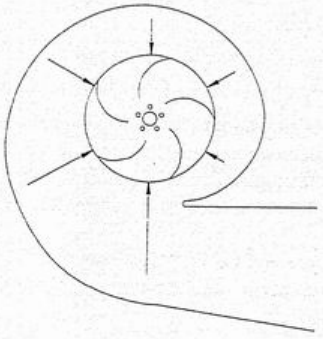


Figure 20 - *unequal pressure on the impeller*

It is becoming more common for centrifugal pumps to be operated at less than rated capacity. Therefore the risk of radial shaft deflection as a result of unequal pressures is much greater. To

compensate, pump manufacturers may incorporate heavier shafts and bearings into their pumps, but this solution is not always economical.

A second, more practical solution is the **twin volute** design. A twin volute pump incorporates a second volute curve inside the pump casing as shown in Figure 21. Both volutes are only 180 degree curves, and join together at the pump discharge.

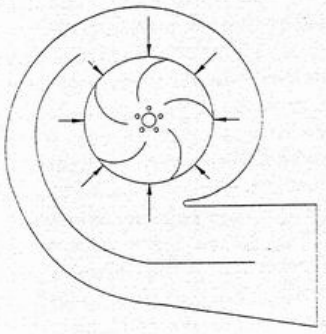


Figure 21 - *a twin volute pump*

The purpose of two volutes is that unequal pressures around the impeller now create two radial forces, each one nearly equal in magnitude to the other but opposite in direction. The two forces cancel each other out, and the net deflection of the shaft at reduced capacity is near zero. Twin volutes also have the advantage of providing additional stiffening to the pump casing.

2.1.2 Diffusion Vanes

Diffusion vanes are a set of stationary vanes that surround the pump impeller as shown in Figure 22. Since the diffusion vanes convert some of the velocity energy to pressure energy, a volute casing is not necessary.

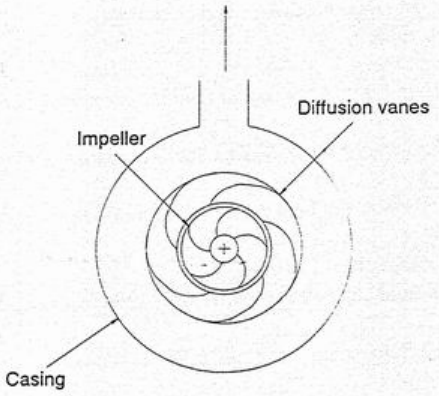


Figure 22 - *diffusion vanes*

Many years ago, it was very difficult to cast and/or machine precise volute casings. Therefore diffusion vanes were very popular, since they were relatively easy and inexpensive to manufacture and they improved the efficiency of the pump. As manufacturing techniques improved, the diffusion vane pump decreased in popularity. Today it offers little advantage over a volute pump; the major advantage of diffuser vanes is that they balance radial forces on the pump impeller.

Pumps with diffusion vanes are very sensitive to the rate of fluid flow. When the pump is operating at its rated flow, the angle of the fluid leaving the impeller matches the angle of the diffusion vanes. Therefore, the fluid meets and interacts with the diffusion vanes with a minimum of turbulence.

When the pump is operating off its rated flow, the fluid meets the diffuser vanes at the wrong angle and causes turbulence. Even a small deviation of 5 to 10% of rated flow can cause enough shock and turbulence to reduce the total head and cause the pump to operate in an unstable condition.

Pump flexibility is also an issue with diffusion vanes. Manufacturers try to provide as complete a line of pumps as possible with a minimum of parts. In order to achieve this, the manufacturers try to make pump components as interchangeable as possible. With volute pumps, the impeller diameter may be reduced by as much as 20% of the maximum value without a serious impact on pump efficiency. Therefore, one impeller design may be used effectively in a variety of volute casings.

The impeller in a diffusion vane pump can only be reduced 5-10% of the maximum value, however, before the hydraulic losses become so severe that the total head is affected. To offer a complete line, then, manufacturers of diffusion vane pumps must stock a greater number of components, which increases the cost of the pump.

2.1.3 Types of Centrifugal Pump

Centrifugal pumps may be classified in many different ways:

1. By type of impeller
2. By orientation of shaft
3. By type of casing split

Type of Impeller

Centrifugal pump impellers are classified according to the type of flow (how the fluid enters the impeller), the type of suction, and their mechanical construction.

The different flow types include

1. Radial flow - The fluid enters the impeller radially (along one of the radii of the impeller).

2. Axial flow - The fluid enters axially (along the axis of the impeller shaft).

3. Mixed flow - A combination of both radial and axial flow.

Impeller classifications according to suction are

1. Single suction - The fluid enters the impeller from only one side.

2. Double suction - The fluid enters the impeller from both sides.

The mechanical designs of pump impellers include

1. Enclosed - The ends of the impeller are shrouded (provides structural integrity and is usually used for clear liquids).

2. Open - The vanes of the impeller are not shrouded (used in inexpensive pumps or for pumping abrasive liquids).

3. Semi-open - The impeller has a partial shroud (provides some support to long vanes and is useful in pumping liquids with solids, like sewage).

Orientation of Shaft

Centrifugal pumps may be classified according to the axis of rotation of their shafts: either horizontal or vertical.

Horizontal pumps are further classified as

1. End suction - where the suction inlet is at the end of the pump

2. Bottom suction - the suction inlet is on the bottom of the pump

3. Top suction

4. Side suction

Vertical pumps, most typically used in pumping wells, may be further classified as

1. Wet pit - where the pump is submerged in its suction supply, as in a sump pump

2. Dry pit - when the suction supply is delivered to the pump through pipes

Type of Casing Split

A centrifugal pump's casing must be split so that the impeller can be installed and removed. The orientation of the casing split provides a final classification for centrifugal pumps:

1. Radially split - the casing is split in a plane perpendicular to the impeller's axis of rotation; that is, the impeller's radius lies in the same (or a parallel) plane as the casing split.

2. Axially split - the casing is split in a plane through the axis of rotation. Axial splits may be horizontal (parallel to the ground) or inclined (usually to accommodate a vertical discharge nozzle).

2.2 Axial Pumps

The flow of both suction and discharge fluid in an axial pump (also called a propeller pump) are parallel to the impeller's axis of rotation. Axial pump impellers resemble propellers or screws on a ship, hence the term "propeller pump." The vanes on an axial pump impeller are often an airfoil in cross section, and run at close clearance to the pump casing.

The vanes of an axial impeller are inclined relative to the axis of rotation as shown in Figure 23. Fluid enters the impeller in an axial direction. The impeller vane cuts into the fluid along the leading

edge of the vane, which is tipped away from the direction of flow. As the impeller rotates, the forward incline of the vane pushes the fluid forward.

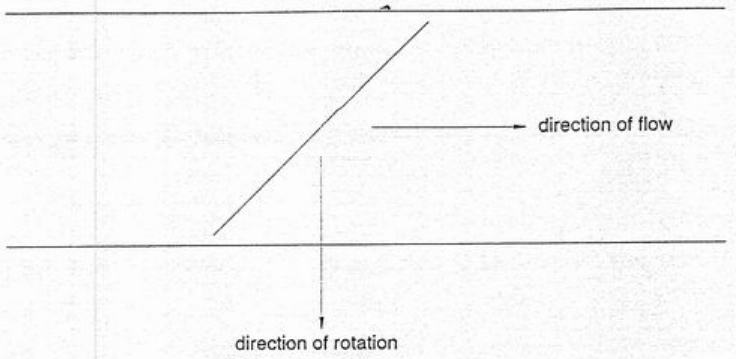


Figure 23 - *incline of axial impeller vanes*

The fluid leaving the impeller has both an axial and a rotational component. Some of the rotational energy of the impeller vanes is imparted to the fluid, causing it to rotate as it moves down the pipe. Consequently, the head developed by the pump is not all available as useful total head.

Therefore, some means is necessary to remove the rotational component from the fluid. In most cases, diffusion vanes are built into the pump casing downstream of the impeller. The diffusion vanes straighten out the flow, and also convert some of the velocity energy into useful pressure energy. For large flows a device called a **scroll** (which is actually a volute curve) may be used instead of diffusion vanes.

Diffusion casings are not used in high head multistage axial pumps. Instead, guide vanes are placed in the suction inlet, ahead of the impeller. The guide vanes prerotate the fluid in a direction opposite to the impeller's rotation (and consequently reduce the pressure upstream of the impeller). As the impeller imparts its rotational component to the fluid, it cancels out the fluid's prerotation and the fluid leaves the impeller axially.

It can be shown mathematically that, for a given size of impeller vane, higher head can be obtained by installing guide vanes upstream of the impeller than by using a diffusion casing downstream.

3.0 OPERATIONAL SOURCES OF VIBRATION

Operational sources of vibration in pumps can be in two different categories: hydraulic and mechanical. Hydraulic vibration sources include

- cavitation
- recirculation
- hydraulic imbalance
- interaction with volute/diffuser

Mechanical vibration sources include

- misalignment
- imbalanced driver

- imbalanced impeller
- impeller instability
- bent shaft
- pipe stresses

3.1 Cavitation

Cavitation occurs when a local pressure drop inside the pump allows cavities filled with fluid vapor to form. These cavities collapse when they contact the high pressure side of the pump, causing both noise and vibration. Pitting of the impeller can also result from prolonged cavitation due to the hammering effect of the collapsing cavity.

Cavities of vapor can only form when the pressure drops below the vapor pressure that corresponds to the fluid's temperature. Pressure drops may affect the entire system or may only be localized. System-wide pressure drops can be caused by

- a decrease in atmospheric pressure resulting from a change in altitude
- a decrease in the absolute pressure on the pumping system
- an increase in the temperature of the fluid being pumped

Local pressure drops may be caused by

- an increase in velocity (speeding up the pump)
- viscosity
- deviation of streamlines in laminar flow (by an obstruction in the pipe or a change in direction, for example)

Vibrations caused by cavitation are usually at 1*RPM, with harmonics up to the number of impeller vanes times RPM. High frequency noise (a loud, steady crackling) is also present as broadband vibration, typically higher than the blade pass frequency.

Since cavitation is caused by decreased NPSH, the solution is to increase the NPSH above the fluid's vapor pressure.

3.2 Recirculation

Recirculation is a hydraulic source of 1*RPM vibration. If a pump has a very high suction pressure, it may pull fluid from the output back through the pump. A high suction pressure accompanied with reduced flow is a major cause of recirculation.

The fluid flows backward through seals, leakage, and over the impeller clearance. Recirculation can be detected by pressure pulsations in the suction piping and in the discharge (it often manifests itself as a loud cracking noise that occurs randomly in the suction piping of the pump). Recirculation can be reduced by increasing the pump output or by installing a bypass. In systems with installed bypasses, it is important that the bypass orifice must be large enough to handle the recirculation flow.

Subharmonics of operating speed (anywhere from 5% to 35% of operating frequency) may also result from recirculation. These subharmonics may excite structural resonances in the pump support and/or suction and discharge piping.

3.3 Hydraulic Imbalance

Hydraulic imbalance is, as its name implies, hydraulically induced (by the fluid being pumped). Therefore, it is impossible to correct hydraulic imbalance by mechanical balancing procedures.

Hydraulic imbalance is typically caused by poor design in the suction piping, such as an elbow very close to the suction inlet of the pump. Even if the fluid flow through the pipe is laminar, the fluid will assume different velocities inside the elbow (the fluid flows faster along the outside curve).

If an elbow is very close to the suction inlet, then the flow does not have time to equalize once it leaves the elbow. This results in unequal flow into the impeller (especially so in double suction pumps) and may cause a high axial vibration at 1*RPM.

A straight pipe flow of 20 to 40 pipe diameters before the suction inlet is recommended. This length allows the flow time to equalize before it enters the pump.

Hydraulic imbalance may also occur if the impeller itself is not symmetrical in the fluid flow areas.

3.4 Interaction with Volute/Diffuser

At its rated flow, the fluid being pumped will meet the diffusion vanes or the volute eye (the wall dividing the volute from the discharge casing) at the correct angle, thus minimizing turbulence.

If the pump is not operating at its rated capacity, then the fluid interacts with diffusion vanes or the volute eye at an incorrect angle, creating turbulence and high axial vibration at subsynchronous frequencies.

Excessive clearance between the impeller and the casing results in high level, random vibration which usually occurs below the operating frequency of the pump.

Acoustic resonance in the piping system can result from the pressure pulsations caused by the interaction of the impeller vanes and the volute or diffuser. The frequency is at the vane pass frequency (the number of impeller vanes times the operating frequency), and is not dependent on the number of diffuser vanes or volutes. Since this is a resonant condition, the vibration amplitude varies dramatically with the operating frequency of the pump.

3.5 Misalignment

Axially split casings (both horizontal and inclined) have a tendency to **breathe** (separate) when they are under very high pressure. This can cause misalignment of the impeller, which leads to high axial vibration and hastens bearing and seal failure (leakage can also result from breathing). When a pump with an axially split casing is to be subjected to high pressures, reinforcing ribs are added both inside and outside the casing.

Thermal growth of pump casings may also lead to misalignment if the pump is handling high-temperature fluids. The pump may start out in good alignment, but may grow out of alignment as the pump heats up from the temperature of the fluid. Pumps handling high temperature fluids should be supported at their centerline as well as their base to prevent misalignment due to thermal growth.

In most cases, the calculated thermal growth is not accurate enough to insure good alignment at operating temperature. If the actual growth is not known, it should be measured as the unit comes up to temperature. There are several good methods to measure the growth, such as optical, Essinger, or Dyalign.

Misalignment can usually be identified if a high 2*RPM vibration is present. Pure angular misalignment is predominately in the axial plane, while pure offset misalignment occurs mostly in

the radial plane. A combination of angular and offset misalignment appears as high vibration in both the axial and radial planes.

3.6 Imbalanced Impeller

Imbalance in the impeller typically creates a high 1*RPM vibration in the radial plane. This imbalance can cause turbulence and cavitation, lead to excessive bearing wear and early failure, and can cause leakage in the pump seals.

If the impeller is balanced at the manufacturer, the complete assembly (including the impeller's half of the coupling) should be balanced. The impeller may then be trim-balanced once it is installed in the pump casing.

3.7 Impeller Instability

The close clearance between the pump impeller and the wear rings and/or the guide bearing tends to raise the critical frequency of the system. If this clearance becomes excessive (that is, if the clearance increases beyond accepted tolerance), the critical frequency of the impeller and shaft may be reduced to close to the operating frequency and the system becomes unstable (small disturbances are magnified by the resonant amplification and appear as 1*RPM vibration).

3.8 Bent Shaft

A bent shaft will cause high 1*RPM vibration in the axial plane. Bent shafts may occur from misalignment, pipe stress (see next section), or poor manufacturing.

If the shaft is bent, then the clearance between the impeller and the pump casing (or the diffusion vanes) will not be equal. Unequal clearance can create pressure pulses at the pump's rotational frequency in the discharge piping, result in inadequate pressure head, and may contribute to recirculation (especially in the case of decreased flow). Vibrations inside the pump casing may also cause cavitation.

If the bend is severe, then the impeller vanes may hit the pump casing. Besides causing damage to the impeller and the casing, the metal dislodged by the impact will be thrown into the discharge, which could lead to damage of other equipment down the line.

3.9 Pipe Stresses

Both suction and discharge piping should be adequately supported (with hangers, expansion joints, etc.) to avoid undue stress on the pump casing. If excessive, pipe stresses can lead to misalignment between the pump and the driver, especially if the pump is loose or has a soft foot.

4.0 STRUCTURAL ASPECTS AFFECTING VIBRATION

This section discusses structural aspects of both centrifugal and axial pumps that can affect vibration. Most of the structural aspects covered here apply to both types of pump; those considerations that are specific to one type of pump will be discussed under the appropriate sub-heading.

A troubleshooting guide that includes the considerations from this section is at the end of this chapter.

4.1 Anti-Friction Bearings

Errors in mounting anti-friction bearings can cause excessive vibration. For example, anti-friction bearings might not be mounted square to the shaft or in the same plane (one bearing is higher or lower than the other). This usually results in elevated bearing temperature and can cause second-, third-, and fourth-order vibration in addition to first-order vibration.

Anti-friction bearings are mounted to a shaft as shown in Figure 24. The inner race is locked to the shaft, while the outer race is held against the bearing housing. The outer race is preloaded so that it does not rotate.

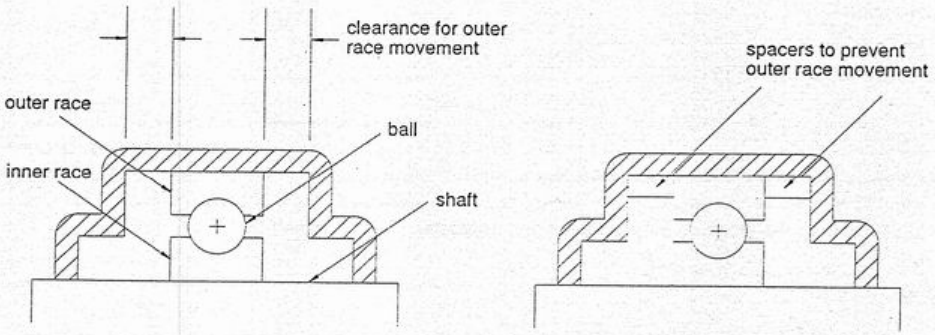


Figure 24 - *floating and fixed bearings*

During thermal growth, the inner race moves with the shaft, since the two are locked together. The preload prevents the outer race from rotating, but not from moving with the shaft and the inner race. Make sure that the outer race does not rub against either side of the bearing housing. Spacers are inserted on either side of the outer race to hold it in place.

A bearing whose outer race is free to move with the shaft is called a **floating bearing**, while a bearing whose outer race is prevented from moving is called a **fixed bearing**. In Figure 24 the bearing on the left is the floating bearing, while the bearing on the right is the fixed bearing.

Only one bearing should be fixed; since some thermal growth is inevitable, one of the bearings must be floating. Normally the fixed bearing is the one closest to the coupling or the pulley that is driving the pump; the floating bearing is the one on the outboard side. This allows the pump shaft to grow into the floating bearing.

If the reverse were true, and a floating bearing was next to a coupling, the shaft could grow into the coupling, use up the coupling clearance, and result in a locked coupling. The pump would run within vibration limits when started cold, but the first- (and maybe the second- and third-) order vibration would increase as the pump came up to operating temperature.

All the cautions that apply to anti-friction bearings should be observed for pump bearings:

- **Bearing lubrication**: Too much lubricant seals in the heat and causes early bearing failure. Too little lubricant does not provide the lubrication necessary to minimize wear.

- **Bearing preload**: The preload on the journal and in the bearing housing clearance must be set to maintain the proper preload on the bearings.
- **Maximum frequency**: The operating frequency of an anti-friction bearing pump should be within the bearing tolerance (7,200/shaft diameter for greased lubricant; 9,600/shaft diameter for oil lubrication).
- **Shaft alignment**: Alignment is a concern when a pump is direct-coupled to the drive motor. One of the first effects of shaft center line misalignment is an increased radial load on the bearings, which can lead to early bearing failure.
- **Resonances**: Pump shaft and bearing pedestal resonances that are close to the pump's operating frequency result in high first-order vibration levels and make field balancing of the pump difficult if not impossible.

  The pump impeller has a fundamental plate resonance as an assembly. The operating frequency and the vane pass frequency forcing functions must not be able to excite this resonant frequency.

  The piping may also have resonances excited by the pump forcing functions, especially in conditions where high pressure pulses exist in the discharge piping.
- **Looseness**: Loose bearing or hold-down bolts can change the system stiffness and result in high vibration levels.

4.2 Sleeve Bearings

One of the major problems encountered in pressure-fed sleeve bearings is the possibility of hydraulic instability of the shaft inside the bearing, called **oil whirl** or **oil whip**.

The rotational velocity of the lubricant next to the shaft is equal to that of the shaft, whereas the rotational velocity of the lubricant next to the bearing surface is zero. If the shaft were perfectly centered in the bearing, and the friction losses were ignored, the average rotational velocity of the lubricant would be exactly 50% of the shaft frequency. However, the frequency actually occurs somewhere between 35 and 49% of the shaft rotational frequency, due to friction losses and shaft eccentricity.

If the shaft is moved off center due to load, eccentricity, or imbalance, then the clearance on one side of the bearing will be greater than it is on the other side. Figure 25 illustrates this condition.

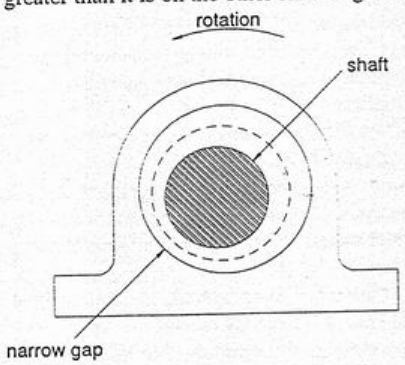


Figure 25 - *oil whirl*

As the lubricant rotates at approximately 50% of shaft frequency, it must squeeze through the narrow area where the shaft is closest to the bearing. The average velocity of the lubricant decreases as the lubricant enters the narrow gap, speeds up inside the gap, and slows down again when it leaves the gap. This speeding up and slowing down creates turbulence on both sides of the gap, and a vortex develops in the high-pressure lubricant zone.

The shaft rides on this oil vortex much like a surfboard rides the surface of a wave. Called **oil whirl**, this causes the shaft to become unstable at somewhat less than half its rotational frequency. The oil whirl stays proportional to the shaft frequency and drops out when shaft RPM drops below the instability threshold.

Oil whirl frequency approaches the first critical of the shaft as the shaft approaches its second critical, creating a resonant condition called **oil whip**. The oil whip frequency remains constant at the first critical of the shaft and drops out when shaft frequency drops below the second critical.

Both of these phenomena can be severe and result in a penetration of the lubricant film (called **wiping**). When this happens, the shaft impacts against the bearing journal, and serious damage can result.

One solution is to change the loading, lubricant, temperature, or clearance so that the shaft does not run in this instability area. Some sleeve bearing designs include axial grooves or offset cylindrical grooves in the journal to break up the oil whirl (refer to section 1.3.3 of the the General Analysis Techniques chapter in Volume 1).

Other concerns for sleeve bearings include

- **Bearing clearance**: Clearance on journal bearings should be between 1 and 1.5 mils per inch of shaft diameter, with a minimum of 2 mils. Insufficient clearance increases bearing temperature as well as the possibility of wiping. Too much clearance decreases the damping and increases the possibility of shaft instability.

- **Cap loading**: Centrifugal pump impellers are often cantilevered. If the pump is mounted in sleeve bearings and is not properly located (moved outboard slightly), the bearing farthest from the impeller can be cap-loaded. This could result in hydraulic instability and early bearing failure if the bearing is not designed for cap loading.

4.3 Resonance

Resonance between a pump and its foundation or piping can cause leakage, warping, misalignment, and equipment failure. Vibrations from the driver can be transmitted through the coupling and cause resonance within the pump system; other nearby machines may also cause pump resonance.

The possibility of resonant amplification of pump structures is very common in vertical pumps, much more so than in horizontal centrifugal pumps. Vertical pumps are tall and slender, and their supports typically have low natural frequencies.

Vibratory forcing functions (imbalance, bearings, misalignment, for example) that are present in a vertical pump can easily excite structural resonances. In fact, rotor imbalance that may be acceptable in horizontal pumps can often cause excessive vibration in a vertical pump due to resonant amplification.

Variable-speed vertical pumps have an even greater problem, since these vibratory forcing functions change as the pump's operating frequency changes. This increases the possibility that one or more forcing functions will pass through the resonant amplification range of the pump structure.

Solutions to resonant amplification include the following:

- Change the pump's operating frequency well above or below the resonant frequency. However, operation too far away from the pump's rated flow can result in cavitation, turbulence, and recirculation. Even if none of these situations occurred, a change in the pump's operating frequency changes the amount of flow. The requirements of the installation might prevent a change in flow.
- Stiffen the structure, thereby raising the natural frequency. Ribs and braces may be bolted or welded to the pump's discharge head, but remember that the heat of welding will deform the pump structure.
- Increase the pump's mass, thereby lowering the natural frequency. This is usually less expensive than stiffening, but should not be done without consideration to the effect on the pump. The increased mass may cause excessive strain on the motor, motor supports, and discharge piping.
- De-tune the system by adding a resilient plate between the motor and the pump. These plates allow for alignment of the motor and pump but lower the natural frequency of the system. In some cases they have not been very successful, but in others they have been a good solution to the problem.
- Add a vibration pad between the pump and the pump base plate in order to lower the natural frequency of the system. This may not be possible if the system requires a pressure seal between the pump and the base, or if the discharge piping is not capable of carrying the hydraulic thrust.
- Add a dynamic vibration absorber to the system. A dynamic absorber consists of a secondary spring-mass system which is mounted to a unit running close to, or at, resonance and is carefully tuned to the operating frequency of the main vibrating system. The absorber is normally attached at the point of greatest amplitude and may consist of a spring steel rod and an adjustable weight so that it can be tuned to the proper frequency.

4.3.1 Dynamic Absorbers

In theory, the dynamic absorber sounds good. If the spring-mass system is carefully tuned to the operating frequency of the pump, the pump vibration should go to zero. This looks like a simple and excellent solution to a serious problem. In most cases, however, the solution looks better on paper than in practice.

The addition of the secondary spring-mass system does greatly reduce the main system amplitude at one frequency. But it also results in a system that now has two resonant frequencies rather than just one. One of these resonances is below the operating frequency, and the other is above. This means, at the least, that the unit will go through the lower resonance each time it is shut down and started up.

This is illustrated in Figure 26. Figure 26(a) shows the response curve for a pump; the frequency F_o is the operating frequency of the pump, and F_r is the pump's natural frequency. Figure 26(b) shows the response curve for the same pump after a dynamic absorber has been added. Note that the amplitude at F_o is much lower in Figure 26(b) than it was in 26(a).

However, the system now has two resonant frequencies since the dynamic absorber has been added. In the original system as shown in Figure 26(a), the operating frequency F_o was below the system's resonant frequency F_r. In Figure 26(b) the pump's operating frequency F_o is now above the first resonance of the system!

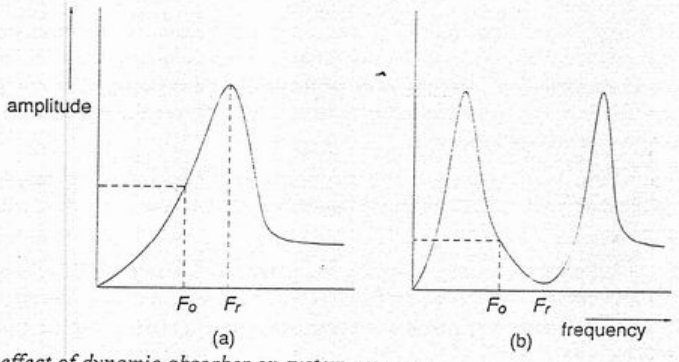


Figure 26 - *effect of dynamic absorber on system response*

If the weight of the mass of the dynamic absorber is 10% of the weight of the vibrating unit, the new upper and lower resonances will be approximately 15% above and below the original resonance to which the absorber was tuned. The spread between the upper and lower resonances increases almost linearly to ±60% as the weight of the absorber mass reaches the mass of the unit. This relationship between absorber mass and upper and lower resonance spread is shown in the graph in Figure 27.

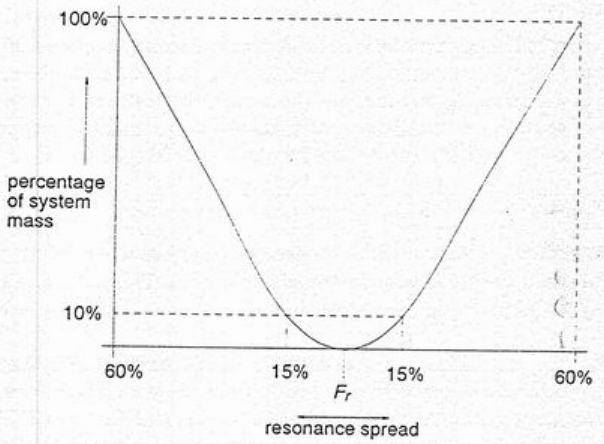


Figure 27 - *absorber mass and resonance spread*

Since it is not practical to add very large mass dynamic absorbers, the spread between the two new resonances is quite small. This makes it very important to use absorbers only where the operating frequency is stable and constant. Suppose we had a 700 pound pump that was resonant at its operating frequency of 1,800 RPM. If we added a dynamic absorber whose mass weighed 70 pounds (10% of the weight of the pump mass), the new resonances would be only 15% above and below the original frequency to which the absorber was tuned (in our case 1,800 RPM).

Now, instead of one resonance at 1,800 RPM (30 Hz) we have two resonances: one at 1,530 RPM (25.5 Hz) and one at 2,070 RPM (34.5 Hz). These new resonances are not very far from the system's original resonant frequency, and now the pump must pass through the entire lower resonant range as it approaches its operating frequency. In fact, the pump's operating frequency may still be within the amplification range of one of these new resonances! Changes in pump speed due to power and demand fluctuations may put the operating frequency even closer to one of the new resonances.

If the only possible solution is the addition of a tuned absorber, and if the operating frequency of the system is not constant, then it is necessary to design a damped tuned dynamic absorber. The damped absorber will reduce, rather than null, the resonant amplitude of the system, but the Q (amplification) of the two new resonances will be greatly reduced as well. The math required for this design is quite complex, so design of damped tuned dynamic absorbers should be done by an expert.

4.4 Looseness

Looseness has many harmonics of 1*RPM frequency (the 2*RPM or 3*RPM frequencies may even be higher than the 1*RPM). Pure looseness has mostly odd-order frequencies (3*RPM, 5*RPM, and so on).

4.5 Misalignment

Misalignment between the pump shaft and the driver shaft is a common cause of vibration. Most coupling misalignment problems can be seen as a high 2*RPM vibration in the axial or radial planes, depending on the type of misalignment. Pure angular misalignment is predominately in the axial plane, while pure offset misalignment occurs mostly in the radial plane. A combination of angular and offset misalignment will appear as high vibration in both the axial and radial planes.

The normal concerns of system alignment also apply to pumps. Checking for a soft foot, the condition of the foundation, the hold-downs, the base plate, and the shim packs all play an important part in the integrity of the system.

The alignment between a pump and a driver changes over time due to foundation settling, bearing and coupling wear, and weakened pipe supports. Thermal growth of the pump, however, is a common cause of misalignment.

If thermal growth is contributing to excessive vibration, the amount of growth was either not calculated correctly or was not considered during the alignment process (the pump was aligned when cold). As the pump warms up, the thermal growth moves the shaft into a greater degree of misalignment.

5.0 ANALYSIS

Refer to the following tables to determine the probable cause of pump vibration.

TIME DOMAIN ANALYSIS		
TYPE OF WAVEFORM	**PROBABLE CAUSE**	**COMMENTS**
Impacts	Looseness	
	Bearing fault frequency	
	Cracked shaft	

TIME DOMAIN ANALYSIS		
TYPE OF WAVEFORM	**PROBABLE CAUSE**	**COMMENTS**
Non-periodic components	Foreign material	
	Outside source of vibration	
Asymmetrical waveforms	Rubs	
	Restrictions to motion	

FREQUENCY DOMAIN ANALYSIS				
FREQUENCY	**HARMONICS**	**AMPLITUDES**	**PROBABLE CAUSE**	**COMMENTS**
1 * RPM	No	1*RPM H > .15 in/sec 1*RPM V > .12 in/sec Low axial both bearings	Imbalance	Consistent 1*RPM phase
		1*RPM H > .15 in/sec 1*RPM V > .12 in/sec 1*RPM A > .1 in/sec	Bent shaft	Look for axial phase 180° out
		1*RPM H > .18 in/sec 1*RPM V < .08 in/sec	Horizontal resonance	High amplitude may be at some other order excited by non-linear system
		1*RPM H < .08 in/sec 1*RPM V > .18 in/sec	Vertical resonance	
		Increases after startup	Thermal bow	
		High motor 1*RPM at pump bearing	Pump mount resonance at motor speed	
		High pump 1*RPM at motor bearing	Motor mount resonance at pump speed	
		High axial	Hydraulic imbalance	- Cannot be corrected by mechanical balancing - Usually caused by poorly designed suction piping
		High radial	Imbalanced impeller	
	2*RPM	1*RPM H > .18 in/sec 1*RPM V < .08 in/sec	Horizontal resonance plus vertical misalignment	Phase across direct drive coupling will either be in phase or ± 10° out of phase
		1*RPM H < .08 in/sec 1*RPM V > .18 in/sec	Vertical resonance plus horizontal misalignment	
	2*RPM, 3*RPM	1*RPM H > .15 in/sec 1*RPM V > .12 in/sec Low axial both bearings	Imbalance plus system non-linearity (looseness, cracks, excessive clearance)	- Consistent 1*RPM phase - Can also produce ½ order harmonics
		High axial on bearing	Bearing cocked on shaft	
	3*RPM, 4*RPM ½ orders		- Locked coupling - External side force	- Locked coupling is usually load-dependent - Asymmetric time signal may also be present

FREQUENCY DOMAIN ANALYSIS				
FREQUENCY	**HARMONICS**	**AMPLITUDES**	**PROBABLE CAUSE**	**COMMENTS**
1*RPM	2*RPM-4*RPM higher than 1*RPM		- Bearings cocked on shaft - Bearings not in same plane	Axial vibration on the bearings will be high
			Cavitation	- High frequency noise also present - Wide skirts
	Sub-harmonics from 5% to 35% of operating frequency		Recirculation	Pulsating and/or loud cracking noise in suction piping
# blades x RPM	No	H, V, A vibration > .15 in/sec	Acoustic resonance	May be caused by interaction of impeller vanes with volute or diffusion vanes
Approximately 40% of 1*RPM	No		Oil whirl	Appears as inside loop on shaft orbit
5-45% of 1*RPM	No	High level, random	- Volute/diffuser vane interaction - Excessive clearance between impeller and casing	Usually caused by operation away from best efficiency point
50% of 1*RPM	½ orders		- Rubs in wear ring area - Anti-friction bearings loose in housing	

SUMMARY

Pumps move fluid from one point to another by adding energy (not pressure) to the fluid. Since energy is defined as the ability to do work, adding energy to a fluid causes the fluid to do work: move through a pipe, fill a tank, or flow to a higher level.

The mechanical energy that is imparted to the fluid is independent of the fluid's density and is called the **pump head**. Pump head is measured in foot-pounds per pound of fluid.

Pump head is measured in feet, and the head generated by a pump operating at a given speed and capacity will be the same for all fluids. Therefore pump heads are expressed in feet of liquid. Head is the height of the fluid, while pressure is the force exerted by the fluid by virtue of its height.

The total head against which the pump must operate is called the **system head**. System head is made up of static head, friction head, and velocity head. The available head (total suction head - vapor pressure in feet) is called the "net positive suction head," and is abbreviated **NPSH**.

The velocity of the fluid must decrease when the tube's cross-section increases. Likewise, the velocity increases when the tube's cross-section decreases. The sum of a fluid's pressure, density, and velocity head are equal at any point along the flow of an ideal fluid. Therefore if a fluid's velocity increases, then the pressure drops; likewise, the pressure increases as the velocity decreases.

There are two main types of pumps:

1. Centrifugal pumps
2. Axial pumps

A centrifugal pump consists of a rotating element called an impeller that is enclosed in a casing. The impeller is mounted to a shaft that is connected to an external power source by a coupling. The impeller rotates at high speed within the casing, spinning the fluid being pumped along with it. Centrifugal force then forces the fluid out of the pump.

The flow of both suction and discharge fluid in an axial pump (also called a propeller pump) is parallel to the impeller's axis of rotation. Axial pump impellers resemble propellers or screws on a ship; hence the term "propeller pump." The vanes on an axial pump impeller are often an airfoil in cross-section, and run at close clearance to the pump casing.

Pump head has both pressure and velocity components. In order to get more useful work out of the pump, some of the velocity energy must be converted into pressure energy. This conversion is accomplished by one of two means:

1. A volute
2. Stationary diffusion vanes

A **volute** is an ever-widening spiral curve (like a snail shell). A volute casing is an ever-widening, spiral-shaped casing that surrounds the impeller. This increasing area helps to convert some of the velocity energy of the fluid into pressure energy.

Diffusion vanes are a set of stationary vanes that surround the pump impeller. Many years ago it was very difficult to cast and/or machine precise volute casings. Therefore, diffusion vanes were very popular. Today they offer little advantage over a volute pump; the major advantage of diffuser vanes is that they balance radial forces on the pump impeller.

REFERENCES

Kovats, Andreas von, Design and Performance of Centrifugal and Axial Flow Pumps, Copyright 1964, MacMillan Company, NY.

Karassik, Igor J. and Carter, Roy, Centrifugal pumps: Selection, Operation, and Maintenance, Copyright 1960, F. W. Dodge Corp., New York, NY.

Karassik, Igor J., Centrifugal Pump Clinic, Copyright 1981, Marcel Dekker, New York, NY.

Spannhake, Wilhelm, Centrifugal pumps, turbines and propellers: basic theory, Copyright 1934

Anderson, Harold Henry, Centrifugal pumps, Copyright 1962, Technical Press, Sutton, Surrey [Eng.]

Lobanoff, Val S., Centrifugal pumps : Design & Application, Copyright 1985, Gulf Publishing Co., Houston, TX.

Stepanoff, Alexey J., Centrifugal and Axial Flow Pumps: Theory, Design, and Application, Copyright 1957, Wiley and Sons, New York, NY.

10 MACHINE-SPECIFIC ANALYSIS: MOTORS

A large hospital in Dearborn, Michigan was experiencing very high vibration from a 400 HP motor generator (MG) set beneath the operating room. The vibration would come and go at random times and, when present, posed a serious threat to the success of any operation in progress.

The authors were contacted to examine the unit and to determine the cause of this random vibration. During the day of data collection, the problem only occurred once and the vibration only lasted about 6 seconds.

In that 6-second period, the vibration from the MG set was so severe that all of the file drawers in the records room next door opened up. The nurses had to go down the rows of file cabinets and push the drawers shut. They indicated that most of the time the vibration was not quite that bad; they only had to shut the drawers a couple of times per month.

Since the instrumentation had not been set up to collect such a short burst of high vibration, autoranging had not finished and no data was taken. Adjustments were made to capture the data, but the phenomenon did not reoccur that day.

The hospital administrator insisted on a preliminary report, and the authors pointed out that this was not possible. It was apparent, however, that the high vibration was coming from the motor. The authors suggested that the hospital contact the motor manufacturer, and ask them to repair the motor.

Two months later the motor manufacturer contacted the authors and informed them that the motor was not the cause of the vibration. The manufacturer complained that the authors had caused them to go to great expense in the removal and testing of the motor.

The authors visited the motor manufacturer and requested that the motor be installed in the dyno room and tested under load for 24 hours. During the test, the motor was fitted with transducers as well as a strobe light to watch the shaft vibration and movement. The vibration reoccured after three or four hours, at 120 Hz with a high noise floor. The strobe light indicated that the rotor was loose on the shaft. The rotor was welded to the motor shaft and the test was repeated. During the 24-hour test, the vibration did not reoccur, and the motor was returned to service.

INTRODUCTION

Tom C. Lloyd dedicated his book on electric motors "To the memory of three unsung martyrs:"

1. Thomas Davenport of Brandon, Vermont, who invented the electric motor (U.S. Patent #132, 1837) and died penniless as a result.
2. Emily Davenport, his wife, who gave up her silk wedding dress for insulation on the windings of the first electric motor.
3. Oliver Davenport, itinerant merchant, who sold his horse and wagon to finance his brother's experiments, thereby putting himself out of business.

This dedication is made all the more poignant when you realize how much industry, and our entire society, depends on electric motors. This chapter discusses electric motors, both AC and DC. Motors share many vibration problems with other types of rotating machinery: imbalance, misalignment, resonance, and bearing faults, to name a few. But motors also exhibit vibration problems resulting from the magnetic fields that exist between the rotor and stator.

Analysis techniques for mechanical sources of motor vibrations are discussed in previous chapters, and are reviewed here. This chapter also discusses some of the analysis techniques used for identifying and isolating electromagnetically-induced vibrations.

1.0 MAGNETS AND ELECTRICITY

You must understand the fundamental concepts of magnets, electromagnets, and induced current before we begin a discussion on motor vibration.

All magnets have a north and south pole. The names of the two poles refer to the effect of the Earth's magnetic field on a compass (the north magnetic pole causes a compass to point north). Like poles repel each other, and opposite poles attract, as shown in Figure 28. Both poles, however, will attract a magnetic metal that is not itself a magnet.

If you bring the north pole of one magnet near the north pole of another magnet, the two poles repel each other and push the magnets apart. Magnetically-levitated trains take advantage of this repulsion. Magnets in the track and the train are of like poles; as they repel each other the train is lifted off the track. The same repulsion occurs between two south poles.

If you bring the south pole of one magnet near the north pole of another magnet, then the two poles attract each other and bring the magnets together. These attractive and repulsive forces vary inversely with the square of the distance between the poles. That is, if the two magnets move from 1 unit to 4 units apart, the attractive (or repulsive) force between them becomes 16 times smaller. Likewise, a move from 4 units to 1 unit apart increases the force by a factor of 16.

Figure 28 hints at the operation of electric motors. Magnetic attraction and repulsion causes a magnet mounted on a central pivot to rotate around in a circle. This is the basic theory behind both AC and DC motors, as we will see in a moment.

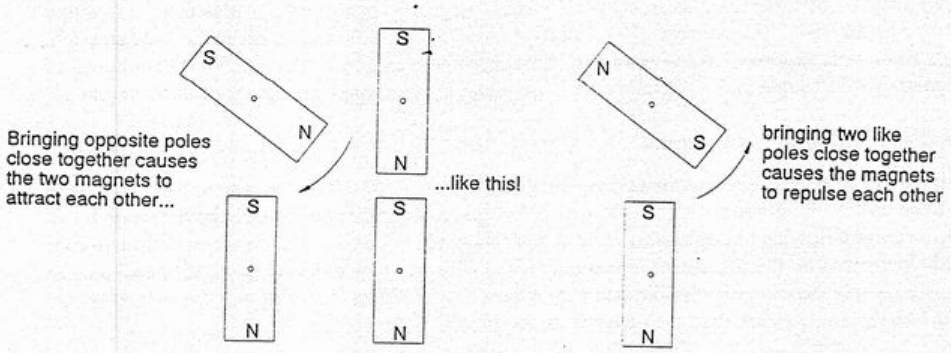


Figure 28 - *magnetic attraction and repulsion*

1.1 Magnetic Fields

A magnet is more than just a north and a south pole. All magnets create a **field**, which is an area in which magnetic force is strong enough to do work (to pull another magnet close or to push one away). A simple experiment allows you to "see" the magnetic field. Spread iron filings on a sheet of cardboard or glass, and place a bar-shaped magnet under the sheet. If you gently tap the sheet, the iron filings arrange themselves in curved paths. These curved paths are called **lines of magnetic force, or flux lines.**

The magnetic field consists of many flux lines. These flux lines are considered to come out of the magnet's north pole, loop through space, and reenter the magnet's south pole. A magnetic field is shown in Figure 29.

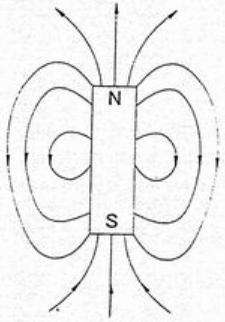


Figure 29 - *a magnetic field*

When two opposite magnetic poles are close to each other, the flux lines exit the north pole of one magnet and enter the south pole of the other. This draws the two magnets together, and creates a single magnetic field around both magnets.

Although we talk of magnetic flux lines leaving the north pole and entering the south pole, the flux lines are not really in motion. In fact, a flux line is an imaginary concept used to illustrate the structure of a magnetic field. The concept of flux lines is used, though, to measure magnetic field strength. Field strength is expressed as the number of flux lines per square inch. A common measurement of magnetic field strength is the kilogauss, which is equal to 1,000 lines per square centimeter (cm^2).

1.2 Electricity and Magnetism

In 1819 a Danish scientist named Hans Christian Oersted (1777-1851) discovered the relationship between electricity and magnetism. Oersted found that a compass needle could be deflected by an electric current flowing through a coil in the manner shown in Figure 30. The direction of the compass needle depended on the placement of the wire: if the wire was above the compass, the north pole of the needle was deflected in the direction of current flow. When the wire was placed below the compass, the needle pointed in the opposite direction.

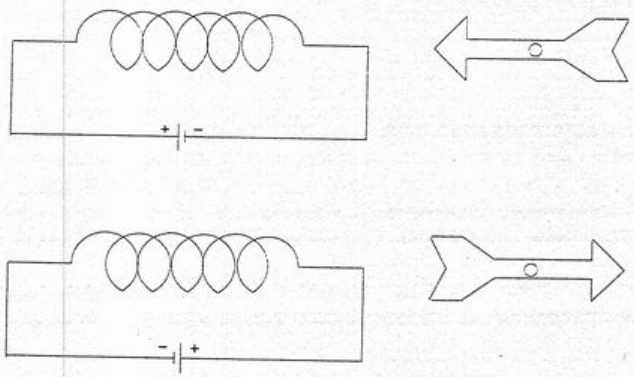


Figure 30 - *effect of electricity on a compass needle*

Oersted's experiment showed that current flowing through a wire creates a magnetic field at right angles to the wire. The "left hand rule" states that if the wire is grasped in the left hand with the thumb pointing in the direction of current flow in the wire, the fingers indicate the direction of the lines of magnetic flux around the wire. The "left hand" simply provides an easy-to-remember method for determining the direction of the magnetic field lines; **DO NOT GRASP ELECTRICAL WIRES WITH EITHER HAND!**

1.3 Electromagnets

An electromagnet is a soft-iron core surrounded by a coil of wire. When the wire leads are connected to a battery, a magnetic field is created and the iron core becomes a magnet. The strength of the magnetic field is equal to the strength of the current in amperes times the number of turns (loops) of the coil. A simple electromagnet is shown in Figure 31.

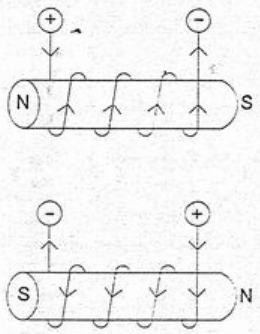


Figure 31 - *a basic electromagnet*

Since either pole of a magnet attracts a magnetic metal with equal strength, most electromagnets are bent into the familiar horseshoe shape so that both poles are used.

Note that, in Figure 31, the poles of the electromagnet reverse when the current flow is reversed. If an electromagnet is connected to an AC source in the U.S., the magnetic polarity (the direction of the north and south poles) will switch back and forth 60 times a second (or 3,600 times a minute). In other countries, with line frequencies of 50 Hz, the magnetic polarity will switch back and forth 50 times a second, or 3,000 times a minute.

1.4 Induced Current

If an electrical conductor moves through a magnetic field, then a current is induced in the conductor. The conductor must move in such a way that it cuts the lines of magnetic force in the field. For example, look at Figure 32. A coil of wire is wrapped around a magnet so that the coil is free to move up and down. A galvanometer connected to the ends of the coil measures the current flow in the coil.

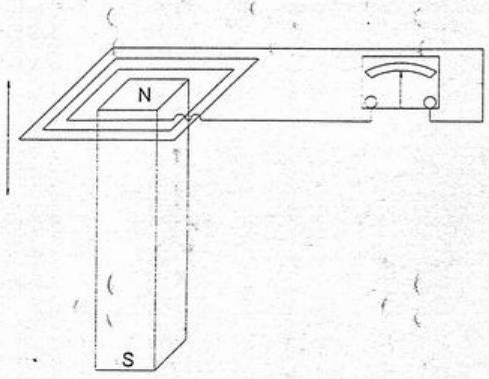


Figure 32 - *induced current*

If the coil remains stationary, then no current is induced and the galvanometer reads zero. Moving the coil down (toward the south pole in our example) causes a current to be induced, since the coil is cutting the magnetic field lines. This causes the needle on the galvanometer to deflect in one direction proportional to the induced current.

Moving the coil in the opposite direction (upward toward the north pole in Figure 32) also induces a current flow, but in the opposite direction. Consequently the galvanometer needle moves in the opposite direction.

The strength of the current flow is proportional to the speed at which the conductor moves through the magnetic field. The direction of flow is always perpendicular to the motion, and its magnetic action tends to resist the motion that produced the current. This is known as *Lenz's law*, named for the German scientist H. F. E. Lenz (1804-1864). The principle of Lenz's law, that current flow is proportional to the speed at which lines of magnetic lines of flux are cut, is the basis for the operation of the seismic velocity transducer.

2.0 PRINCIPLES OF AC MOTORS

The attraction between unlike magnetic poles (combined with the repulsion of like poles) illustrated in Figure 28 is responsible for rotation in AC motors. There are two common types of AC motors:

1. Synchronous motors
2. Induction motors

Both motors are constructed more or less the same, but behave in slightly different ways.

2.1 AC Motor Design and Operation

An AC motor has two major components, both of which are electromagnets:

1. The stator
2. The rotor

2.1.1 The Stator

The stator is made by placing wire wound in coils in the slots of an iron core. This core is securely mounted in the motor frame and remains stationary. A typical stator is shown in Figure 33.

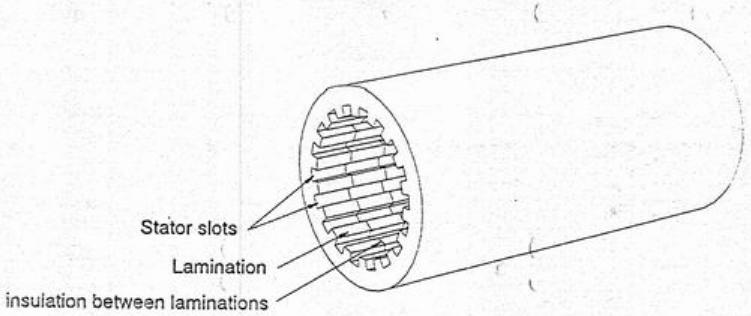


Figure 33 - *a typical motor stator*

The iron core is made up of a stack of thin iron plates, called **laminations**, which are insulated from each other in order to reduce eddy current losses (circular electrical currents that reduce a motor's efficiency). Normally there are several individual windings on the core that are energized from a source of alternating current.

The three-phase system has three electrical phases which are 120 degrees out of phase from each other. The alternating current, AC, changes polarity from positive to negative in a sinusoidal manner at the line frequency. In the United States this change in polarity at the line frequency normally occurs 60 times per second (line frequency in other countries changes polarity 50 times per second).

Alternating current in each of the three phases creates a magnetic field that rotates around the stator 3,600 times a minute divided by the number of pole pairs (in countries with 50 Hz line frequency, the magnetic field rotates around the stator 3,000 times a minute). To understand how the magnetic field rotates, refer to Figure 34, where a three phase, 2-pole, one pole pair, motor stator is illustrated.

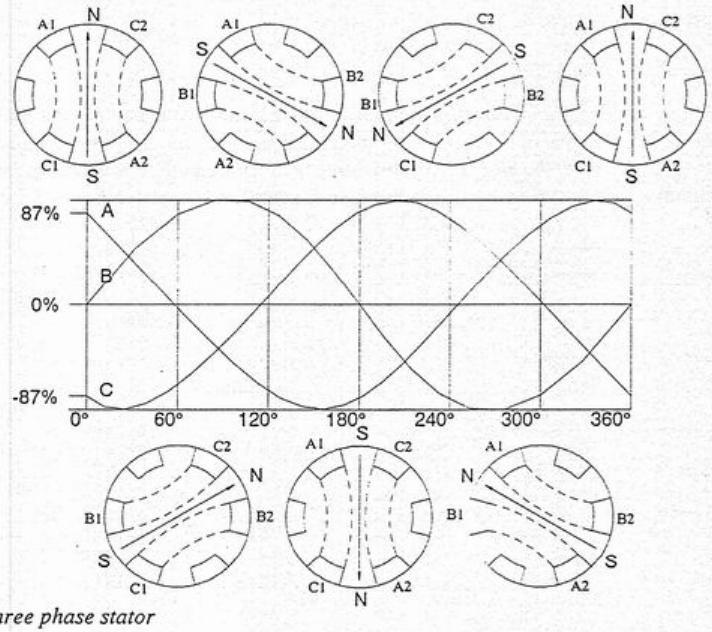


Figure 34 - *three phase stator*

Figure 34 shows three phases, labeled A, B, and C. The AC in each phase is shown in the middle of the figure (if you need an explanation of the Phase Angle shown on the X-axis of the diagram, refer to section 5.0 of the System Response chapter in Volume 1).

At phase angle 0, phase A is at positive 87% of maximum current, phase B is at 0, and phase C is at negative 87% of maximum. This results in a magnetic field whose poles are centered between phases A and C. Phase A has dropped to 0 sixty degrees later, while phase B has increased to positive 87%, and C is still at negative 87%. The poles of the magnetic field are now centered between phases C and B. The field continues to rotate around the stator, as shown in the rest of Figure 34.

The frequency (in Hertz) of the rotating magnetic field is equal to twice the line frequency (in Hertz) divided by the number of stator poles, as shown in Formula 10-1.

$$Fm = 2 * \frac{Fl}{Np} \qquad \text{(Formula 10-1)}$$

where

Fm = frequency of rotating magnetic field
Fl = line frequency (Hz)
Np = number of stator poles (that is, the total number of magnets)

For example, the frequency of the rotating magnetic field in a 4-pole motor energized at a line frequency of 60 Hz is

$F_m = 2 * 60 / 4$
= 30 Hz (1800 RPM)

2.1.2 The Rotor

The other electromagnet, called the **rotor**, is supported by bearings which center the rotor in the stator. The fundamental principal of the motor operation is magnetism, and the greatest resistance in the magnetic path of the motor is the air gap between the rotor and the stator. It is therefore important to have both a minimum and a uniform clearance between the rotor and the stator, as shown in Figure 35.

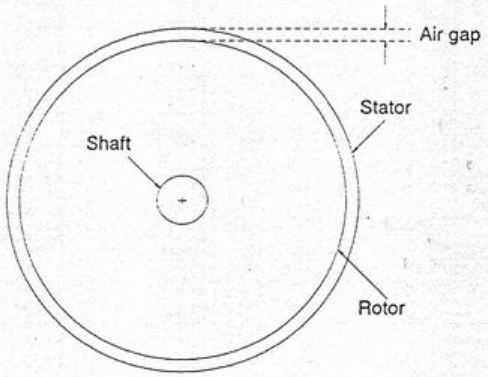


Figure 35 - *a rotor with a uniform air gap*

2.2 Synchronous Motors

If a fixed pole permanent rotor is placed in the rotating magnetic field of the stator, the rotor rotates at exactly the same frequency as that of the stator field. This is known as a **synchronous** motor.

A 2-pole synchronous motor runs at exactly 3,600 RPM with a line frequency of 60 Hertz; a 4-pole synchronous motor runs at exactly 1,800 RPM (at 60 Hz line frequency), as shown by Formula 10-1.

The motor runs at a fixed speed regardless of the load on the motor. If a synchronous motor is overloaded, it either stalls or slips a pole. When a synchronous motor slips a pole, it momentarily stops and grabs the next pole, creating a large jerk which can cause damage to both the motor and the equipment the motor is driving.

The rotating magnet of the synchronous motor can be a permanent magnet but is usually an electromagnet energized from an external direct current source through slip rings.

2.3 Induction Motors

The rotor in an induction motor is an electromagnet, but it is not energized from an external source. The rotor is made up of laminations similar to those in the stator, but in the place of the wound coils in stator slots, there are solid conductors. These conductors are called **rotor bars**, and they run from one end of the rotor to the other and are connected at each end by a shorting ring. Figure 36 illustrates a rotor from a typical induction motor.

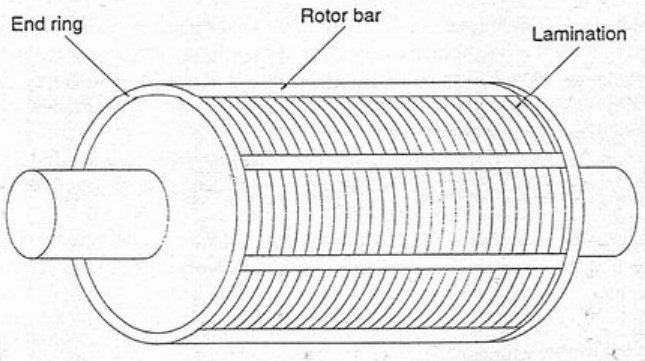


Figure 36 - *an induction motor rotor*

The rotor bars are parallel to each other and are electrically connected by two **end rings** (which are also conductors). If the rotor assembly is rotated within a magnetic field, a current flow in the rotor bars results.

Figure 37 illustrates that the current flow in the rotor is circular. As the rotor turns relative to the fixed poles of the magnetic field, the current flow through the rotor will reverse. In fact, the current flow reverses twice during one revolution of the rotor.

When a source of alternating current is first applied to the stator of the induction motor, the rotating magnetic field cuts one rotor bar as a north pole and the bar on the opposite side of the rotor as a south pole. The current induced in the rotor bars by this rotating field flows through one rotor bar, through the shorting ring, and in the opposite direction in the other rotor bar.

This circular flow of current in the rotor bars causes the rotor to become an electromagnet which then tries to keep up with the rotating magnetic field of the stator. As the rotor accelerates, the relative speed between the rotor and the rotating magnetic field of the stator decreases so that the rotor bar is cutting lines of flux at a slower rate. Therefore, the induced current in the rotor bars decreases.

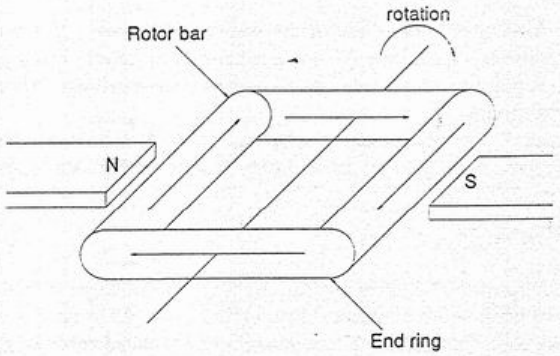


Figure 37 - *circular current flow in the rotor*

If the rotor reached the same frequency as that of the rotating magnetic field of the stator, no current would be induced in the rotor bars (since the bars would not be cutting any lines of magnetic flux). At this point the rotor would cease to be an electromagnet and would slow down. But by slowing down, the rotor bars would once again cut lines of flux. Current would be induced in the rotor bars, making the rotor an electromagnet once again.

Actually, the rotor frequency lags behind the frequency of the rotating magnetic field of the stator so that the torque generated by the induced current equals the torque necessary to drive the motor and the load.

Some induction motor rotors have windings in the rotor which are connected to variable resistors through slip rings instead of an end ring, so that the starting torque of the motor can be externally controlled.

3.0 PRINCIPLES OF DC MOTORS

Whereas magnetic attraction and repulsion create the rotation in AC motors, a magnetic phenomenon called **motor action** creates rotation in DC motors.

Current flowing through an electrical conductor creates a magnetic field around the conductor, at right angles to the current flow. What happens if this conductor is placed inside another magnetic field?

Look at Figure 38. An iron core with a large air gap is wrapped with a coil of wire. Direct current flow in the coil creates a magnetic field in the air gap. If a conductor (with its own magnetic field created by the flow of direct current) is placed inside the air gap, the two fields interfere with each other.

The flux lines of the magnetic field in the air gap "collide" with the flux lines of the conductor's magnetic field. This creates a force which acts against the conductor. If the conductor were free to move, the force would actually push the conductor out of the air gap. When the conductor reached the edge of the air gap, the force would reduce to zero and the conductor would stop moving.

The direction of the conductor's movement depends on the direction of the two magnetic fields. If we reverse the current in the coil (or in the conductor), the field lines reverse. This reversal changes the direction of the force, and moves the conductor in the opposite direction.

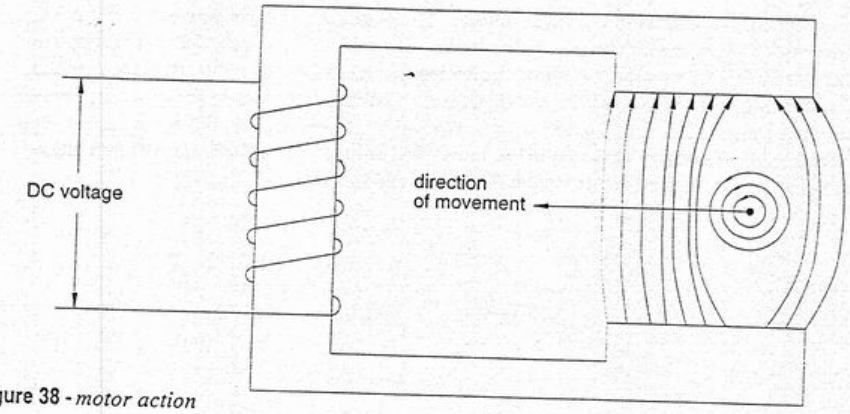


Figure 38 - *motor action*

This motion that results from the interference of two magnetic fields is called **motor action**. The force is proportional to the flux intensity and the current in the conductor. Motor action influences the size of a motor for a given horsepower and speed.

3.1 A Simple DC Motor

The illustration of motor action shown in Figure 38 produces motion along a straight line perpendicular to the air gap field. If the conductor were mounted on a movable shaft, and the shaft were placed inside a field, motor action would cause the shaft to rotate.

Figure 39 shows a simple DC motor. A single coil of wire is mounted on a shaft. The shaft/coil assembly (called an **armature**) is placed in a magnetic field between two fixed magnets. These fixed magnets can be either permanent magnets or electromagnets.

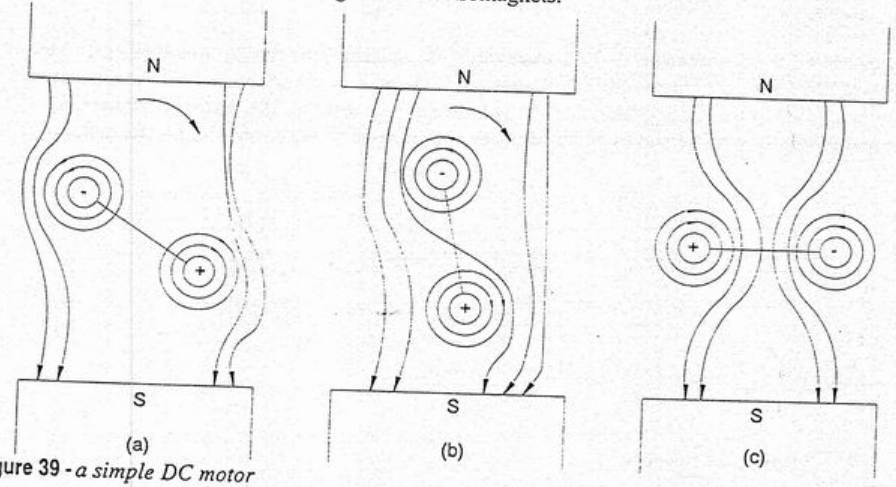


Figure 39 - *a simple DC motor*

Current is supplied to the armature through **brushes** (graphite rods that are connected to a power source and that push against the armature). A flux builds up around the coil once current is supplied. The armature flux interferes with the field flux, producing torque and causing the armature to rotate.

The torque that is generated varies with the armature's position, as shown in the graph in Figure 40. At position (b) the torque is at its maximum. However, at position (c), the force against the armature coil has reduced to zero. This simple DC motor will only rotate through one-half revolution, since no force is applied after position (c) when the torque reduces to zero.

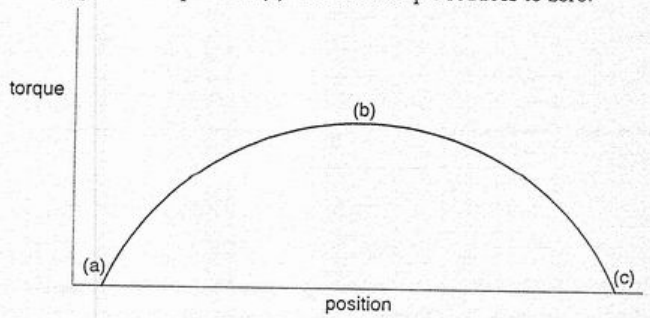


Figure 40 - *torque through one-half revolution*

Recall from the discussion of motor action that the direction of motion depends on the direction of the magnetic field lines. And the direction of current flow determines the direction of the magnetic field. Therefore, if we want this simple DC motor to rotate through one full revolution, we must reverse the current after one-half revolution (at position *c*). Reversing the current changes the direction of the magnetic field, creating an opposite force that carries the shaft around through one full revolution.

3.2 Commutation

A device called a **commutator** is used to reverse the current flow in the armature coil. A commutator is basically a split metal ring that is attached to the ends of the armature coil. An external DC source is applied to the commutator, and the split in the ring reverses the current flow halfway through a revolution. Figure 41 shows a two-segment commutator added to our simple DC motor.

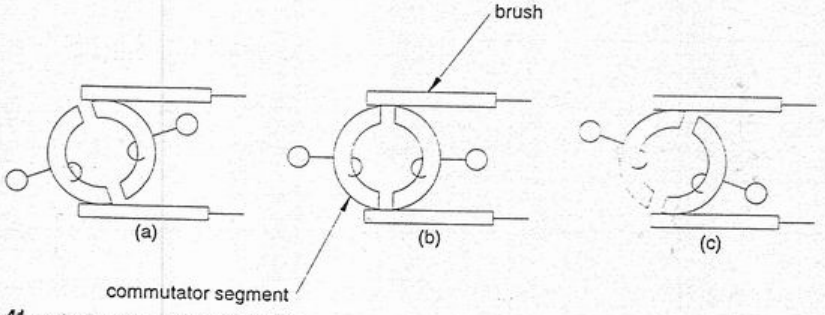


Figure 41 - *a two-segment commutator*

Notice how, in (a), the right commutator segment is in contact with the top brush and the left segment is in contact with the bottom brush. The segments are in contact with these brushes for 180 degrees of rotation.

There is a brief instant, shown at (b), where the brushes are in contact with both commutator segments. The segments are shorted out and there is no current flow through the coil. Consequently, no torque is generated. The momentum of the armature, however, quickly carries the shaft through this position.

Finally, at position (c), each commutator segment is again in contact with only one brush. Now the right commutator segment is in contact with the bottom brush, and the left segment is in contact with the top brush. This causes the flow of current in the coil to reverse, which changes the flux direction and applies torque through another 180 degrees of rotation. When the armature again reaches position (a), the current flow reverses again and a new rotation begins.

3.3 Torque Variation

The two-segment commutator shown in Figure 41 causes the shaft to rotate through 360 degrees. However, the torque varies with the position of the shaft. In the case of a two-segment commutator, the torque vs. position plot is shown in Figure 42.

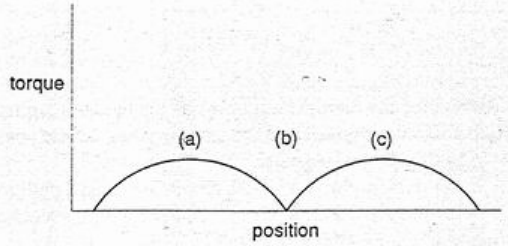


Figure 42 - *torque vs. position plot for a two-segment commutator*

Notice that the torque drops to zero twice during one revolution. Torque variations such as these would create extreme torsional vibration, and would be unacceptable. Torque fluctuations such as those in Figure 42 are minimized by doubling the number of coils and connecting them to additional commutator segments. This design is shown in Figure 43.

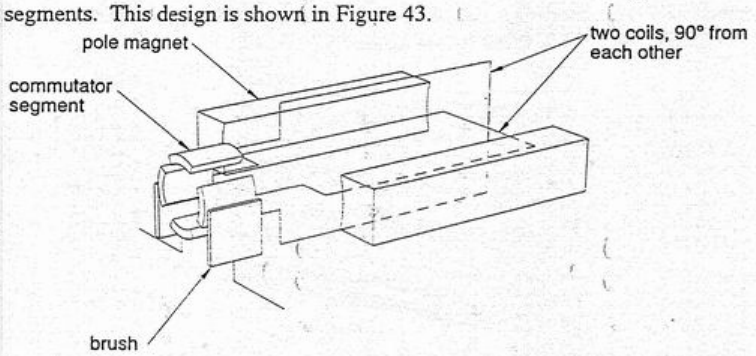


Figure 43 - *a four-segment commutator*

This motor has two coils, placed 90 degrees from each other. Their ends are connected to each segment of a four-part commutator. When one coil is in between the magnet poles (at right angle to the field), the torque generated is zero since its commutator segments are not in contact with any brushes. However, the other coil is in-line with the field, and is in contact with the brushes.

Each coil has a torque curve similar to the one shown in Figure 42, with torque dropping to zero twice during each revolution. However, since the two coils are 90 degrees apart, their torque curves are separated by 90 degrees. As one curve drops to zero, the other curve is at its maximum. Thus the torque never drops to zero, and the output is much smoother.

Torque variations are smoothed out even further by placing many coils on the armature. The motor action force is applied to many coils at once, smoothing out the torque curve and adding to the amount of torque that the motor produces. But each new coil that is added to the armature requires two commutator segments, so that the current can be reversed in each coil.

3.4 Design of DC Motors

A DC motor consists of three parts:

1. A stationary member called a **frame** (also called a **yoke**)
2. A rotating **armature** that carries multiple coils
3. A **commutator**

3.4.1 The Frame

The frame of a DC motor provides mechanical support for the electromagnets that create the magnetic field. In addition to the electromagnets, the frame supports the field windings, which are long loops of wire used to energize the electromagnets.

Besides providing mechanical support for the motor, the frame provides a path for the completion of the magnetic circuit. The field windings are wound so that opposing electromagnets become opposing magnetic poles. When the windings are energized and a field is created, the lines of flux leave the north pole, enter the south pole, and return to the north pole through the motor frame.

Motors are often described by the number of magnetic poles mounted in the frame. A four-pole motor (very common) has four electromagnets mounted in the frame: two north and two south.

3.4.2 The Armature

The armature of a DC motor is a series of toothed iron sheets. When several sheets are laminated together and mounted on a shaft, the armature assembly looks like Figure 44.

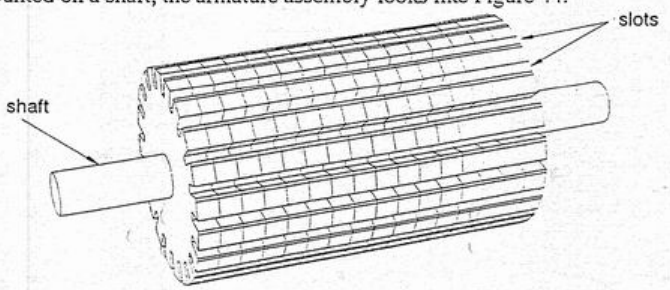


Figure 44 - *a DC motor armature*

The spaces between the teeth form long slots in the armature, and the coils fit inside these slots.

As the armature rotates in the magnetic field, the iron core cuts the lines of magnetic flux the same time as the armature coils. If the armature were made from a solid piece of iron, a circular current called an **eddy current** would be induced inside the armature. Eddy currents reduce the amount of energy available for motor action, and they cause excessive heating of the armature assembly.

Building the armature out of thin sheets reduces the eddy current losses. Since resistance to current flow varies inversely with cross section, the resistance rises as each sheet gets thinner. The sheets are insulated from each other by a layer of mica.

3.4.3 The Commutator

Commutators are made of wedge-shaped segments of copper called **commutator bars**. The commutator bars are held together by clamping rings, and are insulated from each other by thin strips of mica. Since the commutator bars are wedge-shaped, they form a circle when they are placed together. See Figure 45.

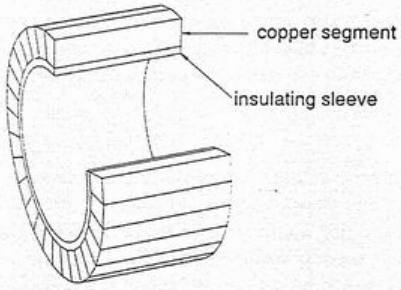


Figure 45 - *a typical commutator*

The commutator is keyed to the motor shaft, and the ends of each of the armature coils are soldered to two of the commutator bars. Depending on how the coils are wound, they can either be soldered to adjacent bars (called **lap winding**) or to opposing bars (called **wave winding**). The difference between the two is that lap winding requires one brush for each pole of the motor, while wave winding only requires half as many brushes as there are poles.

Since each armature coil is attached to two commutator bars, the current reverses direction in each coil once per revolution.

3.5 Brushes

Brushes are electrical conductors that are spring-loaded against the commutator, providing current to the armature coils. Brushes are usually made of graphite, copper, carbon, or some mixture of these materials. A typical DC motor brush is shown in Figure 46.

The brush holder is rigidly mounted in the motor frame, and holds the brush against the commutator. When the motor is operating at full load current, the current flow actually pulls the brush against the commutator. At lower current, the current flow is insufficient to pull the brush against the commutator. Therefore, a tension spring is used to maintain good contact with the commutator.

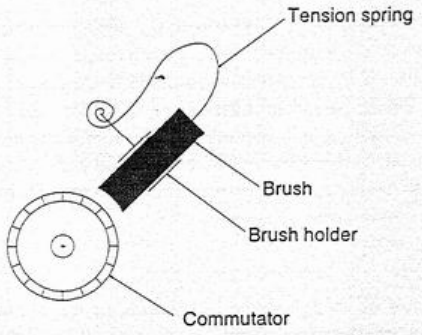


Figure 46 - *brush assembly on a DC motor*

Over time the brush wears out, due to the constant friction between its tip and the commutator. The brush tension spring ensures that the brush is always held tightly against the commutator; as the brush grows shorter, the spring continues to push it down against the commutator.

3.5.1 Brush sparking

No commutator is perfectly smooth. Irregularities in the surface of the commutator segments, eccentricity in the commutator circumference, imbalance and/or misalignment in the shaft, and **high mica** (mica insulation protruding above the surface of the commutator) are all common conditions. So even though the brush tension spring pushes the brush down toward the commutator, these irregularities cause the brushes to bounce up and down as the shaft rotates. Excessive rotor vibration adds to the commutator irregularity, causing the brushes to bounce even more.

Sparking occurs when the brush has momentarily bounced away from the surface of the commutator, leaving a small air gap. The current creates a small spark as it jumps to the commutator. Very little sparking occurs when the brush is in firm contact with the commutator.

In addition to commutator eccentricity, improper brush installation can cause sparking. If the brush holder is too tight, then the brush is not free to move down against the commutator. An old or worn spring does not push the brush against the commutator hard enough under low-current loads.

DC motors that operate in dirty or contaminated atmospheres are prone to sparking. Unless the motor is sealed, impurities in the air coat the commutator. This film has a high resistance, and causes excessive sparking. In some cases, airborne impurities entering a DC motor have caused a direct short between the brushes and ground.

Brush sparking causes extreme wear on the commutator and the brushes, and leads to premature failure of a DC motor. Fortunately, the bouncing of the brushes creates a vibration frequency that can be detected in the frequency domain. See page 86 for more information.

3.6 Armature reaction and brush sparking

It is inevitable that, at some point during the shaft's revolution, a brush will be in contact with two commutator segments. Since the brush material is an electrical conductor, the brush will short out the connected coils while it is touching two commutator segments.

Remember from Section 1.4 that a current is induced in a conductor when the conductor moves through a magnetic field. If any current is present in the coils that are shorted out, a spark will occur at the brush.

Correct brush placement minimizes sparking. Figure 47 illustrates the flux lines from the main field only (there is no magnetic field from the armature coils). The line AB represents the field's **neutral plane** - the plane where no current is induced in the armature coils. The instant a pair of coils passes through this plane, the coils are moving parallel to the flux lines of the main field. Consequently, no current is induced in the coils at this point. If the brushes are placed in the neutral plane, no sparking will occur.

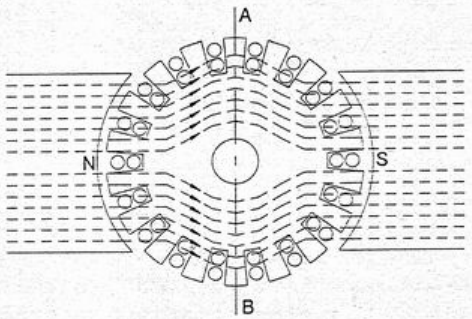


Figure 47 - *main magnetic field*

However, the flux generated around the armature coils distorts the main field, and consequently affects the position of the neutral plane. Figure 48 illustrates the flux lines for the magnetic field generated around the armature coils.

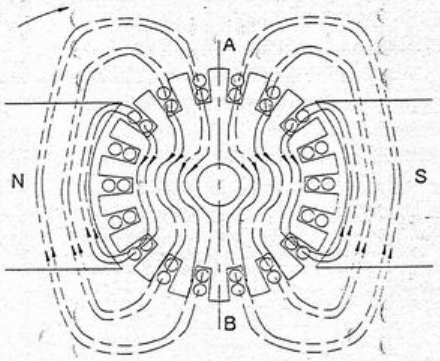


Figure 48 - *magnetic field around the armature coil*

These two fields distort each other, causing the flux lines to twist in the direction of rotation of the armature. This distortion is called **armature reaction**, and shifts the neutral plane to position A'B' (plane A'B' is often called the **load neutral plane**). Plane A'B' is shown in Figure 49.

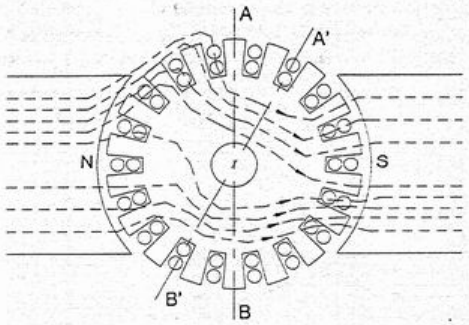


Figure 49 - *the load neutral plane*

If the brushes were left at plane AB, they would prevent sparking only until the motor reached normal load. Then, when the neutral plane shifted to plane A'B', the brushes would spark. Therefore, the brushes should be placed at plane A'B' to prevent sparking during normal load operation. If the brushes are not placed in the load neutral plane, they will spark even if they are pushed against the commutator. Some large DC motors have hardware that moves the brushes in response to changing loads, so that they are always in (or close to) the load neutral plane.

3.7 Interpoles

A DC motor often runs under conditions of changing load. The current in the armature coils varies as the load changes, and this change in current affects the position of the load neutral plane (plane A'B' from Figure 49). Consequently, even though the brushes may be positioned correctly, changes in load will move the neutral plane and sparking will still occur.

Large DC motors often have smaller electromagnets located between the main poles. Called **interpoles** (or **commutating poles** or **compoles**), these smaller magnets greatly reduce the effect of the main magnetic field near the location of the load neutral plane. As long as the load neutral plane falls within the "dead zone" created by the interpoles, very little current will be induced in the armature coils and very little sparking will take place at the brushes.

Interpoles are also used in DC motors that must reverse direction. A change in direction shifts the load neutral plane back in the direction of rotation, which necessitates repositioning the brushes to minimize sparking. The interpoles ensure that, no matter what direction the shaft is rotating, the motor is commutating in the neutral plane.

3.8 Types of DC Motors

There are three main types of DC motors:

1. Shunt wound

2. Series wound
3. Compound wound

3.8.1 Shunt Wound

A **shunt** motor has the field winding connected in parallel with the armature coils. Wiring across the armature coils in this manner is called a *shunt*.

The advantage of a shunt motor is that the speed of the motor is almost constant within the motor's load range. Since the field winding is across the voltage source, the number of flux lines in the field remains essentially constant. Shunt motors develop greater than full load torque on startup, providing that the switching equipment can handle the current.

If a shunt motor is subjected to a sudden load, the armature reaction discussed on page 74 weakens the magnetic field. Since the strength of the magnetic field affects the motor speed, a weakening of the field causes the motor to "run away" (increase in speed without restraint until the motor fails). To understand the cause of motor run away, we first need to examine the factors that influence the operating speed of a DC motor.

The operating speed of a DC motor is directly proportional to the effective voltage and inversely proportionally to the field strength (magnetic flux). Effective voltage is equal to the motor terminal voltage minus the armature losses. The basic equation is:

$$S = \frac{V - Ia * Ra}{K * B} \qquad \text{(Formula 10-2)}$$

where

V = Terminal voltage
Ia = Armature current
Ra = Armature resistance
K = Manufacturing constant of design
B = Magnetic field strength

Formula 10-2 shows that as the magnetic field strength B goes to zero the speed S goes to infinity (since you are now dividing by zero). Therefore, weakening of the field may cause the motor to increase in speed continuously (called **run away**). Opening the shunt circuit will definitely cause the motor to run away.

Most shunt motors have stabilizing winding added in series with the armature. This stabilizing winding usually prevents run away when the shunt field is opened or weakened. However, a shunt motor with stabilizing windings is not as effective as a compound wound motor.

3.8.2 Series Wound

A **series** motor has the field winding connected in series with the armature coil. This type of wiring varies the flux proportionally with the armature current. The torque of a series motor varies as the *square* of the armature current; that is, the torque is proportional to the armature current squared.

The speed of a series motor varies inversely with the current. Rotor speed also varies inversely with load; as the load decreases, the speed increases. Therefore some load must always be kept on a series motor to prevent it from running away. Series motors should never be used in applications where the load can go to zero, even for a moment (such as belt drives).

3.8.3 Compound Wound

A **compound** motor has the field connected in parallel to *and* in series with the armature coil. This compound wiring prevents run away if the load drops to zero (since a shunt winding is present), and prevents run away if the shunt field is opened (since the series winding is present).

4.0 ELECTRICAL SOURCES OF VIBRATION

Both AC and DC motors can excite system resonances at shaft frequency and harmonics. Each motor also has unique electrical sources of vibration.

4.1 Electrical Sources of Vibration in AC Motors

Electrical sources of vibration in AC motors fall into four major categories:

1. Line frequency
2. Slip frequency
3. Slot pass frequency
4. Electrical discharges

4.1.1 Line frequency

The vibration forcing function frequencies associated with the line frequency of an AC motor are:

Fl = Line frequency + harmonics

Fe = Electrical magnetic frequencies + harmonics
$= 2 * Fl$

Fm = Rotating magnetic field frequency + harmonics
$= 2 * Fl / number\ of\ poles$

Fr = Rotor frequency + harmonics

4.1.2 Slip frequency

The difference between the frequency of the rotating magnetic field of the stator and the rotational frequency of the rotor is called the **slip frequency**. Slip frequency is found by the formula:

$$Fs = Fm - Fr$$

where

Fs = Slip frequency
Fm = Rotating magnetic field frequency
Fr = Rotor frequency

Slip frequency has an effect on certain types of motor vibrations. To understand this, consider a 2-pole motor with a rotor speed of 3,420 RPM. The magnetic field in this motor's stator rotates at 3,600 RPM.

The slip frequency for this motor is equal to 180 RPM (since 3,600 - 3,420 = 180). A slip frequency of 180 CPM rotates once for every 20 revolutions of the stator magnetic field (180 x 20 = 3,600); this

is called a slip cycle. Now look at Figure 50, which illustrates a random spot on the rotor as the stator field goes through one slip cycle (20 revolutions).

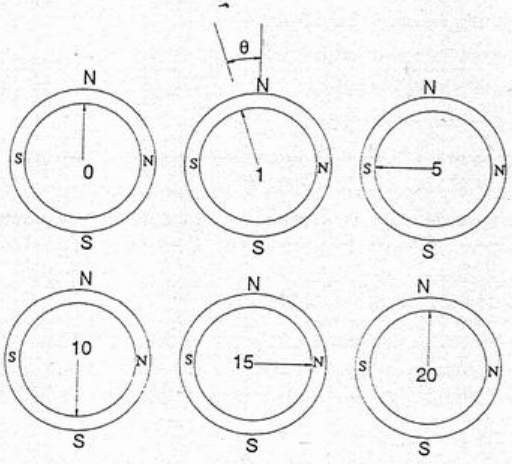


Figure 50 - *one slip cycle*

At the beginning, the spot on the rotor is lined up with the stator's north pole. After one revolution of the stator field, the spot on the rotor lags the stator by 1/20 of a revolution, or 18 degrees (since 360/20 equals 18). After five revolutions, the spot is in between the north and the south poles.

Five revolutions later, the spot lines up with the stator's south pole. After five more revolutions the spot is between poles once again. Finally, after 20 revolutions, the spot is once again lined up with the stator's north pole.

The rotor in a 2-pole motor lines up with a stator pole twice during each slip cycle. Therefore, a rotor bar is subjected to maximum current flow twice for each revolution of slip. Any magnetic problems in the rotor of a 2-pole motor, then, will occur at twice the slip frequency.

We can rephrase the previous paragraph to include motors with more than 2 poles: *A rotor bar will line up with a magnetic pole in the stator at a frequency equal to the number of poles times the slip frequency.* Therefore the maximum current flows in the rotor bar and the rotor is subject to the maximum torque at this frequency.

Since the torque reaches a maximum at this frequency, the rotor will speed up and slow down at a rate equal to the number of poles times the slip frequency. That is, the rotor frequency will be modulated at a rate equal to the number of poles times the slip frequency.

4.1.3 Slot Pass Frequency

In addition to the line frequencies and their harmonics, there are also vibration forcing functions caused by the rotor magnetic field passing the slots in the stator:

Fc = Core slot pass frequency
= *Fr* * *number of stator slots*

Fb = *Fr* * *number of rotor bars*

4.1.4 Electrical Discharges

Electrical discharges can occur in AC motors and are generally of one of the following types:

1. Partial discharge within the stator bar insulation
2. Slot discharge between the stator bar insulation and the stator core
3. Surface discharge over the end winding
4. Discharge between broken conductors
5. Constant surface discharges that ionize surrounding gas (called **corona**)

Since these discharges usually generate very high frequencies, it is normally not possible to detect them in frequency domain spectral analysis. Depending on the fault, it is sometimes possible to see the discharge in the time domain; however, they are best detected using a high-frequency oscilloscope.

4.2 Electrical Sources of Vibration in DC Motors

Creating torque in a DC motor is inherently a "gentler" process than in an AC motor. DC speed control systems bring a DC motor up to speed slowly, whereas AC motors are immediately subjected to full current at startup. The major electrical source of vibration in DC motors is from AC rectification.

4.2.1 AC Rectification

In many cases, the direct current needed to drive a DC motor is not supplied from a DC generator. Since alternating current is usually available, a process called **rectification** is used to convert the AC to DC. When AC is rectified, the negative "humps" are removed, leaving only positive current as shown in Figure 51. This is called **half-wave** rectification, since only half of the wave remains.

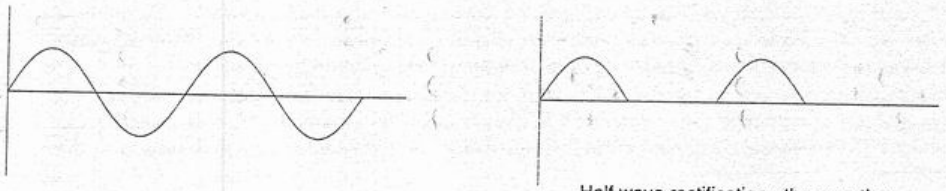


Figure 51 - *half wave rectification*

Half-wave rectification leaves gaps in the current. Devices called **full-wave** rectifiers convert the negative current to positive, so that the average current flow is roughly ⅔ of the maximum current. This is shown in Figure 52. Rectifying three-phase AC, where each phase is offset by 90 degrees, smooths out the ripples and leaves roughly constant current (DC).

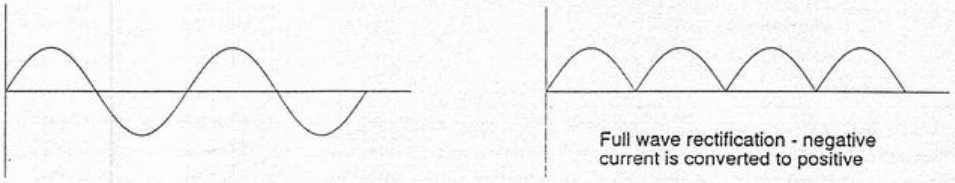


Figure 52 - *full wave rectification*

A common full-wave rectifier is the **silicon-controlled rectifier** or **SCR**. Electrically, an SCR is like three diodes connected in series. When a voltage is applied across the SCR, the center semiconductor junction does not allow current flow. In this state the SCR is said to be switched off. If the voltage is increased, a point is reached where the center junction breaks down and begins to conduct; now the SCR is said to be switched on (or to be **firing**). The SCR is then disconnected so that the junction restores itself and again refuses to allow current flow.

Full-wave rectification on single-phase AC, as shown in Figure 52, leaves 2 peaks per cycle (one for each time the SCR fires). If three-phase current is rectified, 6 peaks per cycle are now present. Since 60 Hz is normal AC line frequency in the U.S., 360 peaks per cycle are present in the rectified DC output of an SCR. In countries with 50 Hz line frequency, the rectified DC output has 300 peaks per cycle.

These peaks cause a pulsation in the DC that supplies the motor's main field. Consequently, the motor's speed varies by the same pulsation frequency. This SCR firing frequency is a source of vibration, both from exciting components to resonance and from the speed variations (torsional vibration) that it introduces into the driven process.

4.3 External Sources of Vibration

Although most vibration problems associated with electrical systems are motor-related, there are other sources which should not be overlooked. Vibration can be caused by some of the following:

- the dress of the conductors in the raceway
- loose lamination in power transformers
- unbalanced phase currents
- high current pulses from welders or solenoids

5.0 MECHANICAL SOURCES OF VIBRATION

Many mechanical forcing functions discussed in previous chapters can also occur in electric motors:

1. Imbalance
2. Thermal bow in the rotor
3. Shaft or stator resonances
4. Misalignment, both mechanical and electrical
5. Defective bearings
6. Looseness
7. Rubs

When analyzing motor vibration problems, you must determine at the outset whether the problem is mechanical or electrical. However, there is not always a clear-cut separation between the two. For example, a cracked rotor bar may cause spot heating in the rotor, resulting in a thermal bow. The rotor bow will appear as rotor imbalance (mechanical), when in fact the basic problem is the rotor bar (electrical).

Another example can be seen in a beat frequency in the motor (where the amplitude increases and decreases as the result of addition of two different frequencies). The frequencies that result from the sum and difference of these two frequencies are called **sidebands**. A beat usually indicates a magnetic problem which is electrical. However, another machine running close to the same speed or even a belt in the drive may be the cause of the beat; in this case, it's a mechanical problem.

Since a beat in the motor can be an indication of an electrical problem, it is very important to determine whether the beat is due to any other vibration source close to the running frequency of the motor. If the vibration disappears when the motor is shut off, then the beat has an electrical source. However, if the beat continues during coastdown (after the motor has been shut off), then the vibration has a mechanical source.

Once other sources of vibration have been eliminated, the beat can be checked by comparing a peak hold spectra with an average spectra, or by observing the maximum and minimum points on an instantaneous spectra. The difference between the peak hold spectra and the average spectra is an indication of the electrical, or magnetic, level, and the average spectra is an indication of the mechanical level.

It may be possible to detect the presence of electrical problems by taking data during a shutdown. The power should be cut near full load, if possible, and data taken during the coastdown. If you use a spectrum analyzer, take data in the time domain at a rate that ensures adequate information at the trip point. This normally requires a very large input buffer.

The difference in vibration just before trip and just after trip is a good indication of the electrical plus mechanical vibration versus the mechanical vibration. If the motor is shut down under load, it may stop in such a short time that you may not be able to distinguish between the electrical and the mechanical vibration. In addition, a resonance close to the running speed of the motor will distort the data.

Rotor Loose on Shaft

A combination mechanical-electrical problem (mentioned in the introduction to this chapter) occurs when the rotor is not tight on the rotor shaft. High, broadband vibration is present when the rotor moves on the shaft, and is not present when the rotor frets and seizes.

Changes in the load or current can cause the rotor to break free and the vibration reoccurs. The pattern is usually not periodic and the duration of the vibration may be only a few seconds, with several hours between occurrences.

Thermal Growth

Run the motor at no load and check for rotor balance. Then record amplitude and phase readings at rotor frequency as the motor is brought up to full load and temperature, to determine the presence and magnitude of a thermal vector (a vibratory force brought about as a result of uneven heating in the motor). The presence of a thermal vector is a good indication of uneven heating in the rotor, which may be caused by a rub, shorted lamination, or one or more broken rotor bars.

Uneven Air Gap

If the air gap between the rotor and the stator is uniform, the magnetic forces acting on the rotor are balanced. A uniform air gap is especially important for induction motors, since the air gap affects the amount of induced current in the rotor. However, since the magnetic force on the rotor varies directly as the square of the stator current and inversely as the square of the air gap, an uneven air gap can result in very high imbalanced magnetic forces.

Check the rotor to stator gap at eight equally-spaced locations at each end of the rotor to ensure concentricity, and also check for an out-of-round condition or a rotor bow. Also, check the stator for run-out and coil resistance, and check the motor for frame twist by checking for a soft foot. A soft foot is a poorly shimmed foot on the motor that, if tightened, deforms the motor enough to cause stator eccentricity.

The vibration caused by a rotor that is not centered in the stator occurs at a frequency equal to twice the line frequency, regardless of the number of stator poles. The vibration level increases with load and disappears the instant the power is shut off.

Vibration readings at 2**line frequency* should be taken in a circular pattern around the frame of the motor at locations which are not equally spaced and not spaced in increments divisible by three (since most motors are three-phase). If the vibration levels at each location are plotted with respect to their location around the motor, the maximum vibration in the absence of a resonance will occur where the rotor is closest to the stator, since the force between the stator and the rotor is greatest at this point.

Out of Round Rotor

Even though the rotor is centered in the stator it may not be perfectly round. In this case, the vibration is at the rotor frequency and beats (i.e. increases and decreases in amplitude) at a frequency equal to the *number of poles * the slip frequency*. Even though an increase in load on the motor increases the slip frequency, it does not have any effect on the vibration, including the beat, caused by an out-of-round rotor.

As in the case of the non-uniform air gap, the vibration caused by an out-of-round rotor disappears the instant the power is cut. Non-uniformities in the air gap also result in vibration at the rotor bar pass frequency F_b with side bands at twice line frequency and at rotor frequency.

Stator Looseness

Vibration at twice the line frequency, regardless of the number of poles, may also be caused by a stator loose in the motor frame, or by loose lamination or coils in the stator. This vibration disappears as soon as the power is cut.

Rotor Bars

Since one of the causes of motor failure (other than bearings) is related to the rotor bars, it is important to detect rotor bar faults at an early stage of development. This is especially important for motors larger than 250 HP which are subjected to many starts under load.

When the motor is started, especially under load and across the line, high currents flow in the rotor bars, which results in very high rotor bar stress. Given enough starts, rotor bar problems will occur.

This type of motor failure usually progresses as follows:

1. A rotor bar cracks because of repeated high current stress.
2. Spot heating occurs at the crack which may cause a rotor bow. This looks like imbalance, so the motor may be rebalanced rather than analyzed for rotor fault.
3. The bar breaks and arcing occurs. This causes additional heating and rotor bow. Even though the motor is rebalanced, the rotor may eventually rub against the stator.
4. The adjacent bars carry more current so that they are subjected to even higher thermal and mechanical stresses.
5. Rotor lamination are damaged, leading to motor failure.

As we saw in Section 4.1.2, the difference in frequency between the running speed of an induction motor and the synchronous magnetic frequency of the stator is known as the slip frequency, and can be calculated as:

$$Sl = \left(2 * \frac{Fl}{P}\right) - Fr \qquad \text{(Formula 10-3)}$$

where

Sl = Slip frequency in Hz
Fl = Line frequency in Hz
P = Number of poles in the motor
Fr = Rotational frequency in Hz (RPM/60)

On the other hand, electrical engineers usually calculate the per unit slip as a unit-less number as

$$s = 1 - \left(\frac{Fr}{Fs}\right) \qquad \text{(Formula 10-4)}$$

where

s = Per unit slip
Fr = Rotational frequency in Hz (RPM/60)
Fs = Synchronous magnetic frequency in Hz ($2 * {}^{Fl}/_{P}$)
Fl = Line frequency
P = Number of poles in the motor

When a broken rotor bar is present in a motor, then torque, magnetic forces, and rotor frequency are all modulated. The change in current pattern at the broken bar produces harmonic fluxes which induce current in the stator winding at line frequency harmonics. There are also additional vibratory

peaks called **sidebands** that are separated from the line frequency by plus and minus the number of poles times the slip frequency.

If *s* (per unit slip) is used, the sidebands are at ± 2*s* times the line frequency and its harmonics. For example

Line frequency = 60 Hz
Number of poles = 4
Magnetic frequency = 1800 CPM (30 Hz)
Rotor frequency = 1740 RPM (29 Hz)
Per unit slip *s* = 1 - 1740/1800 (.0333)

Side bands are at the number of poles times (30 - 29) = 4 Hz, so that at 60 Hz the upper and lower side bands occurs at 56 and 64 Hz. Calculated using per unit slip:

sidebands = 60 Hz ± (2 * .0333 * 60)
= 56 and 64

Since the broken rotor bar produces changes in these different parameters, there are several methods you can use to determine the presence, as well as the number, of broken bars:

1. Spectral analysis of the supply current, using a current transformer on one of the supply lines
2. Spectral analysis of the stator core vibration at slot pass frequency
3. Analysis of the axial flux from a coil around the motor shaft
4. Analysis of the motor speed variations using synchronous time averaging at slip frequency

One or more broken rotor bars will cause the motor to vibrate in a manner very much like the vibration caused by an out-of-round rotor. The significant difference is that the amplitude of the beat frequency at the number of poles times the slip frequency is proportional to the load. In fact, the beat is not there when the motor is running solo, whereas it will be there with an out-of-round rotor.

Since the vibration level caused by broken rotors bars may be quite low, spectral analysis of the supply current using a clip-on current transformer on one or more of the supply lines can be a simpler way to detect the presence of broken rotor bars. No transducers need to be mounted to the motor and, since the current can be analyzed at the switchgear room, you can sometimes check several motors at one location.

In addition, research has shown that current analysis can provide data reliable enough to indicate the presence of one or more broken rotor bars as well as an estimate of the number of such bars.

The general equation for the frequencies present in the air gap flux or in the current spectra can be written as

$$Fn = N * Fl \qquad \text{(Formula 10-5)}$$

where

Fn = Harmonic frequency
N = Harmonic index
Fl = Line frequency

The harmonic frequency *Fn* will contain sidebands at

$$Fns = Fl * \left(N * (1 - s) \pm s\right) \qquad \text{(Formula 10-6)}$$

where

Fns = Harmonic frequency sidebands
Fl = Line frequency
N = Harmonic index
s = Per unit slip

In theory, only the frequencies where *N* is equal to an odd number not divisible by the number of line current phases appear in the current spectra. For example, on a three-phase motor, frequencies where *N* equals 1, 5, 7, 11, and so on, would be present. Frequencies where *N* equals 3 and 9, for example, would not be present, since those numbers are divisible by 3. Single- and two-phase motors are generally so small that they do not have broken rotor bars, so this calculation is usually only carried out on three-phase motors.

When stator winding configuration and manufacturing asymmetries are taken into account, the spectral components in the current that are of most value in the detection of broken rotor bars are

$$F(1) = Fl$$
$$F(2) = P * Fr - Fl$$
$$F(3) = Fl + 2 * P * Fr$$
$$F(4) = Fl + 3 * P * Fr$$
$$F(5) = Fl + 5 * P * Fr$$
$$F(6) = Fl + 6 * P * Fr$$

where

F( ) = Current harmonic [that is, *F(1)* is harmonic number 1]
Fl = Line frequency
P = Number of poles in the motor
Fr = Rotor frequency in Hz (RPM/60)

If the motor is loaded at 90% or higher of rated load and the *F(2)* spectral component is 55 dB lower than the fundamental *F(1)*, there are probably no broken or cracked rotor bars. On the other hand, if the *F(2)* component is less than 46 dB lower than *F(1)*, there are rotor bar problems. Levels between 46 dB down and 55 dB down from *F(1)* are motors with possible rotor bar problems.

To increase confidence in the analysis, check the levels at *F(3)* through *F(6)*. If these frequencies are present in the current spectra and their levels are higher than 60 dB down from *F(1)*, there is a certain rotor bar problem.

Once you identify a broken rotor bar problem, you can calculate the estimated number of broken bars from the following formula:

$$N = \frac{(4 * R)}{(D + P)} \qquad \text{(Formula 10-7)}$$

where

N = Estimated number of broken rotor bars
R = Number of rotor slots
D = 10 raised to the power of C [C is described next]
P = Number of poles in the motor

If the amplitude levels at $F(1)$ and $F(2)$ are read in dB, then

$$C = \left| \frac{(A - B)}{20} \right|$$

and if the amplitude levels are read in engineering units, such as amperes, then

$$C = \log\left(\left| \frac{A}{B} \right| \right)$$

where

A = The level read at $F(1)$
B = The level read at $F(2)$

In addition to the line frequency and its harmonics and sidebands, the current spectra also contains components at the slot pass frequency and its harmonics and sidebands. To examine these higher frequencies, you must filter out the component at line frequency.

The general equation for these components is

$$Fs = Fl * \left[\left(2 * \frac{R}{P} \right) * (1 - s) \pm N \right] \qquad \text{(Formula 10-8)}$$

where

Fs = Slot pass harmonic
Fl = Line frequency in Hz
R = Number of rotor slots
P = Number of poles in the motor
s = Per unit slip
N = Harmonic index (1, 3, 5, 7, and so on)

When broken rotor bars are present, these components are modulated so that sidebands at ± ($2 * s * Fl$) are present in the spectra. Although these sideband amplitudes are almost independent of load, it is much more difficult to analyze the spectra at these higher frequencies. Since you can get a good estimate of the rotor bar condition at the lower frequencies, you rarely need to analyze the slot pass harmonics and sidebands.

In addition to the current analysis, stator core vibration spectra at slot pass frequency can be analyzed for sideband activity to predict the presence of broken rotor bars and rotor eccentricity.

The general equation for these spectral components is

$$Fv = Fl * \left[\left(2 * \frac{R}{P} \right) * (1 - s) \pm N \right] \qquad \text{(Formula 10-9)}$$

where

Fv = Vibration harmonic
Fl = Line frequency
R = Number of rotor slots
P = Number of poles in the motor
s = Per unit slip
N = Harmonic index (0, 2, 4, 6, and so on)

Once again, when broken rotor bars are present these harmonic components in the frequency spectra are modulated with sidebands at $\pm (2 * s * Fl)$.

Although the sidebands are present, a high resolution zoom analysis is required to identify them, and it is very difficult to estimate the number of broken bars. In most industrial situations, it is not practical to mount accelerometers to the stator core.

Two common static tests which are used to find broken rotor bars are the growler test and the single phase test. The growler test (so named because the rotor "growls" during the test) is run on the rotor out of the motor and uses an energized coil of wire around the rotor as a means for checking the magnetic field around each rotor bar. The single phase test is run with the rotor in the stator. A low-voltage source is applied to one of the phases of the motor and the rotor is slowly turned while the current in the phase coil is monitored.

Both of these static tests are usually run on a cold motor, and most of the time a cracked or broken rotor bar is not evident until the rotor reaches operating temperature.

Shorted Lamination

Another common problem in the rotor or stator is caused by shorted lamination. Such shorts may be caused by a rub between the rotor and the stator, by inappropriate handling or machining of the rotor or the stator, for example.

Shorted lamination causes localized heating which may distort the stator or cause the rotor to bow. Even though the bowed rotor may be balanced, the shorted lamination continues to heat the rotor and reduces the efficiency of the motor.

Commutation

Commutation in DC motors is a source of vibration due to the mechanical contact between the brushes and the commutator segments. Since no commutator is perfectly smooth, the brushes bounce up and down as the motor spins. The bouncing of the brushes creates a vibration frequency at the *number of commutator segments* * RPM. This frequency, or its harmonics, can excite other components within the system to resonance.

You can detect brush vibration at the *number of commutator segments* * RPM in the frequency domain. The presence of brush vibration indicates that the brushes are still in good condition, and that the spring is still exerting tension against the commutator. When the bouncing vibration disappears, the brush is no longer being pushed against the commutator. Consequently, an air gap exists permanently between the brush and the commutator, causing excessive sparking.

High mica (or high commutator bars) causes excessive sparking and brush bouncing. The vibration from the bouncing brushes can often cause an audible chatter, especially during startup or coastdown.

6.0 ANALYSIS

The following tables can help you determine the probable cause (or causes) of the vibration you measured.

TIME DOMAIN ANALYSIS		
TYPE OF WAVEFORM	**PROBABLE CAUSE**	**COMMENTS**
Impacts	Looseness	
	Bearing fault frequency	
	Cracked shaft	
	Rotor loose on shaft	
Non-periodic components	Foreign material	
	Outside source of vibration	
Asymmetrical waveforms	Rubs	
	Restrictions to motion	

FREQUENCY DOMAIN ANALYSIS				
FREQUENCY	**HARMONICS**	**AMPLITUDES**	**PROBABLE CAUSE**	**COMMENTS**
1 * RPM	No	1*RPM H > .15 in/sec 1*RPM V > .12 in/sec Low axial both bearings	Imbalance	Consistent 1*RPM phase
		1*RPM H > .15 in/sec 1*RPM V > .12 in/sec 1*RPM A > .1 in/sec	Bent shaft	Look for axial phase 180° out
		1*RPM H > .18 in/sec 1*RPM V < .08 in/sec	Horizontal resonance	High amplitude may be at some other order excited by non-linear system
		1*RPM H < .08 in/sec 1*RPM V > .18 in/sec	Vertical resonance	
		Increases after startup	Thermal bow	
	No	High motor 1*RPM at bearing	Motor mount resonance at motor speed	
	2*RPM	1*RPM H > .18 in/sec 1*RPM V < .08 in/sec	Horizontal resonance plus vertical misalignment	Phase across direct drive coupling is either in phase or ± 10° out of phase
		1*RPM H < .08 in/sec 1*RPM V > .18 in/sec	Vertical resonance plus horizontal misalignment	
	2*RPM, 3*RPM	1*RPM H > .15 in/sec 1*RPM V > .12 in/sec Low axial both bearings	Imbalance plus system non-linearity (looseness, cracks, excessive clearance)	- Consistent 1*RPM phase - Can also produce ½ order harmonics
		High axial on bearing	Bearing cocked on shaft	
	3*RPM, 4*RPM ½ orders		- Locked coupling - External side force	- Locked coupling is usually load-dependent - Asymmetric time signal may also be present
	2*RPM-4*RPM higher than 1*RPM		- Bearings cocked on shaft - Bearings not in same plane	Axial vibration on the bearings will be high
	Slot pass freq. with sidebands at 2*line freq.		Eccentric rotor	- Vibration will beat at freq. equal to # poles * slip freq. - Disappears when power is cut
2*slip frequency	Sidebands of line freq. at ± the # poles * slip		Broken rotor bars	Similar to eccentric rotor, but beat at # poles * slip is proportional to load
2*line frequency			- Rotor not centered in stator - Stator loose in motor frame - Loose laminations or coils in stator	- Vibration is regardless of number of poles - Vibration increases with load and disappears when power is cut
Rotor frequency * # of stator slots			Core slot pass frequency	May excite system resonance

SUMMARY

The principles that govern the design and operation of motors and generators are identical; therefore, the types and causes of vibration are the same for both.

The fundamental concepts of magnets, electromagnets, and induced current must be understood before a discussion on motor/generator vibration can begin. All magnets have a north and south pole. Like poles repel each other, and opposite poles attract. Both poles attract a metal that has no magnetism.

Current flowing through a wire creates a magnetic field at right angles to the wire. An electromagnet is a soft-iron core surrounded by a coil of wire. When the wire leads are connected to a battery, a magnetic field is created and the iron core becomes a magnet. The strength of the magnetic field is equal to the strength of the current * the number of turns of the coil.

The poles of the electromagnet reverse when the current flow is reversed. If an electromagnet is connected to an AC source in the U.S., the magnetic polarity switches back and forth 60 times a second. In other countries with 50 Hz line frequency, the magnetic polarity will switch back and forth 50 times a second.

If an electrical conductor moves through a magnetic field, then a current is induced in the conductor. The conductor must move so that it cuts the lines of magnetic force in the field. The strength of the current flow is proportional to the speed that the conductor moves through the magnetic field. The direction of flow is always perpendicular to the motion, and its magnetic action tends to resist the motion that produced the current.

There are two types of AC motors:

1. Synchronous motors
2. Induction motors

Both motors are constructed more or less the same, but behave in slightly different ways.

An AC motor has two major components, both of which are electromagnets. One electromagnet, called the **stator**, is made by placing wire wound in coils in the slots of an iron core. This core is securely mounted in the motor frame and remains stationary. Normally there are several individual windings on the core which are energized from a source of alternating current.

The three-phase system has three electrical phases which are 120 degrees out of phase from each other. The alternating current, AC, changes polarity from positive to negative in a sinusoidal manner at the line frequency. Alternating current in each of the three phases create a magnetic field that rotates around the stator 3,600 times a minute.

The other electromagnet, called the **rotor**, is supported by bearings which center the rotor in the stator. The greatest resistance in the magnetic path of the motor is the air gap between the rotor and the stator. It is therefore important to have both a minimum and a uniform clearance between the rotor and the stator.

If a fixed pole rotor magnet is placed in the rotating magnetic field of the stator, the rotor will rotate at exactly the same frequency as that of the stator field. Such an assembly is known as a **synchronous** motor. The rotating magnet of the synchronous motor can be a permanent magnet but is usually an electromagnet energized from a direct current source.

The rotor in an induction motor is an electromagnet, but it is not energized from an external source. The rotor is made up of laminations similar to those in the stator, but in place of the wound coils in stator slots there are solid conductors, called **rotor bars**, running from one end of the rotor to the other and connected at each end by a shorting ring.

When a source of alternating current is first applied to the stator of the induction motor, the rotating magnetic field cuts one rotor bar as a north pole and the bar on the opposite side of the rotor as a south pole. The circular flow of current that results causes the rotor to become an electromagnet which then tries to keep up with the rotating magnetic field of the stator.

As the rotor accelerates, the relative speed between the rotor and the rotating magnetic field of the stator decreases so that the rotor bar is cutting lines of flux at a slower rate. Therefore the induced current in the rotor bars decreases.

If the rotor were to reach the same frequency as that of the rotating magnetic field of the stator, there would be no induced current in the rotor bars. Thus the rotor would cease to be an electromagnet and would slow down. When this happens the rotor bars once again cut lines of flux, and current is induced in the rotor bars, making the rotor an electromagnet.

The rotor frequency lags behind the frequency of the rotating magnetic field of the stator, so that the torque generated by the induced current equals the torque necessary to drive the motor. The difference between the frequency of the rotating magnetic field of the stator and the rotational frequency of the rotor is called the **slip frequency**. Any magnetic problems in the rotor occur at the number of poles * the slip frequency.

Motor action, the motion that results from the interference of two magnetic fields, is responsible for rotation in a DC motor. The motor action force is proportional to the flux intensity and the current in the conductor, and influences the size of a motor for a given horsepower and speed.

A DC motor has multiple coils of wire mounted on a shaft. The shaft/coil assembly (called an **armature**) is placed in a magnetic field between two fixed magnets. These fixed magnets can be either permanent magnets or electromagnets.

Current is supplied to the armature through **brushes** (graphite rods that are connected to a power source and that push against the armature). A flux builds up around the coil once current is supplied. The armature flux interferes with the field flux, producing torque and causing the armature to rotate.

Rotation from the motor action force would only cause the rotor to rotate through one half revolution, unless the current flow through the coil was reversed. A device called a **commutator** is used to reverse the current flow in the armature coil. A commutator is basically a split metal ring that is attached to the ends of the armature coil. An external DC source is applied to the commutator, and the split in the ring reverses the current flow halfway through a revolution.

Torque varies with the position of the shaft. For each coil, the torque drops to zero twice during one revolution. Torque variations such as these would create extreme torsional vibration, and would be unacceptable. Torque fluctuations are smoothed out by placing many coils on the armature. The motor action force is applied to many coils at once, smoothing out the torque curve and adding to the amount of torque that the motor produces. But each new coil that is added to the armature requires two commutator segments, so that the current can be reversed in each coil.

REFERENCES

R. W. Thompson, Understanding Induction Motor Operation and Flaw Diagnosis, 1989 PDM Users Seminar

Gee, Lamont and Roper, Bill, Kiemle-Hankins Company, personal interview.

Baxter, Nelson, ABM Technical Services, personal interview.

Hill, Rowland, National Electric Coil, personal interview.

Anderson, Edwin P. and Miller, Rex, Electric Motors, 4th ed., Copyright 1983, T. Audel, Inc., Indianapolis, IN.

Andreas, John C., Energy-efficient Electric Motors: Selection and Application, Copyright 1982, Marcel Dekker, Inc. New York, NY.

Lloyd, Tom C., Electric Motors and their Applications, Copyright 1969, John Wiley and Sons, Inc.

Pansini, Anthony J., Basics of Electric Motors, Copyright 1989, Prentice-Hall, Inc.

Traister, John E., Handbook of Electric Motors, Copyright 1984, Prentice-Hall, Inc.

Tippens, Paul E., Physics, Fourth Edition, Copyright 1991, Glencoe/McGraw-Hill.

11 MACHINE-SPECIFIC ANALYSIS: COMPRESSORS

INTRODUCTION

This chapter discusses vibration considerations specific to compressors. The more common designs of compressors are discussed, along with vibration problems arising from each design.

1.0 PRINCIPLES OF COMPRESSOR OPERATION

An anonymous quotation can be found in one of the references listed at the end of this chapter:

> *"Mankind lives at the bottom of an ocean of air compressed by the weight of the air above it. Since he first blew his breath on a glowing ember in starting his fire and later fashioned a crude bellows of animal hides, man has been adding to the uses and applications of compressed air."*

The movement of any gas or gases, not just air, is a primary need in industry. All gases and fluids move from an area of high pressure to an area of low pressure. So in order to move a gas, pressure must be produced.

Compressors operate on one of two principles:

1. Reducing the volume of a trapped gas, usually by means of a piston moving in a cylinder. As the volume is reduced, the gas is compressed. When the volume increases, a vacuum is created that pulls more gas in for compression.

2. Accelerating a gas to a high velocity and then rapidly decelerating the gas. This converts the kinetic energy of the gas into potential energy (pressure).

As we shall see in the next section, it is not always possible to achieve the desired compression in one step. Sometimes the compression is done in stages, where each stage is the compression done between a gas inlet and outlet.

By increasing the pressure of the gas, a compressor adds energy to the gas. Since energy is defined as the ability to do work, adding energy to a gas causes the gas to do work: move through a pipe, drive air tools, or accelerate a chemical reaction.

A compressor transforms mechanical energy from the rotating shaft of its driver into the kinetic and potential energy required to move the gas. The mechanical energy that is imparted to the gas (in foot-pounds per pound of fluid) is independent of the gas's density and is called **head**.

Head is measured in feet, and the head generated by a compressor operating at a given speed and capacity will be the same for all gases.

The terms *head* and *pressure* are often used interchangeably, but this is not correct. By expressing head in feet you are measuring the energy produced by the compressor, since a certain amount of energy is required to move the gas. Gases can be considered as fluids, so a discussion of head and pressure using fluid applies equally well to a gas. To understand the difference between head and pressure, consider a column of fluid as shown in Figure 53.

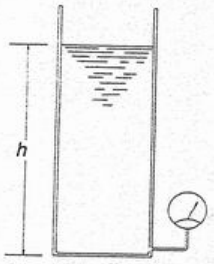


Figure 53 - *head and pressure*

The column of fluid in Figure 53 is a certain height *h* above the bottom of the column. Due to its height, the fluid exerts pressure (equal to its mass times gravity) on the bottom of the tube. Head is the height of the fluid; pressure is the force exerted by the fluid by virtue of its height.

A column of water 2.31 feet high produces a pressure of 1 psi (pound per square inch) at its base. Therefore, any pressure measured for a column of water can be converted to feet of water by multiplying the pressure by 2.31. For fluids other than water, the height needed to produce 1 psi is found by dividing 2.31 by the specific gravity of the fluid.

2.0 TYPES OF COMPRESSORS

There are four basic types of compressors:

1. Reciprocating
2. Rotating

3. Centrifugal
4. Axial

Both reciprocating and rotating compressors use volume reduction to produce pressure, while centrifugal and axial compressors use the acceleration/deceleration design.

2.1 Reciprocating Compressors

The design of a reciprocating compressor is similar in many respects to an internal combustion engine: a piston, driven by a crankshaft, moves up and down (reciprocates) inside a cylinder. Intake valves are mounted at the top of the cylinder to let low pressure air in, and exhaust valves let the compressed air out.

As the piston moves down in the cylinder, the volume inside the cylinder increases. This increase in volume momentarily lowers the pressure inside the cylinder, creating a vacuum that sucks gas into the cylinder. The intake valve then closes, trapping the gas inside the cylinder. As the piston moves up, the volume is decreased and the gas is compressed. The exhaust valve is opened near the top of the piston's stroke, and the compressed gas (at a higher pressure) is released. Figure 54 illustrates a typical reciprocating compressor.

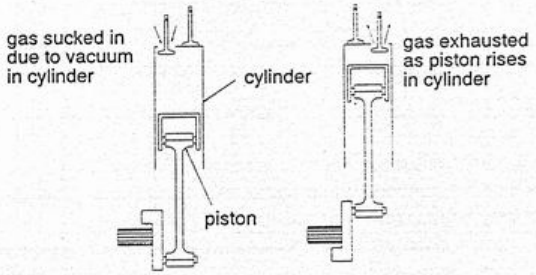


Figure 54 - *a reciprocating compressor*

The volume of gas that can be held in the piston is the same for each cycle. Therefore, the capacity of a reciprocating compressor is directly proportional to the speed of the piston; moving the piston faster compresses more gas per unit time.

Theoretically, only one stage would be needed to compress a gas to any pressure. However, compressing a gas raises its temperature. This fact is exploited in diesel engines, where the heat of compression is sufficient to ignite the fuel. The temperature created by compression limits the amount of compression that can be done in one stage. If more compression is desired than can be done in one stage, the outlet of the cylinder is directed to the inlet of another cylinder (the next stage).

2.2 Rotating Compressors

Rotating compressors work by trapping gas in a "pocket" formed by rotating components. As the components rotate, the volume of the pocket is decreased, which compresses the gas. Once the gas has been compressed, the pocket lines up with an outlet and the gas is released.

A common type of rotary compressor called **sliding vane** is illustrated in Figure 55. A cylindrical casing contains a smaller cylindrical rotor that is mounted off-center. Slots cut into the rotor hold vanes that can slide in and out of the slot. The vanes are held against the casing by centrifugal force, pins, or springs. This forms a pocket between the vanes, the casing, and the surface of the rotor.

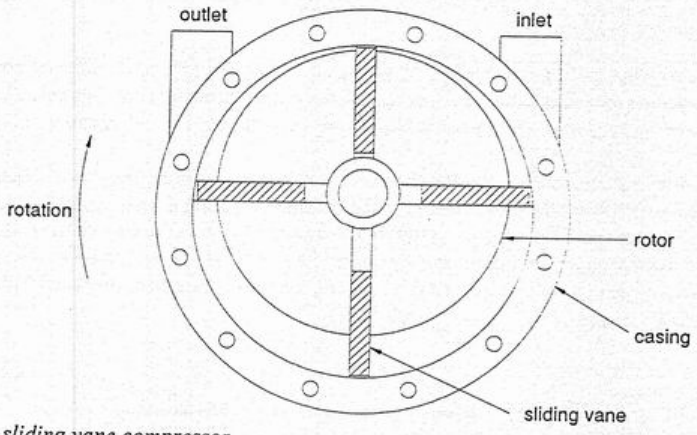


Figure 55 - *a sliding vane compressor*

As the rotor turns, a pocket passes the compressor inlet. The pocket grows in volume, creating suction which pulls gas into the pocket. When the second vane rotates past the inlet, the pocket is sealed and now behaves very similar to a reciprocating compressor.

The rotor continues to turn, and the pocket grows smaller as the vanes slide up toward the discharge outlet. When the pocket is at its smallest volume, the leading vane passes the discharge outlet and the gas is released. If more compression is required, the discharge outlet is connected to the inlet of the next stage.

There are many different types of rotary compressors: variations on the sliding vane design, many-bladed rotors contained in a water jacket, and multi-lobed impellers or screws that rotate against each other. It is beyond the scope of this chapter to describe them all in detail.

2.2.1 Screw compressors

Rotary screw compressors are typically multi-stage and employ rotating screws to create pressure. The screws are usually shafts with two or more lobes, and the shafts are placed so that the lobes fit inside each other without touching. Figure 56 illustrates a rotary screw compressor.

The position of the lobes creates pockets that pass by the inlet and discharge outlet the same as in a sliding-vane compressor. The lobes cannot touch each other, since the two shafts must rotate at high speed. However, to maintain adequate pressure, the clearances between the lobes must be very tight. Lobe clearance is maintained by timing gears, so gear mesh frequencies are a very prominent component in the vibration signature of a rotary screw compressor.

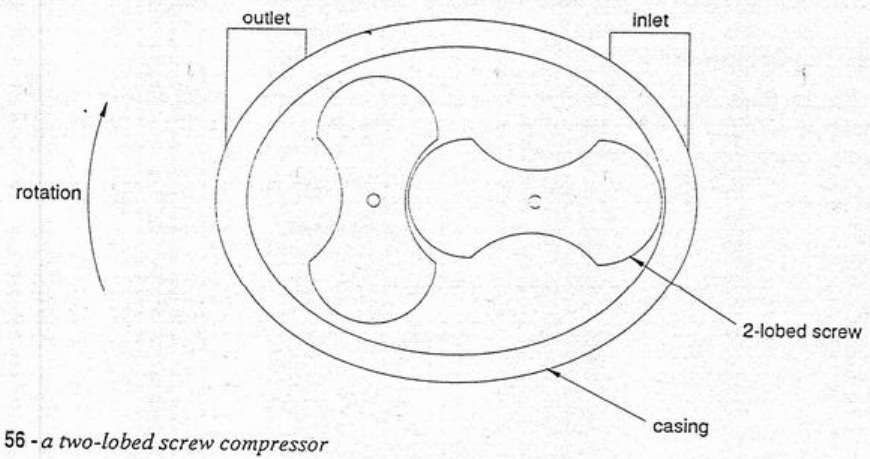


Figure 56 - *a two-lobed screw compressor*

2.3 Centrifugal Compressors

Centrifugal compressors create high pressure by accelerating and decelerating the gas. A typical centrifugal compressor is shown in Figure 57. A multi-vaned impeller rotates inside a housing. The gas enters the center of the impeller (called the eye), and is forced to the outside of the impeller by centrifugal force. Guide vanes are installed in some centrifugal compressors to evenly spread the gas around the entire impeller eye.

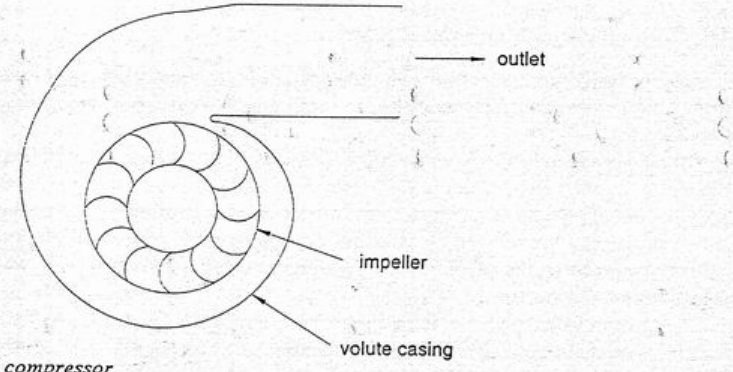


Figure 57 - *a centrifugal compressor*

The gas has been accelerated quite a bit by the time it reaches the periphery of the impeller. Once it leaves the impeller, it enters an area called the **diffuser**. The diffuser is usually a constantly expanding area inside the casing called a **volute** (a volute is an ever-widening spiral, much like a snail shell).

Inside the volute the gas decelerates, and the kinetic energy from the gas's motion is converted into pressure. For greater compression, the outlet of a volute is directed to the eye of the next impeller.

2.4 Axial Compressors

The gas flows along the axis of the rotor in an axial compressor. Axial compressor blades are usually an airfoil in cross-section and run at a very close clearance to the side of the casing. Figure 58 illustrates a typical axial compressor.

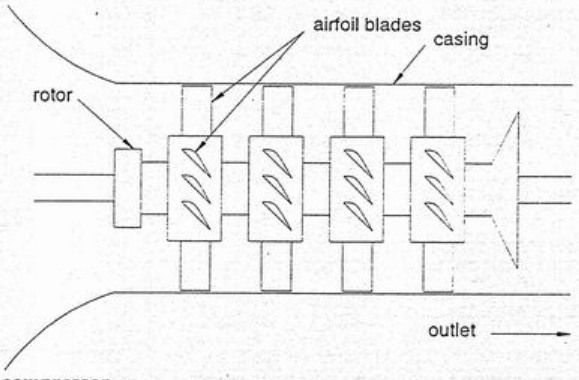


Figure 58 - *an axial compressor*

3.0 STRUCTURAL ASPECTS AFFECTING VIBRATION

3.1 Thermal Growth and Misalignment

All types of compressors heat up during operation. The major sources of heat are from the process (for example, heat generated as gases are compressed) and from friction (bearings and piston rings, for example).

All metals expand as their temperature increases. This change in length, called **thermal growth**, can cause the shaft center lines to shift from their cold positions. In some cases this shift is minimal, and the cold alignment is satisfactory for operation at normal load and temperature. In other cases, however, thermal growth can be extensive enough to cause serious misalignment problems. In the case of compressors, the attached piping undergoes thermal growth as well, which may put increased pipe strain on the system.

The change in temperature is rarely uniform throughout a machine. For example, the exhaust manifolds on reciprocating and rotary compressors are much hotter than other areas on the machine. This uneven heating causes the machine to pitch and bend rather than grow straight up.

We are primarily concerned with vertical growth. When a shaft is centered in a machine, the metal on the left of the shaft grows to the left, the metal on the right grows to the right, and the shaft remains centered.

The metal below the shaft tries to expand downward, but can't, since the machine is sitting on a solid foundation. Therefore the thermal growth pushes the shaft upwards. One of the first effects of

shaft center line misalignment is an increased radial load on the bearings, which can lead to early bearing failure.

3.2 Anti-Friction Bearings

Errors in mounting anti-friction bearings can cause excessive vibration. For example, anti-friction bearings might not be mounted square to the shaft or in the same plane (one bearing is higher or lower than the other). This usually results in elevated bearing temperature and can cause second-, third-, and fourth-order vibrations in addition to first-order vibrations.

Anti-friction bearings are mounted to a shaft as shown in Figure 59. The inner race is locked to the shaft, while the outer race is held against the bearing housing. The outer race is preloaded so that it does not rotate.

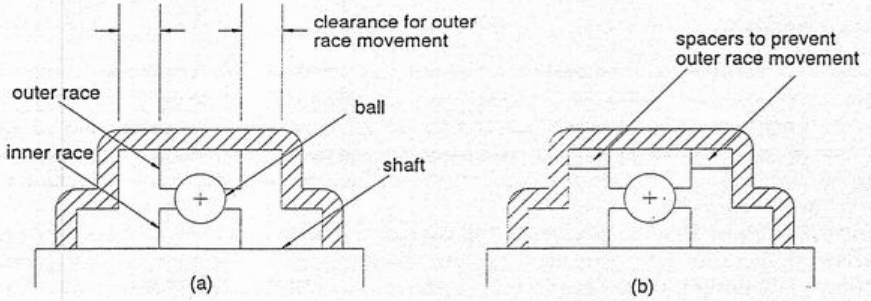


Figure 59 - *fixed and floating bearings*

During thermal growth, the inner race moves with the shaft, since the two are locked together. The preload prevents the outer race from rotating, but not from moving with the shaft and the inner race. Make sure that during the thermal growth of the shaft, the outer race is not restricted from growing by either side of the bearing housing.

A bearing whose outer race is free to move with the shaft is called a **floating bearing**, while a bearing whose outer race is prevented from moving is called a **fixed bearing**. Spacers are inserted on either side of the outer race of the fixed bearing to hold it in place. Figure 59(a) illustrates a floating bearing, and a fixed bearing is shown in Figure 59(b).

Only one bearing should be fixed; since some thermal growth is inevitable, one of the bearings must be floating. Normally the fixed bearing is the one closest to the coupling or the pulley that is driving the compressor; the floating bearing is the one on the outboard side. This allows the compressor shaft to grow into the floating bearing.

If the reverse were true, and a floating bearing was next to a coupling, the shaft could grow into the coupling. Excessive growth could use up the coupling clearance and result in a locked coupling. The compressor would run within vibration limits when started cold, but the first- (and maybe the second- and third-) order vibration would increase as the compressor came up to operating temperature.

All the cautions that apply to anti-friction bearings should be observed for compressor bearings:

- **Bearing lubrication**: Too much lubricant seals in the heat and causes early bearing failure. Too little lubricant does not provide the lubrication necessary to minimize wear.
- **Bearing preload**: The preload on the journal and in the bearing housing clearance must be set to maintain the proper preload on the bearings.
- **Bearing clearance**: Clearance on journal bearings should be between 1 and 1.5 mils per inch of shaft diameter, with a minimum of 2 mils. Insufficient clearance increases bearing temperature as well as the possibility of wiping. Too much clearance decreases the damping and increases the possibility of shaft instability.
- **Maximum frequency**: The operating frequency of a compressor with anti-friction bearings should be within the bearing tolerance (7,200/shaft diameter for grease lubricant; 9,600/shaft diameter for oil lubrication). For example, if the compressor has a 5-inch shaft with grease lubricated bearings, then the shaft frequency should not be higher than 1440 RPM.

3.3 Resonances

Compressor shaft and bearing pedestal resonances that are close to the compressor's operating frequency result in high first-order vibration levels and make field balancing of the compressor difficult, if not impossible. The compressor rotor (or crankshaft) itself has a fundamental plate resonance as an assembly. In addition, each blade or vane has its own resonant frequency. The operating frequency and the blade pass frequency forcing functions must not be able to excite any of these resonant frequencies.

Compressor drive components produce their own forcing functions, which must not excite any resonances in the compressor. Since the compressor is direct-coupled to the driver, then the operating frequency of the driver is the same as the compressor, and any 1*RPM forcing function in the motor (imbalance, angular misalignment, bent shaft, for example) will be transmitted to the compressor. The suction and discharge piping may also have resonances excited by the compressor forcing functions, especially in stall or surge conditions.

3.4 Looseness

Loose bearing or hold-down bolts can change the system stiffness and result in high vibration levels. A rotor loose on the shaft (excessive clearance between the wheel bore and the shaft) allows the rotor to shift on the shaft between runs, so acceptable balance may not be possible.

4.0 OPERATIONAL SOURCES OF VIBRATION

4.1 Pulsations

Because of their suction and discharge cycles, reciprocating and rotating compressors can induce pulsations into the process. Piston movement in a reciprocating compressor can also cause starvation in the suction piping and high pressures in the discharge piping.

These pulsations can be eliminated by installing pulsation filters or surge chambers in the piping. Pulsation filters and surge chambers are vessels that help to equalize the pressure pulses and smooth out the flow to the process.

4.2 Flow Characteristics

Flow characteristics at the compressor inlet should be as smooth as possible. Analysis of compressor efficiency and flow involves the measurement of the flow by an expert.

Every attempt should be made to have laminar (non-turbulent) flow for as long a distance as possible at the input. It may also be necessary to use an inlet cone to reduce turbulence. The compressor outlet should have some reasonable run of laminar flow before a transition (a change in the diameter of the piping, a turn, an elbow, for example). If a transition is close to the compressor outlet, then steering vanes may be added to reduce turbulence.

4.3 Surge

A blockage on the compressor outlet can cause surge. Surge occurs when the pressure developed by the compressor is not equal to or greater than the downstream pressure in the piping. This causes gas to surge back across the clearances in the compressor. Surge results in a high noise level and high pulsations in the piping; these pulsations normally occur between 30 to 45% of compressor speed, with higher harmonics and wide skirt noise. Surge may be caused by many things: a blockage on the compressor outlet, too long a flow in a diffuser, or improper settings of variable inlet guide vanes.

4.4 Rotary Screw Compressors

Rotary screw compressors present a very complicated vibration problem. A typical drive train arrangement for a two-stage rotary screw compressor is shown in Figure 60. A large gear called a **bull gear** is connected to the motor shaft. The bull gear meshes with two **pinion gears**, each one attached to the male lobe of the compressor. At the other end of the male lobe, a **timing gear** drives the female lobe. The male and female lobes do not necessarily rotate at the same frequency.

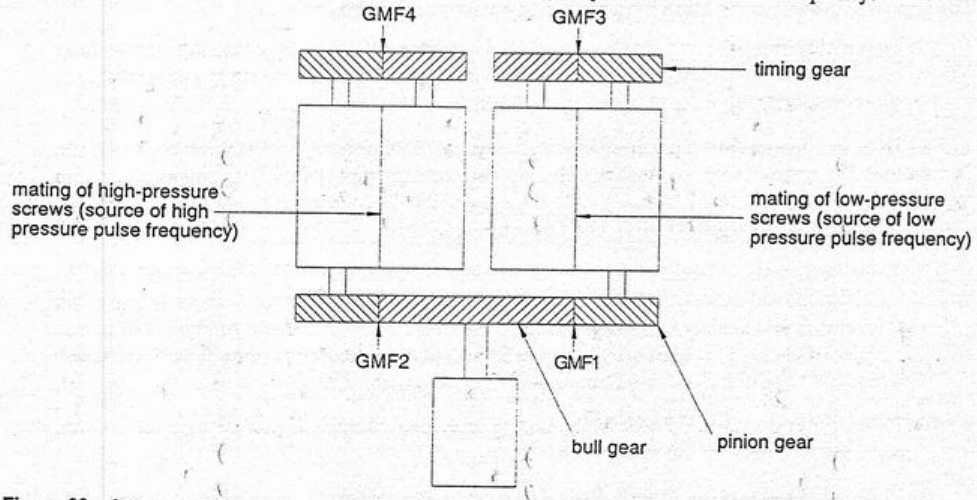


Figure 60 - *drive train of a rotary screw compressor*

Every gear mesh location in the rotary screw compressor drive train produces a gear mesh frequency. In the example shown in Figure 60, there are four gear mesh frequencies being generated:

1. Between the bull gear and the low-pressure male pinion gear
2. Between the bull gear and the high-pressure male pinion gear
3. Between the male low-pressure timing gear and the female low-pressure timing gear
4. Between the male high-pressure timing gear and the female high-pressure timing gear

Depending on the design of the compressor, all the gear mesh frequencies that are present could be different, and any one of them could present a vibration problem. You will need physical data on the compressor (the rotational frequency of each gear and the number of teeth on each) to isolate a gear mesh frequency to a specific lobe.

Pressure pulses are produced in the compressed air due to the meshing action of the lobes. These pulses occur at a frequency equal to the *number of lobes * the rotational frequency*. This frequency, called the **pulse frequency**, is generated by each stage of the compressor. Again, depending on the design of the compressor, the low-pressure pulse frequency could be different than the high-pressure pulse frequency.

These pulse frequencies cannot be removed from the air flow leaving the compressor. Due to the high rotational speed of the lobes, the pulse frequency is too high to be effectively minimized by pulsation filters or surge chambers.

5.0 VIBRATION ANALYSIS OF COMPRESSORS

Vibration analysis and the solution of vibration problems is a logical process, where the analyst follows a procedure to gather the information necessary to pinpoint the vibratory forcing function. The logical approach to vibration problem solving can be broken into five steps:

1. **Determine the problem** - Ask *what, where, and how:* What is the problem? Where is the problem? How was the problem identified? Always solve one problem at a time; if there is more than one, solve the most severe problem first.
2. **Inspect the system** - Do a complete walk-around to determine the condition of the system; look for wear, cracks, looseness. Identify the number and type of components (couplings, bearings, motors, for example). Take physical data on the system, such as spacing and clearances, diameters, RPM(s), and type of mounting.
3. **Take static data** - Calculate resonances within the system (shaft, bearing pedestal, housing, base, blade, and others). Calculate other possible forcing functions, such as bearing fault frequencies and blade/vane pass frequency. Section 1.3 of the General Analysis Techniques chapter (Volume 1) provides information on calculating possible forcing functions. Check alignment within the system. Record the background vibration.
4. **Take dynamic data** - Record vibration data during startup, at normal load, and during coastdown in both the time and frequency domains.
5. **Determine the forcing function(s)** - Examine the data for excessive amplitude at any of the frequencies you calculated in Step 3. Check for synchronous, sub-synchronous, and non-synchronous vibrations. Look for impacts, non-periodic events, and non-symmetric waveforms.

5.1 Determine the Problem

The first step in diagnosing compressor vibration is to determine the problem by asking "what, where, and how:"

- **What is the problem?**
  - The compressor doesn't "feel good."
  - Someone feels the compressor vibrates excessively.
  - There are high levels of vibration in the piping.
  - The compressor makes unusual or excessive noises. It is difficult to say absolutely that a noisy compressor is the responsibility of a vibration specialist. The noise study of a compressor requires analysis by an expert: the gas flow is calculated to determine if it is too fast for that size of pipe, the piping is analyzed to determine if it is properly designed and supported.
  - The compressor noise is transmitted through the building.
- **Where is the problem?** Or, at least, where does someone *feel* the problem is located: the compressor, the motor, the bearings, the piping, the floor, or elsewhere.
- **How was the problem identified?** If there is a vibration data collection program in effect, you may be able to study historical data on this unit.

5.2 Inspect the System

Then make a general walk-around inspection, carefully noting the following:

- The condition of the unit:
  - Does the compressor appear to be well maintained or neglected?
  - Is the grouting in good shape or is it crumbling and cracked?
  - Are the hold-downs tight?
  - Are there shim packs that appear to be solid? Not many thin ones, but the fewest number of shims in order to obtain the proper height and location of the unit.
  - Are the shim packs in good condition?
- The type of compressor:
  - Reciprocating, rotating, centrifugal, axial
  - The number of blades or vanes
  - The operating frequency
  - The shaft diameter and shaft material
  - The distance between bearing centers

- The mounting of the compressor

- The type of motor:
  - AC, DC, variable speed, synchronous
  - The motor's operating frequency
  - The type of bearings in the motor
  - Whether the actual bearing type is available, so that fault frequencies can be calculated and checked

- The coupling:
  - The coupling type
  - The condition of the coupling
  - How the coupling is aligned, and how often that alignment is checked

- The type of bearings:
  - Anti-friction or sleeve
  - How the bearings are lubricated, and how often
  - Whether the bearing clearances been checked recently, and what the clearances are
  - What the bearing fault frequencies are, if possible

If the compressor is not running, then static measurements can be made first. Otherwise, running checks can be made first, and static measurements can be made once the compressor is shut down.

5.3 Take Static Data

Static data (that is, data taken when the compressor is not moving) is used to calculate system resonances and possible forcing functions. Dynamic vibration data taken when the compressor is running is then analyzed to determine if any of these calculated frequencies are present.

Remember, the major vibration within the compressor may be a harmonic of one of these forcing functions. System resonances can amplify one of the harmonics of the forcing function, causing the amplitude of that harmonic to exceed the amplitude at the forcing function frequency. So when the dynamic vibration data is analyzed, look for excessive amplitudes at *harmonics* of the calculated forcing function frequency as well as *at* the forcing function frequency.

5.3.1 Resonant Frequencies

Impact tests should be performed to determine the resonant frequency of various compressor components such as bearing pedestals, shafts, the foundation, and the air duct. Bearing pedestals and motor supports should be tested for resonant frequencies in the horizontal, vertical, and axial direction.

Very large compressor systems, with drive motors in the hundreds of horsepower, are often mounted on heavy concrete bases. Their mass makes it nearly impossible to determine their resonant frequency from an impact test: it is simply too difficult to hit hard enough with a force hammer to

make the system vibrate. You can often determine the resonant frequency of these systems by analyzing the vibration data during startup. Section 5.4.1 describes this technique in detail.

5.3.2 Blade Pass Frequency

For rotating, centrifugal, and axial compressors, count the number of compressor blades so that you can determine the blade pass frequency. High blade pass frequencies can occur when a compressor wheel is not centered in the duct (especially so for axial compressors), or when a cutoff plate is installed too close to the compressor wheel on a centrifugal compressor.

Both centrifugal and axial compressors can exhibit high blade pass frequencies when the compressor is operated well below or above its best efficiency point.

5.3.3 Rotational and Reciprocating Forces

Reciprocating engines are often used to drive compressors. The rotation of the crankshaft and the reciprocating motion of the pistons create vibratory forces at crankshaft rotating frequency and 2 * *crankshaft frequency*. These forces may be transmitted to the compressor, exciting some resonance in the system.

Sections 2.1.3 through 2.1.5 of the Engines chapter discuss these forces in great detail.

5.3.4 Mechanical Data

For all types of compressors:

- Check the clearances on the compressor bearings, and determine if the bearings are level and square with the shaft.
- Identify which is the fixed bearing, and which is the floating.
- Check for proper lubrication and preload on each compressor bearing.
- Check for any looseness (bearing bolted to the pedestal, the bearing cap, pedestal to the foundation, and elsewhere).
- Check shaft alignment.
- Check the coupling clearance.
- Check condition of coupling.
- Check the motor hold downs for looseness.

5.3.5 Background Vibration

Measure the background vibration in both the time and frequency domains. Check the time domain for impacting and other non-periodic components that would not appear in the frequency domain spectra. There are many cases where excessive vibration has been attributed to a compressor when it is actually due to the background vibration.

5.4 Take Dynamic Data

Before any dynamic data can be recorded, transducers must be mounted at the appropriate locations. Dynamic vibration data is recorded during compressor startup, normal load operation, and coastdown.

5.4.1 Startup Data

Startup data is taken in the time domain. Sometimes the startup torque shock can act as a high-energy impact, and a "ringdown" from this shock can establish a first mode resonance in the motor or the compressor mounting. This resonance can be clearly seen in time domain data taken during startup. Examining this ringdown for the first-mode resonance is useful on very large compressor systems whose mass makes it difficult to calculate the resonance from an impact test.

If a multi-channel data recorder is available, record a tach pulse and startup vibration from as many locations as possible (horizontally, vertically, and axially at each bearing). If a multi-channel data recorder is not available, check phase and amplitude at running frequency on the drive side bearing in the horizontal direction.

5.4.2 Normal Load Data

Data taken during normal load operation of the compressor is taken in both the time and frequency domains. Use time domain data to check for impacts, non-periodic components, and non-symmetrical waveforms that can indicate looseness, rubs, electrical problems, or surge. Use frequency domain data to check the amplitudes of the various resonant and fault frequencies calculated from the compressor static data.

Time domain data

Once the compressor is running under normal load, use an accelerometer to examine the time domain for impacts, non-periodic components, and non-symmetrical waveforms. If there are impacts in the time domain, look for

- looseness
- bearing fault frequencies
- a cracked shaft
- the compressor wheel loose on the shaft

If there are non-periodic components in the time domain, look for

- foreign material in the compressor
- an outside source of vibration causing non-periodic events, like bad SCR timing in the electrical drive
- surge across the compressor

If there are asymmetries in the time domain, look for rubs or restrictions to motion in the compressor assembly.

Frequency Domain Data

First, take low-resolution broadband spectra in the horizontal, vertical, and axial directions on all bearings, the bearing pedestals, the compressor housing, the piping, the motor, and the motor foundation. These broadband low-resolution spectra can be taken very quickly, allowing you to easily determine where the vibration energy lies in the frequency domain. A more detailed investigation can then be made at the frequency location of those energy levels.

Many times an analyst fails in attempting to solve a vibration problem simply because they did not take enough high-frequency data to provide the information necessary for solving the problem. Low resolution, high baseband spectra allow you to identify very quickly where the energy levels are. Then you can examine each of these energy areas in more detail.

Then take high-resolution data in those energy areas. Take readings in each direction at each bearing, the bearing pedestal, the motor, and the motor base. Take phase and amplitude readings with a tach pulse in the horizontal, vertical, and axial directions.

Also, take readings in the axial direction at the top, bottom, left, and right sides of each bearing to determine how the bearing is moving in space (whether it is cocking or rocking, for example). If possible, take these phase and amplitude readings at first-, second-, and third-order to check for mechanical non-linearities in the system.

Phase readings at operating frequency are very important in identifying vibrations due to resonance and misalignment. To identify resonance, check the phase from horizontal to vertical on each bearing. You can identify misalignment by checking phase from horizontal to horizontal and vertical to vertical between bearings and across the coupling.

A useful test you can perform with a compressor (or with any rotor) is to obtain 1, 2, 3, and 4 pulses per revolution. One way is to use a tracking adaptor and 4 equally-spaced pieces of reflecting tape. Many times a coupling has 4-8 bolts around it that can be sensed using a proximity pickup. With a tracking adapter, you can divide the data down into 1 pulse per revolution, 2 pulses per revolution, etc. Once you have this tach data, you can use the vibration analyzer to look at the vibration component at each order. It is not always necessary to use a tracking adapter; some analyzers allow you to look at multiple orders from a single tach pulse.

5.4.3 Coastdown Data

Take coastdown data as the compressor coasts to a stop after shutdown. During coastdown, take phase and amplitude on the worst bearing at first-, second-, and third-order if possible. If a multi-channel data recorder is available, use that to record data from as many locations as possible.

5.5 Determine the Forcing Function(s)

If all of the preceding data is taken, it should be possible to pinpoint vibration problems in any compressor installation.

5.5.3 Examination of Time Domain Data

Examine the time domain carefully for impacts, non-periodic events, and for non-symmetrical waveforms.

Impacting in the time domain is usually a good indication of some mechanical non-linearity in the system. When examining impacts, it is important to determine that they are, in fact, periodic. For example, in some cases the phase firing of the triacs in a DC drive are not timed properly. These non-timed firings can appear to be impacts in the time domain, but they are not periodic. This non-periodicity indicates that these events will not have a high energy level in the frequency domain.

Non-periodic information in the time domain coupled with noise and vibration in the piping can indicate surge, which indicates that the compressor is operating into a pressure which is higher than that developed by the compressor.

If the vibration is erratic (it comes and goes, and the amplitude varies dramatically), and there is also the presence of some second- and maybe half-order vibration, then it is possible that the sleeve bearing is loose.

If the coupling locks up, there can be high second-, third-, or fourth-order vibrations. However, a locked coupling is usually a load-dependent problem, and there should be a sudden drop out of the high vibration when the unit is shut down.

5.5.2 Examination of the Frequency Domain

Imbalance

If the primary vibratory energy is at operating frequency, and there is not great disparity between horizontal and vertical readings (2 to 1 or less), then the compressor rotor is not in balance. Imbalance causes the phase angles to move approximately 70 to 90 degrees when you go from horizontal to vertical on a bearing (if there is no resonance amplification).

If many harmonics of running frequency are present, then there is some kind of non-linearity in the system (usually some kind of looseness or excessive bearing clearance). Some typical examples of non-linearities include the compressor blades being loose on the shaft, a cracked shaft, excessive coupling clearance, or the compressor is rocking on its foundation.

Misalignment: Coupling and Bearing

The presence of second-, third-, and fourth-order components can indicate some kind of alignment problem for direct-coupled machines. The misalignment may also be bearing misalignment (the bearings are not at the same level or are not normal to the shaft). If both bearings are fixed, leaving no possibility for thermal growth, it is possible to get high axial vibration. Bearings that are cocked on the shaft result in high axial vibration, and many times high second- and third-order vibration as well.

Resonance

Look at the relationships between horizontal, vertical, and axial amplitudes. For example, if the horizontal amplitude is much higher than the vertical, then probably the system is resonant in the horizontal direction. If so, the system will probably not be resonant in the horizontal direction at 2, 3, or 4 times operating frequency.

Cavitation

Cavitation is usually accompanied by orders of vibration harmonically related to running speed with higher frequency noise and relatively wide skirts.

Mechanical Non-linearities

Another way to check the compressor for mechanical non-linearities is by adding a known weight as an imbalance weight. This weight is a first-order disturbance: it is a rotating vector at rotor frequency, so it should not affect any of the higher orders. The weight should be calculated to produce an imbalance of one to two mils. Refer to the Field Balancing chapter in Volume 1 for more information on calculating trial weights.

Checking the phase and amplitude at first-, second-, third-, and maybe fourth-order with and without the weight immediate indicates whether one of the higher orders of vibration is being affected by a first-order disturbance. A first-order disturbance such as imbalance should not affect the higher orders. However, if the system has a mechanical non-linearity, the higher orders are affected.

Oil whirl

Oil whirl causes vibration at less than half-order in sleeve bearings. If shaft orbits are available, the oil whirl appears as an inside loop on the orbit.

High Blade Pass Frequency

If a very high blade pass frequency is present, you should determine where the compressor is running in relation to its best efficiency point.

Once the compressor efficiency has been determined, you should determine if there is a resonance close to blade pass frequency that is amplifying the blade pass frequency. Look for the resonance in the piping, the foundation, the bearing pedestals, and the compressor shaft itself. Impact tests should be performed on the shaft in both the vertical and axial directions. Mount transducers on the bearings and impact the rotor in the vertical, horizontal, and axial directions.

Shaft Orbits

Shaft orbits can be very helpful in diagnosing compressors in sleeve bearings. Many units have eddy current probes already installed in the bearings; however, not all such installations include two probes mounted at 90 degrees to each other in each bearing.

If the weight of the rotor is equal to at least 50% of the total weight of the unit, it is rarely necessary to use non-contact probes in sleeve bearings. If the rotor weight is small compared to the total weight, it becomes more important to examine the movement of the shaft in the bearing clearance as well as the bearing casing vibration.

With probes mounted at 90 degrees to each other in the bearing, the orbit of the shaft movement in the bearing can be examined. Use a tach signal to trigger the orbit. The information provided by analyzing shaft orbits includes the following:

- The direction of the maximum movement of the shaft.
- The movement of the shaft centerline from start to running condition. This also indicates the remaining clearance between the shaft and the bearing.
- The stability of the phase of the orbit.
- The presence of contact areas or rubs.
- The presence of misalignment, indicated by banana shaped or figure eight-shaped orbits.
- Secondary loops indicate shaft resonances.
- Unstable orbits indicating hydraulic instability such as oil whirl.

SUMMARY

Compressors operate on one of two principles:

1. Reducing the volume of a trapped gas, usually by means of a piston moving in a cylinder

2. Accelerating a gas to a high velocity and then rapidly decelerating the gas

It is not always possible to achieve the desired compression in one step. Sometimes the compression is done in stages, where each stage is the compression done between a gas inlet and outlet.

There are four basic types of compressors:

1. Reciprocating
2. Rotating
3. Centrifugal
4. Axial

Both reciprocating and rotating compressors use volume reduction to produce pressure, while centrifugal and axial compressors use the acceleration/deceleration design.

A reciprocating compressor is similar in many respects to an internal combustion engine: a piston, driven by a crankshaft, reciprocates inside a cylinder. As the piston moves down in the cylinder, the volume inside the cylinder increases. This increase in volume momentarily lowers the pressure inside the cylinder, creating a vacuum that sucks gas into the cylinder. As the piston moves up, the volume is decreased, and the gas is compressed.

Rotating compressors work by trapping gas in a "pocket" formed by rotating components. As the components rotate the volume of the pocket is decreased, which compresses the gas. Once the gas has been compressed, the pocket lines up with an outlet and the gas is released.

Centrifugal compressors create high pressure by accelerating and decelerating the gas. A multi-vaned impeller rotates inside a housing. The gas enters the eye of the impeller and is forced to the outside of the impeller by centrifugal force. Guide vanes are installed in some centrifugal compressors to evenly spread the gas around the entire impeller eye.

The gas flows along the axis of the rotor in an axial compressor. Axial compressor blades are usually an airfoil in cross-section, and run at a very close clearance to the side of the casing.

REFERENCES

Nisenfeld, A. Eli, Centrifugal Compressors: Principles of Operation and Control, Copyright 1982, Instrument Society of America, Research Triangle Park, NC.

Feller, Eugene W., Air Compressors: Their Installation, Operation, and Maintenance, Copyright 1944, McGraw-Hill Book Co., New York, NY.

Compressed Air and Gas Institute, Compressed Air Handbook, Copyright 1947, Compressed Air and Gas Institute.

12 MACHINE-SPECIFIC ANALYSIS: ENGINES

The authors received a call from an automotive manufacturer to assist in a lawsuit. An attorney had purchased a car which vibrated so badly at stop lights that his wife had become physically sick. The manufacturer requested that the authors locate and repair the cause of vibration.

Prior to seeking help, the automotive manufacturer concluded that the vibration was related to either fuel flow or ignition, and had replaced the carburetor several times in an effort to remedy the problem. When the vibration persisted, the manufacturer replaced the spark plugs, wires, coil, points, and distributor, all to no avail. It was at this point that the authors were called.

The authors took vibration readings on the engine, and recorded a half-order vibration of 64 mils. Since this vibration was not torque-sensitive (it only occurred when the car was stopped), the authors concluded that the half-order vibration must be caused by either the fuel flow or the engine ignition.

When the manufacturer stated that replacing the carburetor and the ignition system had not solved the problem, the authors decided to disassemble the engine. Inside the engine, under the intake manifold, were two pounds of baked beans blocking the number eight intake valve! After we cleaned out the intake valve, the engine ran perfectly.

1.0 ENGINES

Engines obtain power (and hence their ability to do work) by the burning of fuels such as gasoline, diesel fuel, jet fuel, coal, oil, and even wood. As these fuels are burned they release rapidly expanding

gases, which may act directly on a piston or turbine rotor. The fuel may also be used to heat another liquid such as water, causing it to expand against a piston or turbine.

Work can be done in an engine by two methods:

1. External combustion
2. Internal combustion

External combustion engines burn fuel outside of the cylinder or turbine casing. The fuel is used to heat another liquid (usually water), causing the liquid to turn to steam. The expansive force of the steam is directed against a piston or turbine, which causes motion.

The external combustion engine can range in size from tiny toy steam engines to large reciprocating steam engines developing several thousand horsepower (HP), on up to steam turbines developing over 1 million HP.

Internal combustion engines burn their fuel inside the cylinder or turbine casing. The fuel-air mixture burns rapidly, and the expanding gases push against the piston or turbine.

Internal combustion engines range in size from small reciprocating engines used in model aircraft (developing far less than 1 HP) to large reciprocating engines developing over 10,000 HP and turbine engines (such as jet engines) ranging from 100 HP to more than 60,000 HP.

This chapter will concentrate on the reciprocating internal combustion engine. A **reciprocating** engine is one with a piston that travels up and down (or back and forth) in a cylinder (from the word **reciprocate**, which means to move back and forth alternatively).

The earliest engines were first built in the 1800's by a French engineer named Alphonse Eugene Beau De Rochas (1815-1893), who developed the basic principles of an efficient reciprocating internal combustion engine. Those principles were:

- Use the smallest possible surface volume.
- Use the most rapid expansion process.
- Use the maximum expansion process.
- Use the maximum pressure at the beginning of the expansion process.

These principles outlined the basic requirements for an efficient 4-cycle engine (also called a 4-stroke engine). Each traverse of the cylinder by the piston is called a **cycle** or **stroke**. In a 4-cycle engine, there are four distinct cycles, each one with a unique purpose:

1. **Intake stroke** - The piston moves down to the bottom of the cylinder, creating a vacuum which draws the air/fuel mixture into the cylinder.
2. **Compression stroke** - The intake valve closes and the piston moves toward the top of the cylinder, compressing the air/fuel mixture to a high pressure.
3. **Power stroke** - When the piston is at the top of the cylinder the compressed air/fuel mixture is ignited, forcing the piston back to the bottom of the cylinder.
4. **Exhaust stroke** - The exhaust valve opens and the piston moves back toward the top, expelling the combustion gases from the cylinder.

Another type of reciprocating internal combustion engine is the **2-cycle** engine, which has two distinct piston cycles:

1. As the cylinder moves upward on the compression stroke, the air/fuel mixture is drawn into the crankcase. Ignition occurs at the top of the compression stroke, which is followed by the power stroke.

2. As the cylinder moves down on the power stroke it compresses the air/fuel mixture in the crankcase. A sliding valve is opened when the piston is at the bottom of the stroke, and the exhaust gases are allowed out the valve. At the same time the compressed air/fuel mixture enters the cylinder, which helps to push the remaining exhaust gases from the cylinder. The valves close and the piston begins another compression stroke.

A 2-cycle engine provides a power stroke on each cycle of the engine, whereas a 4-cycle engine provides a power stroke on every other engine cycle.

The first successful Beau De Rochas engine was not constructed until a German named Nikolaus August Otto (1832-1891) built such an engine in 1876. It had a very small output, and was extremely heavy, but it initiated the development and production of spark-ignited engines. Almost at the same time, Rudolf Diesel (1858-1913) was working on the thermodynamic principles of compression ignition (now known as the diesel cycle). Diesel produced his first successful engine in 1892.

Before Beau De Rochas' and Diesel's development of spark- and compression-ignited engines (both of which were originally 4-stroke), two Englishmen named Barnett and Sir Dugald Clerk (1854-1932) were developing the 2-stroke engine. There are many 2-stroke engines in production today; however, the otto cycle 4-stroke engine is by far the most common design.

1.1 Pressure Volume (PV) Curves

You can get a great deal of information on an engine's performance by plotting the pressure within the cylinder relative to the cylinder's volume. Since the piston is always moving, both the pressure and the volume within the cylinder are always changing. These plots, called **pressure volume** (or **PV**) **curves** are different for spark- and compression-ignited engines, and they contain much information on an engine's timing and ignition.

Ideal Otto Cycle

Figure 61 shows the ideal otto cycle (also called **explosive cycle**) for a normally-aspirated four-stroke engine. Beginning at point 1, the fuel-air mixture is inducted into the cylinder, the intake valve closes, and compression begins. The portion of the curve between points 1 and 2 represents the piston's compression cycle, since, as the piston moves upward, volume decreases while pressure increases.

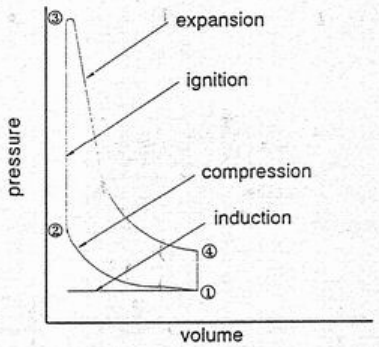


Figure 61 - *the PV curve for an ideal otto cycle*

Ignition occurs at point 2 on the PV curve in Figure 61. Point 2 is at the completion of the compression cycle, just before top dead center. When ignition occurs there is a sudden, rapid rise in pressure with little or no change in volume (since the piston is essentially at top dead center); this is called **constant volume ignition**. Note that the PV curve rises straight up to point 3, since pressure is increasing dramatically while volume is virtually unchanged.

Point 3 on the PV curve is the beginning of the power stroke, where the expanding combustion gases force the piston down to the bottom of the cylinder. As the piston descends, the volume increases while the pressure decreases. Expansion of the combustion gases occurs **adiabatically** (i.e. without the gain or loss of heat since combustion is so rapid) and continues to push the piston down until the exhaust valve opens and the exhaust gases are vented to the atmosphere.

The opening of the exhaust valve occurs at point 4 on the PV curve. As the gases are expelled from the cylinder, the pressure volume decreases. The curve moves downward from point 4 and meets point 1, since the piston is now at the beginning of its next intake cycle.

There is much information in the "heel" and the "toe" of the curves that relates to valve timing, ignition timing, and the expansion of the gases in the cylinder. If you imagine the PV curve as the side view of a foot, the heel will be the area at point 2 while the toe will be the area at points 1 and 4.

The toe of the curve contains information on valve overlap: whether the intake and exhaust valves are opening separately or are open at the same time, and how well gases are being exhausted and fuel is being introduced.

The heel of the curve contains information on the dynamics of fuel ignition: if fuel is ignited evenly and at the right time or if preignition is present (preignition is explained in section 2.2).

Ideal Diesel Cycle

Figure 62 illustrates the ideal diesel cycle (also called the **non-explosive cycle**) for a 4-cycle diesel engine. At point 1, the cylinder is at the bottom of the cylinder, beginning its compression cycle.

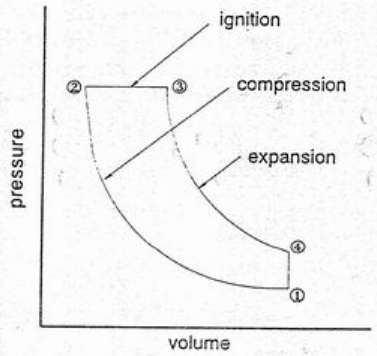


Figure 62 - *the PV curve for an ideal diesel cycle*

The cylinder begins to move upward, compressing air, and reaches maximum pressure (and minimum volume) at point 2. Fuel is then injected into the cylinder, igniting immediately due to the heat generated by the compression of air in the cylinder.

Fuel is burned between points 2 and 3 on the curve. Combustion creates an increase in volume while pressure remains constant; this is called **constant pressure ignition**. The expansion of the combustion gases occurs between points 3 and 4, as the cylinder is forced down and the volume increases.

The toe of this PV curve (the area of points 1 and 4) also contains information on valve closure and efficiency, while the heel of the curve (the area of points 2 and 3) provides information on the combustion process.

PV curves are a source of important information concerning the operating condition of an internal combustion engine. To obtain a PV curve for modern high-speed engines, you need a way to insert a pressure transducer into the cylinder (called a **cylinder tap**). Attempts have been made to do this using specially-designed spark plugs with an internal pressure transducer, but they have not been very successful. Normally a high-temperature, high-pressure transducer is inserted into the cylinder tap, and the PV curve is obtained by comparing that pressure with the angle of crank rotation (which is directly convertible into cylinder volume).

Although most large reciprocating engines have cylinder taps, virtually none of the smaller engines (such as automobile engines) are equipped with taps. Therefore the analysis of PV curves is normally limited to large, stationary reciprocating engines.

1.2 Compression Ratio

The **compression ratio** of an engine is defined as the largest volume in the cylinder (when the piston is at bottom dead center) divided by the smallest volume in the cylinder (when the piston is at top dead center).

Modern spark-ignited engines run with compression ratios from 4 or 5 up to 10 or 12, whereas diesel or compression-ignited engines can run with compression ratios as high as 20 to 25.

The total work done is proportional to the compression ratio: the higher the compression ratio the more efficient the engine. There are limits to compression ratio, some of which include

- the mechanical design of the system
- the loads on the crankshaft and connecting rod bearings
- the forces on the piston

In addition, high compression ratios result in higher ignition temperatures, which increases the temperature stress on both the piston and the cylinder wall.

2.0 VIBRATORY FORCES IN RECIPROCATING ENGINES

To understand the forces that exist in a reciprocating engine, we must look at the slider crank mechanism (normally called a single-cylinder engine), because all larger reciprocating engines are simply multiplications of the single-cylinder principle.

2.1 The Slider Crank Mechanism

The slider crank consists of a crankshaft with an off-center crank pin, as shown in Figure 63.

A connecting rod is attached to the crank pin which goes to the wrist pin of the piston. As the crankshaft rotates, the connecting rod forces the piston up and down in the cylinder. The maximum volume in the cylinder occurs when the piston is farthest from the top of the cylinder, and the minimum

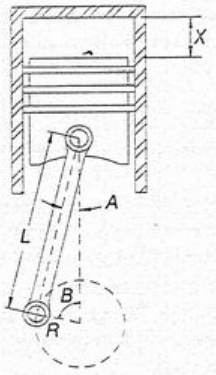


Figure 63 - *the slider crank mechanism*

volume occurs when the piston is nearest to the top of the cylinder. The compression ratio would be the maximum volume divided by the minimum volume.

Rotating and Reciprocating Forces

The slider crank mechanism has two first-order (1*RPM) vibrations: a rotating force due to crankshaft imbalance and a reciprocating force due to the up-and-down movement of the piston in the cylinder. The first-order rotating force can be corrected by balancing the crankshaft, but correcting the reciprocating force is not as easy.

To successfully diagnose vibration problems in internal combustion engines, we must understand these forces and how they are developed. And in order to define and understand these forces, we will use a very basic formula of physics, $F=ma$, which says that force equals mass times acceleration. To define and understand these forces, we must

1. Determine the instantaneous acceleration of the piston in the cylinder
2. Determine the rotating and reciprocating masses
3. Multiply these two values to find the force

2.1.1 Determine the Instantaneous Acceleration

To find the instantaneous acceleration of the piston, we must first derive a formula to calculate the position of the piston in the cylinder at any instant. The derivation of this formula is shown in the box on page 117. For our purposes, we can simply discuss the ramifications of this formula.

Calculating the momentary position of the piston results in an expression which has an infinite number of terms. The first term represents the crankshaft frequency, and each successive term represents a harmonic order of the crankshaft frequency; the second term is the second order, the third term is the third order, and so on.

The third and greater orders are of small magnitudes because of the R/L ratio (the radius of the crank pin divided by the length of the connecting rod). Each order is determined in part by R/xL, where x is the number of the order: the ratio is $R/3L$ for the third order, $R/4L$ for the fourth order, and so on. Since

FINDING THE POSITION OF A SLIDER CRANK MECHANISM

The value of a slider crank's position x at any moment is given by

$$X = L + R - (L * \cos(A)) - (R * \cos(B)) \qquad \text{(Formula 12-1)}$$

From the Law of Sines:

$$L * \sin(A) = R * \sin(B) \qquad \text{(Formula 12-2)}$$

And from the general trigonometric relationships:

$$cos(A) = \sqrt{(1 - \sin^2(A))} \text{ so that } cos(A) = \sqrt{1 - \left(\frac{r}{l}\right)^2 * \sin^2(B)} \qquad \text{(Formula 12-3)}$$

Substituting these relationsips into formula 1 gives

$$X = L + R - L * \sqrt{1 - \left(\frac{R}{L}\right)^2 * \sin^2(B)} - R * \cos(B) \qquad \text{(Formula 12-4)}$$

Expanding and neglecting all but the first two terms:

$$X = R * [1 - \cos(B) + \left(\frac{R}{2L}\right) * \sin^2(B)] \qquad \text{(Formula 12-5)}$$

Setting $B = \omega t$ and differentiating twice to get acceleration gives

$$A = \omega^2 * R * [\cos(\omega t) + \left(\frac{R}{L}\right) * \cos(2\omega t)] \qquad \text{(Formula 12-6)}$$

Since force = mass * acceleration ($f=ma$) it can be seen that forces exist in the slider crank system at rotational frequency and at twice rotational frequency.

L keeps getting larger while R stays the same, the higher orders have lower and lower magnitudes. This allows us to ignore the third and greater orders and concentrate only on the first and second orders (which makes things much easier!).

If we use only the first two terms, Formula 12-2 gives us an expression for the instantaneous position of the piston with respect to the cylinder head. If we change the angle B in Figure 63 to ωt for the rotational position of the crankshaft, and then differentiate Formula 12-2 twice, we go from distance x to acceleration a. Formula 12-3 gives us the instantaneous acceleration of the piston for any angle of the crankshaft rotation.

If we substitute the formula for instantaneous acceleration into $F=ma$, we can calculate the forces generated by the sliding mechanism at any angle of crankshaft rotation by multiplying the instantaneous acceleration times the mass.

But how do we find the mass? And is the mass the same for both the rotating *and* the reciprocating force?

2.1.2 Determining the Mass

The slider crank mechanism consists essentially of two masses: a **rotating mass** and a **reciprocating mass**. If we placed the connecting rod on two scales as shown in Figure 64, making sure that the rod is perfectly level, the weight shown on the scale at the crankshaft end would be the rotational mass of the connecting rod, and the weight at the other end would be the reciprocating mass of the connecting rod. We only need to divide the weight by gravity to obtain the mass.

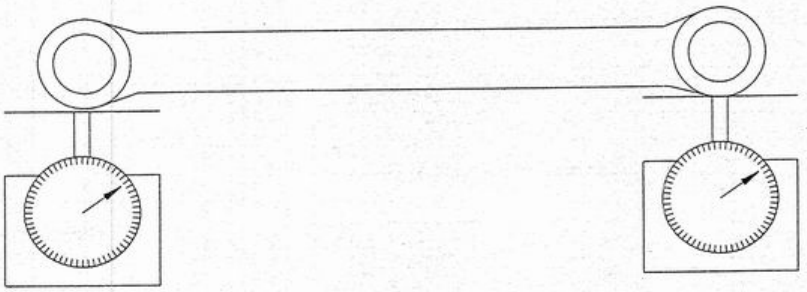


Figure 64 - *rotating and reciprocating mass*

The total reciprocating mass is the reciprocating mass of the connecting rod, the wrist pin, and the piston itself. The mass of the piston rings is usually ignored, since the friction force between the piston rings and the cylinder wall cancels out the effect of the piston rings in the total reciprocating mass. The total rotating mass is the rotating mass of the connecting rod plus the connecting pin on the crankshaft.

2.1.3 Determine the Rotating and Reciprocating Forces

Now that we have both mass and instantaneous acceleration, we can calculate the forces that are generated as a result of the engine's operation.

What the previous equations tell us is that we have three forces generated by a slider crank mechanism:

1. A rotating force at 1*RPM of the crankshaft
2. A reciprocating force that is equal to the rotational frequency of the crankshaft
3. A reciprocating force equal to 2 * *crankshaft rotational frequency*

2.1.4 Correcting for First-Order Reciprocating Forces

The reciprocating force is in-line with the cylinder bore and perpendicular to the rotational axis of the crankshaft. The first-order reciprocating force can be reduced by adding a counterweight to the crankshaft, which generates a force equal and opposite to the force generated by the total reciprocating mass. It is difficult to add that mass directly in-line with the piston unless the correction weight is split on each side of the connecting rod, as shown in Figure 65.

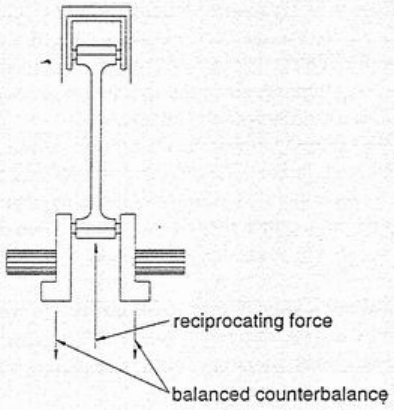


Figure 65 - *reciprocating piston with no moment*

Since the downward force of the counterweights is on either side of the piston, the crank pin will move straight down. If you have trouble visualizing this, lay a pencil on this page in line with the crank pin. Now, place your fingers on the pencil at the two points indicated by the arrows and push down equally. The pencil slides down the page and remains straight, without tipping or rocking.

Instead of being split, the correction weight may be added on only one side of the piston, as shown in Figure 66.

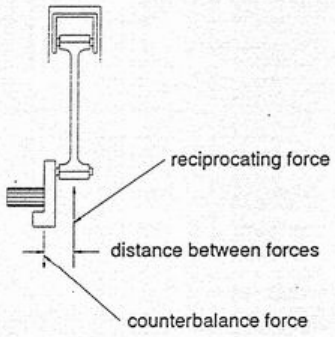


Figure 66 - *an unbalanced counterbalance force*

If all of the correction weight is added on one side of the crank pin, the two masses (the piston and the correction weight) create forces that do not act in-line with each other, developing a moment. The distance between these two forces is called a **moment arm**, and a **moment** is the product of the moment arm and one of the two forces.

If the correction force is directly in-line with the imbalance force, the two forces add to zero. If the correction force is not directly in-line with the imbalance force, then we have a moment which is

equal to one of those forces (not both) times the distance between them. Even though the in-line force may be balanced out, the moment force is not balanced out and causes the engine to rock.

Moment arms provide mechanical advantage. For example, bolts can be loosened and tightened easily by using a long wrench, since the moment arm between the center of the bolt and the end of the wrench provides a mechanical advantage.

In Figure 66, the upward force of the piston acts on one side of the crank pin, while the downward force of the counterweight is applied at the opposite end of the crank pin. Even if these forces are equal, the moment still imparts a rocking motion to the crank pin. Now lay your pencil over the crank pin in Figure 66 and put a finger at each end. Push up on one end and down on the other. Instead of moving straight across the page, the pencil tends to remain in the same place and rotate around its center.

Normally, in a single-cylinder engine, the correction weight is split and added on each side of the crank pin. In some engine designs, however, this split is not possible. In these cases a perfect correction for the fundamental reciprocating vibration is obtainable, but there is no correction for the resulting moment.

2.1.5 Side-to-Side Forces

Unfortunately, using counterweights to correct for the first-order reciprocating force causes the entire engine to move side to side, regardless of whether the counterweights are split. The reason for this is illustrated in Figure 67, which shows the end view of a crankshaft with a counterbalanced piston.

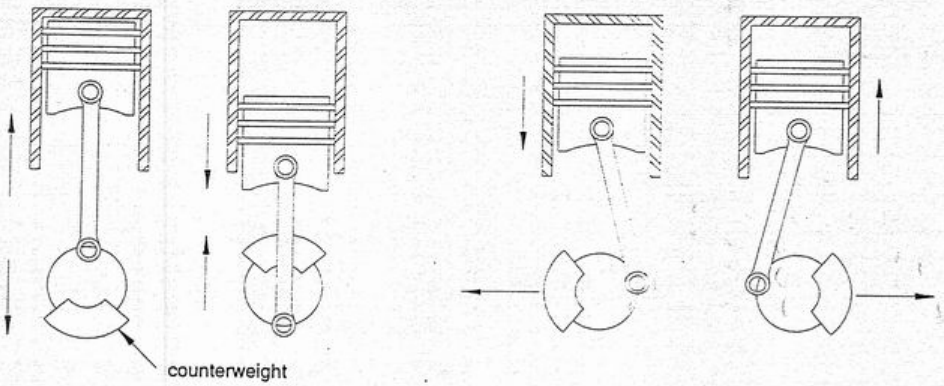


Figure 67 - *the origin of side-to-side motion*

The presence of the counterweight creates a first-order rotating force that is canceled out only when the piston is at the top and bottom of its stroke. When the piston is at the top of the cylinder, the counterweight is at the bottom of the crankshaft and the reciprocating force is canceled out.

However, 90 degrees later, the counterweight has moved and is now pointing out to the right. And here there is no opposing reciprocating force from the piston to cancel out the rotating force of the counterweight. In the absence of a canceling force, the counterweight creates a 1*RPM force that pushes the engine to the side.

Therefore, if a first-order weight were added to the crankshaft to null out the first order reciprocating component, it would only move the vibration from in-line with the cylinder bore to 90 degrees to the cylinder bore, perpendicular to the axis of the crankshaft.

That is, the correction weight added to the crankshaft would properly cancel out the reciprocating first-order forces of the piston and its masses in the direction of the reciprocating motion. But there is little value in this, since the addition of the weight would move the vibration to 90 degrees to the cylinder bore, and a second weight cannot be added to cancel out this vibration.

Since it is impossible to completely remove the first-order reciprocating force without introducing excessive side-to-side forces, most engine designers opt for removing only 50% of the reciprocating force (the counterweights cancel out only half of the reciprocating force). This is a tradeoff: the 1*RPM reciprocating force is only half as small as it could be, but the side-to-side force is only half as high as it could be. The vibration in-line with the cylinder bore would be reduced by 50%, and a 50% force would be added at 90 degrees to the cylinder bore, resulting in a small first-order orbit at crankshaft rotational frequency.

The presence of the side-to-side force is the reason that motor mounts are made of rubber. Rubber mounts can absorb most of the side-to-side force without transmitting it to the frame.

2.1.6 Correcting for Second-Order Reciprocating Force

The second order of the sliding mass is also a force that is in-line with the cylinder bore and is 90 degrees to the crankshaft. In most single-cylinder engines, no attempt is made to counteract this second-order reciprocating force.

There are engine designs that allow for compensation of this force. The most obvious method would be to build the engine with horizontally opposed cylinders; as one cylinder moves up, its opposite counterpart is moving down. However, this is not a practical design for all applications.

Some small engines are made with counter-rotating shafts that run at twice the crankshaft frequency. These counter-rotating shafts have imbalances which are phased so that they counteract the second-order reciprocating force in-line with the cylinder bore, as well as each other at 90 degrees to the cylinder bore. This system is known as the Maxwell system.

There are some engine designs that do not require compensation for the first- and second-order reciprocating forces. These configurations are listed in Table 1.

TYPE	CRANK THROWS	PRIMARY INERTIA FORCES	SECONDARY INERTIA FORCES	PRIMARY COUPLES	SECONDARY COUPLES
1 cylinder	1	No	No	None	None
2 cylinder - vertical	2 @ 180	Yes	No	No	None
2 cylinder - opposed	2 @ 180	Yes	Yes	No	No
4 cylinder - vertical	2 @ 180	Yes	No	Yes	Yes
4 cylinder - vertical	4 @ 90	Yes	Yes	No	Yes
6 cylinder - vertical	3 @ 120	Yes	Yes	Yes	Yes
6 cylinder - vertical	6 @ 60	Yes	Yes	No	Yes
8 cylinder - vertical	4 @ 90	Yes	Yes	Yes	Yes
8 cylinder - 90V	4 @ 90	Yes	Yes	No	Yes

Table 1 - *engine types and reciprocating forces*

2.2 Knock and Preignition

One of the problems that can occur in higher compression ratio engines is called **compression knock** (often erroneously called **preignition**). Compression knock is influenced by many factors:

- The type of fuel being used
- The fuel-air ratio
- The compression ratio
- The ignition timing
- The design of the combustion chamber
- The valve timing

It is possible to have both the intake and exhaust valves open at the same time. In fact, it is usually preferable to have some valve overlap, allowing hot exhaust gases to enter with the fuel-air mixture. If not properly adjusted (if the exhaust valve opens too soon, for example), valve overlap may result in compression knock, causing serious damage to the bearings and the top of the piston. Normally compression knock results in high shock vibration in the engine, can be heard as a loud knocking noise, and is easily seen with accelerometers mounted to one of the head bolts in the engine. It is important not to allow combustion knock to continue for a long time.

Preignition is caused by a hot spot on the piston igniting the fuel/air mixture prematurely. Hot spots can be caused by a piece of foreign material (carbon in the fuel or the air, for example) sticking to the piston, or by an irregular combustion pattern excessively heating one portion of the piston's surface. Preignition results in excessive pressures and increased temperatures in the combustion chamber, and also causes a considerable loss in engine power. Preignition sounds like a "ping," and can be detected with accelerometers mounted on the engine's head bolts. Head bolts are normally high-temperature locations, and care must be taken in the selection of the accelerometers used.

In the case of a diesel engine there may be **diesel engine knock**, which causes the engine to run very roughly. There is usually a significant amount of half-order vibration on the 4-stroke diesel engine.

2.3 Gas Pressure Forces

Gas pressure forces result from the expansion of combustion gases in the cylinder. These forces can be resolved into vertical components (which act in-line with the center line of the cylinder along the connecting rod) and horizontal components (which act perpendicular to the center line of the cylinder).

The in-line forces act upon the main bearings of the crankshaft, the crank-end rod bearing, and the piston-end rod bearing. The perpendicular forces act against the cylinder wall and can result in piston slap, particularly if the skirt clearance is excessive. The **piston skirt** is that portion of the piston below the centerline of the wrist pin. Its primary function is to guide the piston squarely in the cylinder so that the piston does not rock back and forth.

The forces on the crankshaft are different between the piston's compression stroke and power stroke: the force is much higher on the power stroke due to the explosive expansion of the combustion gases. Therefore, some engines have the wrist pin offset from the center of the piston, so that the connecting rod is more vertical on the power stroke than it is on the compression stroke.

Piston slap sounds like a tick in the engine, and can be detected with accelerometers mounted on the crankcase side of the cylinder wall.

Gas pressure forces in 4-cycle engines normally occur as a family of harmonics, typically the first through the sixth order. In most engines these forces occur as a harmonic family at first order, and many times with half-order components (1.5 order, second order, 2.5 order, etc.). In 2-cycle engines the half-order components are rarely present.

2.4 Torsional Vibration Forces

Due to the periodic variation of the gas pressure forces on the piston, there is an increase and decrease in the torsional force on the crankshaft and any shafts driven by the engine. Torsional vibration in a 4-cycle engine will also occur at half-orders. In a 2-cycle engine, the half-order components are not present.

It is inevitable that, at some operating frequency, one of these forces or their harmonics will excite a natural resonance in the system. It is best to design the engine so that any resonances are at the higher harmonics, where the amplitudes are low enough so that the resonant amplification is not a serious problem.

In the case of torsional vibration, one of these harmonics will coincide with the first-mode torsional resonance of the crankshaft. Because torsional damping is not high, this can easily result in crankshaft failure. Therefore most engines have torsional dampers attached to one end of the crankshaft and a flywheel at the other. The torsional damper acts as a vibration absorber at the torsional resonance of the crankshaft (refer to the Pumps chapter, section 4.3.1, for a discussion on dynamic absorbers).

When internal combustion engines are used to drive relatively long shafts (such as stationary engines driving gear boxes or pumps), it is important to provide sufficient torsional vibration damping so that the engine crankshaft and the drive shaft are not damaged.

2.5 Crankshaft Balance

The crankshaft is a large rotational component which must be balanced. Depending on the type of crankshaft (the number of throws, their angular location, and the number of cylinders in the engine), the crankshaft may or may not need to be balanced in a balancing machine with **bobweights**, which are weights mounted at the crankshaft throw to represent the rotational mass (and one-half of the reciprocating mass) of the connecting rod. These crankshaft configurations and the necessity for bobweights are illustrated in Table 2.

Number of cranks	Number of cylinders	Cylinder pattern
1	1	–
2	2	in-line
2	2	in-line
1	2	V 90°
2	4	V 90°
4	8	V 90°
3	3	in-line
5	5	in-line
4	4	Narrow V

Table 2 - *crankshafts requiring bobweights during balancing*

If the engine is disassembled and the crankshaft is balanced in a machine, you must determine whether bobweights are needed (and their correct value) before you perform the balancing procedure.

3.0 ANALYZING ENGINE VIBRATION

Whether the vibrations already discussed are excessive depends on the fuel, ignition, ignition timing, and the load on the engine. Half-order vibration in a 4-stroke engine is usually an indication of a fuel or ignition problem; check to determine the source of the vibration. Refer to the accompanying table for help in analyzing common causes of vibration in engines.

Acceptable vibration amplitudes vary depending on the engine design and its application. Engines mounted in vehicles are resiliently mounted, particularly those engines where the couple or inertia forces cannot be totally balanced out. Sometimes it is important to design crankshaft counterweights so that the vibration is reduced in one direction and increased in the other.

For example, an automobile manufacturer found that their four-cylinder engine moving up and down would impact the hood of the car, so the counterweights were changed to cancel out the vertical inertial forces, which resulted in the engine moving side to side in the vehicle. In motorcycles, it is usually preferable to have the engine move side to side instead of up and down. On some permanent installations, depending on the foundation resonance, it may be necessary to overcompensate for the inertial forces so that the primary vibration is transverse instead of in-line with the cylinders.

In some engine installations, it is not possible to provide enough counterweight in the crankshaft, so additional weights are added in the damper and the flywheel. If the damper or the flywheel has been changed, and the engine owner does not realize that a counterweight is incorporated in that component, the engine may not be in balance when the new component is added.

3.1 Reciprocating Loads on Bearings

During piston cycles in reciprocating engines, there are dramatic changes in the loads on the connecting rod bearings at the wrist pin and at the crank pin. To some degree, these reciprocating loads are also transmitted to the main crankshaft bearings.

These large reciprocating loads limit the life of these bearings. A simple sleeve bearing, radially loaded with clean lubricant, theoretically has an infinite life; this is not true in a reciprocating engine, where wear causes the bearing clearances to increase.

As bearing clearances increase, the reciprocating loads cause an increase in the pounding effect (from the engine's power cycle) which adds to the increase in bearing clearance. Increased bearing clearance also causes an increase in the harmonic content of the vibration in the engine. In some cases bearing clearance is so great that the impacts caused by this excessive clearance can be heard with the human ear. It is important to detect the increase in the impacting due to increased bearing clearance and correct the problem before the engine is destroyed.

3.2 Vibrations from Engine Accessories

Other sources of engine vibration include accessories such as generators, alternators, water pumps, oil pumps, fuel pumps, cooling systems, and turbochargers and superchargers. Engine-driven cooling systems, usually belt-driven fans rotating near a radiator, are a source of fan vibration (see the chapter on Fans) as well as belt vibration (covered in the Power Transmission Components chapter).

Turbochargers and superchargers are also sources of vibration. A turbocharger, which is a turbine driven by the engine's exhaust, can be a source of blade pass frequency. In some cases blade pass

frequency from turbochargers has been recorded as high as 100,000 RPM (approximately 1,667 Hz). Superchargers are either gear- or belt-driven, and may have excessive gear mesh frequencies or belt pass frequencies that excite some system resonance. Also, the drive belt on a belt-driven supercharger will have a belt strand resonance which may be excited by some other forcing function in the system.

An engine usually drives something: automobile engines drive the transmission, drive shaft, and rear axle and wheels, while stationary engines can drive compressors, pumps, and many other types of machinery. Engines are also torsional vibration generators, due to the jerky motion of the crankshaft. This is caused by the hammering of the power cycles of the pistons. A flywheel on the end of the crankshaft helps to smooth out the jerks, but some torsional vibration may be imparted to the driven machine.

COMMON CAUSES OF ENGINE VIBRATION			
ENGINE ORDER	4 CYLINDER IN-LINE	6 CYLINDER IN-LINE	V8 (90 DEGREE V)
½-order torque sensitive	Single-cylinder misfire	Single-cylinder misfire	Single-cylinder misfire
1st-order imbalance	Unusual	Unusual	Unusual
1 ½-order torque sensitive	—	—	—
2nd-order	Normal	Unusual	Unusual
2nd-order torque sensitive	Normal	—	Bank-to-bank EGR
3rd-order torque sensitive	—	Normal	—
4th-order torque sensitive	Normal (small amount)	—	Normal

SUMMARY

Work can be done in an engine by two methods:

1. External combustion
2. Internal combustion

External combustion engines burn fuel outside of the cylinder or turbine casing. The fuel is used to heat another liquid (usually water), causing the liquid to turn to steam. The expansive force of the steam is directed against a piston or turbine, which causes motion.

Internal combustion engines burn their fuel inside the cylinder or turbine casing. The fuel-air mixture burns rapidly, and the expanding gases push against the piston or turbine.

REFERENCES

Dott. Ing. Luigi Buzzi, Crankshaft Balancing, Booklet #19, 1982, CEMB S.p.A., Mandello del Lario (Co) Italy.

Lichty, Lester C., Internal Combustion Engines, Copyright 1951, McGraw-Hill, New York, NY.

Hickling, Robert and Kamal, Mournir M., Engine Noise: Excitation, Vibration, and Radiation, Copyright 1982, Plenum Press, New York, NY.

Heywood, John B., Internal Combustion Engine Fundamentals, Copyright 1988, McGraw-Hill, New York, NY.

13 MACHINE-SPECIFIC ANALYSIS: TURBINES

INTRODUCTION

A turbine is a form of heat engine. Heat energy of steam is transformed into kinetic energy as the steam flows through nozzles. This kinetic energy is transformed into work as the steam reacts with the turbine rotor, causing it to rotate.

Heat energy for a turbine does not necessarily have to come from steam. Jet engines, for example, make use of the heat energy available in the combustion of jet fuel. Turbines that derive heat energy from sources other than steam are called gas turbines.

The analysis and solution of vibration problems in gas turbines is a highly specialized science, and is beyond the scope of this text. Therefore, this chapter will deal with vibration problems that are specific to steam turbines.

1.0 PRINCIPLES OF TURBINE OPERATION

A steam turbine has four fundamental parts:

1. **Rotor** - a turbine rotor is a shaft with an array of blades mounted axially. The blades are usually airfoil in cross section, but they may be deep buckets instead. The steam blowing past the rotor blades causes the shaft to turn. Turbine rotors have several rows of blades, so that the maximum amount of work can be extracted from the steam.
2. **Stator** - the turbine stator is a stationary frame that houses the rotor. The stator serves to confine and direct the flow of steam through the turbine, and to increase the turbine's efficiency. As the steam flows past the rotor blades, a small rotational component is imparted to the steam flow. This rotation of the steam would decrease the amount of available energy

to the remaining rotor blades, decreasing the turbine's efficiency. The stator is lined with blades that break up the rotational component of the steam, providing more energy for the remaining rotor blades.

3. **Nozzles** - the nozzles are responsible for maximal conversion of available steam energy into kinetic energy. Due to the decreasing cross-section of the nozzle, a given volume of steam entering the nozzle exits at a high velocity. Nozzles are designed to minimize friction between the steam and the nozzle wall, and to prevent the formation of eddy currents in the flowing steam.
4. **Frame** - the frame supports both the stator and the rotor, and holds the rotor bearings. Rotor bearings are usually sleeve or tilt pad design.

The entire turbine assembly is mounted to a large, stable foundation to prevent warpage and settling. The turbine feet usually rest on soleplates, which are flat metal plates or castings. The turbine is bolted to the soleplate, and the soleplate rests on shims and is bolted (and grouted) to the foundation.

Steam first enters the high pressure (HP) section of the turbine and flows into the nozzles. Control valves are used to carefully time when steam exits the nozzles and flows into the rotor. Correct steam timing is important for optimal efficiency and maintaining proper bearing loading.

As the steam flows through the HP section, it causes the rotor to turn. But the steam begins to expand once it leaves the nozzles. Since expansion represents an energy loss (the steam cools as it expands), the steam can do less work as it flows through the turbine. To compensate for this energy loss, successive rotor blading stages are at a larger diameter. This creates a larger moment arm, giving the steam an extra mechanical advantage in rotating the shaft.

Once the steam leaves the HP section it can take two different courses. Some turbine designs return the steam to the boiler for reheating (called **superheating**, since the steam never has a chance to return to water). The superheated steam is returned through one or more reheat sections of the turbine before entering the low pressure (LP) section. Other turbines dispense with the reheat sections and duct the steam directly to the LP section from the HP section.

One or more LP sections may be present in the turbine. Rotor blades in the LP sections are at successively larger diameters (just like the HP section) to compensate for the expansion of the steam. Since the steam is at a lower pressure in the LP section, the rotor blades must be much larger than the HP section to provide an even greater mechanical advantage.

By the time the steam reaches the LP sections, it has expanded and cooled to a point where water begins to condense within the steam. There is still sufficient steam to perform work; the steam, however, is laden with water. This condition is often referred to as "wet steam," due to the presence of water from condensation. Steam in the HP sections, where condensation has not yet occurred, is referred to as "dry steam."

After the steam has passed through the LP section(s), it is now at too low a temperature and pressure to do any useful work. Some turbines, called **condensing** turbines, place the LP section directly above a condenser. The condenser is a large chamber that allows the steam to condense back into liquid water, where it is returned to the boiler for reconversion into steam. As steam condenses into water its volume decreases, creating a partial vacuum in the condenser (usually 1-2 inches of Mercury above vacuum).

In other types of turbines (called **non-condensing**), the steam is piped from the LP section to other parts of the plant, where it may be used for heat or for the manufacturing process. The steam is eventually routed to a condenser elsewhere in the facility.

2.0 TURBINE VIBRATION

Turbine vibration is almost always first-order (1*RPM), with imbalance and rotor bow being the two most common sources of vibration. Due to their size, and the fact that they operate at high speeds, turbines usually run above their first-mode resonance (often above the second and third modes as well). Coal-fired turbines usually run at 3,600 RPM, above the third critical. Turbines at nuclear power plants usually run at 1,800 RPM, above the second-mode resonance.

Therefore, a turbine will run through at least one resonance during startup and coastdown. This, coupled with the tight clearance between the rotor and stator blades, necessitates extremely accurate alignment of the rotor shaft within the stator.

It is very important to do all vibration analysis on turbines when the turbine is thermally stable. Long turbine/generator shafts that are rigidly coupled can grow more than one inch axially due to thermal growth. Allow time for oil temperature to equalize throughout the turbine, and for the shaft alignment to assume its normal operation position.

Some of the more common causes of vibration in turbines are listed in this section.

2.1 Imbalance

Mechanical imbalance in a turbine rotor is caused by the presence of a heavy spot somewhere on the rotor (usually in the blades, rather than the shaft). Mechanical imbalance can have a number of causes:

- Errors in assembly.
- A broken blade or bucket, causing a heavy spot 180 degrees away. Broken blades also cause a cranking vector at running speed due to steam imbalance (cranking vectors will be explained in a moment).
- Balancing a turbine rotor that is not thermally stable, such as immediately after the installation of a shrunk-on part. The rotor may operate smoothly at startup, but will be out of balance once it has stabilized at its operating temperature.
- Balancing bowed rotors. This is especially a problem with spare rotors that have been stored horizontally for long periods of time.

The LP rotors usually require balancing more often than HP rotors, for two reasons:

1. LP rotors are subjected to wet steam, which is very corrosive and erodes rotor blades.
2. LP rotors operate close to their second bending mode, and are therefore sensitive to imbalance.

2.1.1 Cranking Forces

Another type of imbalance in turbines is called **steam imbalance** (sometimes called a **cranking vector**). It was stated previously that the opening of control valves is closely timed for efficient operation and optimal bearing loading. Optimal bearing loading means that the force from the impact of the steam against the blades is evenly distributed around the rotor.

Certain conditions can cause the force of the steam against the rotor to become imbalanced. The tips of rotor blades can often break off due to high centrifugal forces. The steam that enters the rotor at this point does little or no work. On the other side of the rotor, however, the blades may not be broken and are experiencing the full force of the steam.

This imbalanced application of force against the rotor creates a 1*RPM imbalance force that rotates around the shaft. It is called a cranking force because it has the same effect as a hand crank. Imagine

a hand crank used to start antique automobiles. A tangential force is applied continuously to the crank in order to turn it. This tangential force acts on the shaft in the same direction, but acts through the shaft centerline against the bearings. As the crank turns, the tangential force rotates around the shaft.

Likewise, steam imbalance acts like a large crank, applying a cranking force that rotates around the shaft at 1*RPM. This cranking force only appears under load, and disappears when load is removed. However, the imbalance resulting from the broken blade does not disappear.

Broken blades are not the only cause of cranking forces. A control valve that fails to open can also cause a cranking force. Another cause of cranking forces is coupling keys that are driving the shaft. Keys in couplings are not meant to experience torque. Torque from the turbine is transmitted by the tight fit of the coupling hub to the shaft. If the interference fit of the hub is not tight enough, the hub will slip, and the key will be forced to experience the torque.

Loose coupling bolts, loose disks in disk couplings, and loose impellers in flexible couplings can all cause cranking forces. Worn nozzle blocks and/or improper control valve timing are additional sources of steam imbalance that lead to cranking forces.

2.2 Rotor Bow

Turbine rotors, especially those in the HP section, are subjected to extremely high temperatures (over 1,000° F). Over time, these high temperatures can cause rotors to become bowed. Rotors can also become bowed if they are stored horizontally for long periods.

Rotor bow appears to be a static imbalance, since the phase at both ends of the rotor is essentially the same. Installing balance weights at the rotor end planes, however, does not solve the imbalance that results from the rotor bow. This is because an internal bending moment is created between the center of the rotor (where the bow is greatest) and the end planes.

The presence of a bent shaft can be determined by taking axial vibration readings at the 3, 6, 9, and 12 o'clock positions around the shaft. Phase readings that are 180 degrees apart between the 12 and 6 o'clock (or the 3 and 9 o'clock) positions are a good indication of a bent shaft.

It is necessary to install balance weights in the center plane of a bowed rotor. This is not too difficult, since the center plane of most HP rotors is accessible. The analyst must exercise extreme care when mounting weights in the center plane, due to the high heat. Balance weights should be heated before they are installed. Otherwise they might expand and seize before they are completely installed.

2.3 Misalignment

Alignment is very critical for long turbine life. In order to convert as much heat energy to work as possible, the clearance between the rotor and stator blades must be small. Severe misalignment can cause rotor blades to impact the stator, which can lead to steam imbalance.

Misalignment occurs most often in the couplings. Coupling misalignment causes the driven shaft to accelerate and decelerate during each revolution, which increases wear on both the coupling and the bearings. There are many causes of coupling misalignment:

- Foundation settling on either the driver or driven machine
- An improper alignment procedure
- Thermal growth that was not factored into the alignment procedure
- Stresses from attached piping that pulls one of the machines off of alignment

Misalignment may also be caused by bearings that are not on the same level or are not normal to the shaft. If both bearings are fixed, leaving no possibility for thermal growth, it is possible to get high axial vibration. Bearings that are cocked on the shaft cause high axial vibration, and many times second- and third-order vibrations as well.

Coupling misalignment usually causes high second-, third-, and fourth-order frequencies, although 1*RPM vibration is also present. Pure angular misalignment occurs mostly in the axial plane, while pure offset misalignment occurs mostly in the radial plane. A combination of angular and offset misalignment causes high vibrations in both the axial and radial planes.

High vertical-to-horizontal amplitude ratios are common in machinery vibration analysis. For example, vibrations in the vertical plane may be 1/3 or 1/4 of the horizontal amplitude, because machines usually have more freedom to vibrate horizontally than they do vertically.

Horizontal-to-vertical amplitude ratios of 3:1 or greater are often due to resonance. However, shaft misalignment can quite often produce ratios of 3:1 or greater. Therefore, when high ratios are measured, check for a resonance condition first. If resonance does not appear to be present, then shaft misalignment is likely.

The phase relationship across the coupling is also indicative of misalignment. If the phase readings on the coupling side bearing on each machine are essentially in phase or 180 degrees out of phase both horizontally and vertically, then shaft misalignment is almost certainly present.

Refer to the Shaft Alignment chapter in Volume 1 for more information on vibration from misalignment.

2.3.1 Drawdown

Drawdown affects LP rotors that are mounted above a condenser. The near vacuum of a condenser can create a suction that actually pulls the LP rotor off of alignment. This change in alignment can, in extreme cases, cause rotor blades to impact the turbine stator. Misalignment also subjects the bearings to unacceptable loads, and can cause oil whirl in some cases.

2.4 Thermal Growth and Distortion

Large steam turbines may have rotors nearly 200 feet long. These rotors can grow over one inch axially during operation, so allowance for thermal growth must be present. If thermal growth is restricted, or is uneven, then distortion of the turbine occurs.

Most steam turbines are used in the generation of electricity. Generator rotors present many special problems, one of which is additional thermal growth. The electric current generated during operation heats up the rotor, causing it to grow. A generator rotor is a composite of different materials, and each material has its own thermal growth characteristics. Rotor problems such as shorted field windings can cause additional heat, significantly increasing the amount of growth.

2.4.1 Distortion

Distortion does not cause vibration directly. Rather, it creates conditions such as misalignment or rubs which lead to vibration. Solving the vibration does not remedy the problem; you must determine the cause of the distortion and eliminate it.

There are many causes of distortion due to thermal growth. The casing for the HP rotor must grow significantly, due to the extremely high temperatures of the steam first entering the turbine. As the casing heats up, it expands. If the casing is restricted from expansion, vibration on the HP rotor will

increase and appear to be caused from imbalance. Use dial indicators during startup to ensure that the casing expands sufficiently.

Steam turbines that are not protected from exposure are extremely susceptible to distortion. Sunshine on one side of the turbine may heat that side unevenly. Likewise, wind and/or rain can cool one side of a turbine more than the other, causing the casing to warp (and possibly crack).

Heat from nearby piping or leaky seals can cause uneven heating, leading to extreme misalignment. Uneven heating can cause turbine feet to lift completely off the soleplates. If the soleplates themselves become too hot, they can break loose from the foundation.

Piping forces can also cause frame distortion. If the pipes are not adequately supported, their weight can pull on the turbine casing unevenly and cause warpage and distortion.

Condensing turbines can experience water accumulation after shutdown that can lead to distortion of the casing. There is no way to prevent water from accumulating inside a turbine once the steam has been shut off. Depending on the amount of steam in the system, the water may rise above the shaft center line.

Condensing turbines that exhaust upwards depend on incoming steam to create pressure, pushing the wet steam upwards into the condenser. When the steam is shut off the pressure disappears, and all the condensed water runs back down into the turbine casing, sometimes completely filling the casing. If the turbine is still hot as it begins to fill with water, the casing and/or the rotor may be severely warped or cracked.

2.5 Resonance

Resonance is a condition where the system amplifies the vibrations that are present in the turbine. The amount of amplification can be severe and can lead to permanent damage of the turbine.

3.0 BALANCING STEAM TURBINES

From the preceding section, it is obvious that the high temperatures associated with turbine operation can be a major source of vibration. Changing temperatures as the turbine heats up also affect the turbine's response to vibratory forces. Therefore, make sure that the turbine is as thermally stable as possible before you begin any analysis. It takes time for the alignment to stabilize, the oil temperature to equalize (which minimizes distortion), and for the generator rotor (if attached) to stabilize.

A significant portion of turbine balancing work is performed immediately after an outage. Situations such as boiler problems and control difficulties can cause a turbine to trip off after only running for a few minutes. If the turbine had been running with high vibration, customers often request that the rotor be balanced based on the short run data rather than from data taken while the system is stable. The reason for this is that startups can cost a customer hundreds of thousands of dollars:

- When a turbine is not running, the loss of productivity is very costly. Turbines are expensive items, and need to be running nearly continuously in order to pay for themselves (generating electricity, driving pumps, for example).
- Turbine rotors can weigh hundreds of tons. As a result, they will sag under their own weight if they sit still for a short period of time (often as short as 2 minutes). This sag must be worked out of the rotor before it is taken up to running speed by a process called slowrolling. The rotor is spun at low RPMs for an appropriate amount of time until the bow has disappeared. Depending on the turbine's size, slowrolling can take over 12 hours.

If the vibration problem can be solved while the turbine is down for another reason, then the customer will usually opt to do so. Vibration data is usually taken on the rotor bearings, since these provide the best transmission path from the rotor to the outside of the turbine. When the analysis of the vibration indicates that imbalance may be the cause, weights are usually added in the end planes of the rotor, near the bearings. Figure 68 illustrates a sample turbine driving an electrical generator. Unacceptably high levels of vibration were found on the HP rotor and on the generator.

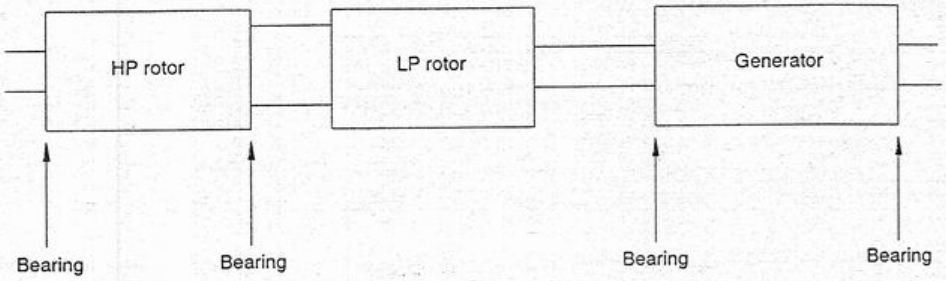


Figure 68 - *a turbine/generator set with high vibration*

Figure 68 shows four bearings with high levels of vibration. It would seem logical, then, to use a multi-plane balance technique as was described in Volume 1 in the General Analysis Techniques and the Field Balancing chapters: as-is and trial weight data would be taken in each plane (in this case each bearing), influence coefficients between each plane would be derived, and a final balance solution for each plane would be calculated. There are three reasons why this is not the best technique for balancing steam turbines:

1. The number of runs required for solution of a multi-plane balancing problem equals the number of planes plus 1 (the as-is run). In the case of Figure 68, five runs would be required. Since each startup can cost a customer hundreds of thousands of dollars, the cost of this technique is prohibitive.
2. Due to the immense size and weight of turbine (and generator) rotors, the influence between planes is very small. Although computer-based analysis techniques could handle these small values, their effect on the vibration is not enough to offset the cost of the extra startups.
3. Due to the multitude of thermal effects, and the amount of time at running speed required for the turbine to attain thermal stability, the turbine's characteristics might be changing from run to run. The vector effects of the thermal changes may be much larger than the effects of imbalance in nearby planes. These thermal vectors can be mistakenly interpreted as influence coefficients by multi-plane balance programs, producing incorrect results.

3.1 Acceptable Methods

There are two balancing techniques that, in most cases, provide an adequate solution to turbine imbalance without an excessive number of runs:

1. The Static Couple method

2. The Intermediate Solution method

3.2 Determining Which Method to Use

To decide which method to use in balancing a turbine rotor, you must first understand the two types of imbalance that will be present: static and couple imbalance.

Pure **static** imbalance is a single mass imbalance acting through the shaft centerline in a single direction. Vibration readings taken at both ends of a turbine rotor with static imbalance would be in phase, since the imbalance acts in the same direction at both ends. Static imbalance causes a rotor to vibrate as shown in Figure 69.

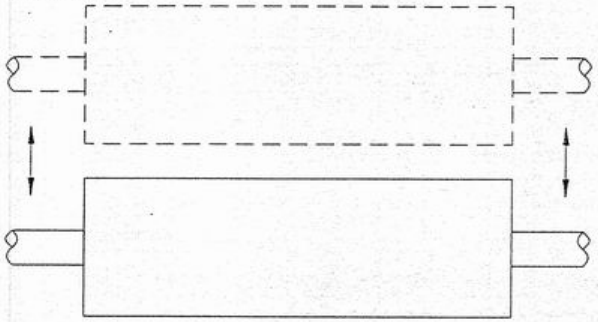


Figure 69 - *pure static imbalance*

Pure **couple** imbalance results from two mass imbalances that are 180 degrees apart. Vibration readings taken at both ends of a turbine rotor with pure couple imbalance would be 180 degrees out of phase. A rotor with pure couple imbalance would vibrate as shown in Figure 70.

Figure 70 - *pure couple imbalance*

In most rotating machines, the imbalance is a combination of pure static and couple imbalance, and is called **dynamic**. Multi-plane techniques are used to solve dynamic imbalance, correcting both

the static and couple components. The number of runs that are required to do so, however, make multi-plane techniques highly uneconomical for balancing turbines.

The technique you use depends on the relative size of the static and couple components of the turbine's vibration. If the magnitudes of the static and couple imbalance are vastly different, use the static couple method to correct the greatest imbalance. If the magnitudes are equal (or very close), use the intermediate solution method.

Making the Compromise

Although these two methods can balance turbine rotors in a few runs, both involve compromise: some residual imbalance is left. The philosophy is that if the remaining imbalance is within tolerance, then it is not worth the cost of an extra run to eradicate it completely.

When deciding which technique to use, ask these three questions:

1. **What does a run cost?** If stopping and starting the turbine is a million dollar proposition, then take every step to minimize the number of runs. If startups are not too costly, then perhaps the customer will be willing to pay for a multi-plane balance.
2. **Can the customer live with compromise?** That is, how much residual imbalance can they tolerate in their system? If the application is so critical that only .5 mils can be tolerated, then residual imbalance is not acceptable. However, if 3 mils of residual imbalance is within tolerance, then one of these methods is appropriate.
3. **Is static couple the best compromise?** This depends on the relative magnitude of the static and couple vectors. If they are vastly different, then static couple balancing is a good compromise. If they are equal, or very close to equal, then static couple is not a good compromise.

4.0 THE STATIC COUPLE METHOD

Static couple balancing is an effective method of balancing turbines without the high number of runs associated with multi-plane methods. This method identifies the static and couple components of the turbine's imbalance, allowing you to solve for the one with the greatest magnitude. The component with the greatest magnitude is solved for, leaving the other untouched. Leaving residual imbalance may not seem like an ideal solution, but the static couple method can often solve the worst of the vibration and, most importantly, save runs.

The static couple method works well on steam turbines for several reasons:

- Steam turbines, especially those used for electrical generation, run at only one speed. The rotor's vibration as it passes through resonance is of less concern than its vibration at running speed. Since the rotor spends very little time at these resonant modes, vibration here is not as important.
- The static couple method is ideally suited to the limited accessibility of turbine and generator rotors. Balance weights for static and couple imbalance are installed at the end planes of turbine and generator rotors, which are the only planes available on LP and generator rotors.
- Low pressure rotors usually require balancing more often due to erosion caused by wet steam. Most LP rotors are highly susceptible to couple imbalance since they operate near their second vibration mode.
- Most turbine rotors operate near a static or a couple mode.

There are cases where the static couple method is not appropriate. Rotors that are not symmetrical, or rotors where the bearing stiffness is not equal at both ends, should not be balanced with the static couple method. In these cases, refer to the intermediate solutions method described later in this chapter (and in Section 5 of the Field Balancing chapter in Volume 1).

4.1 Performing Static Couple Balancing

Consider the example rotor shown in Figure 71. The bearing on the left is designated as plane *A*, while the right bearing is designated plane *B*.

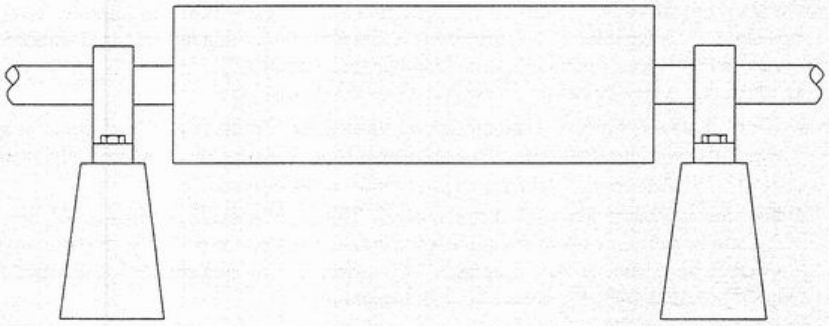


Figure 71 - *a turbine rotor with static imbalance*

An as-is vibration reading is taken at plane *A*, and the imbalance vector *A1* is recorded. The imbalance vector *B1* is recorded at plane *B*. The static imbalance is found by the formula

$$Static\ Imbalance = \tfrac{1}{2}\left(\overline{A1} + \overline{B1}\right) \qquad \text{(Formula 13-1)}$$

and the couple imbalance is found by the formula

$$Couple\ Imbalance = \tfrac{1}{2}\left(\overline{A1} - \overline{B1}\right) \qquad \text{(Formula 13-2)}$$

If you're not sure how to add and subtract vector quantities $\overline{A1}$ and $\overline{B1}$, refer to the **VECTOR ADDITION AND SUBTRACTION** box on page 137 for a quick review. If the magnitude of the static imbalance is significantly greater than the couple imbalance, then correct the static component. Likewise, correct the couple component if it is significantly greater than the static component.

Once you have decided which of the imbalance vectors you will correct, you must estimate the phase lag of the system. The estimated phase lag is used to determine the location of your balance weight for correcting either static or couple imbalance. To estimate the phase lag, you must know where the turbine's operating speed lies in relation to the bearing support resonance. If the operating speed is below resonance, then the phase lag will be between 0 and 90 degrees; expect a phase lag between 90 and 180 degrees for turbines operating above resonance. Refer to Section 13 of the System Response chapter (Volume 1) for more information on phase lag.

VECTOR ADDITION AND SUBTRACTION

A vector has both a magnitude and a direction. The unbalance reading 2.7 oz.in. @ 45 degrees is a vector; 2.7 oz.in. is the magnitude, and 45 degrees is the direction. If you plot the vectors on polar paper, they can be added graphically following the technique shown in Section 13 of the System Response chapter in Volume 1. Adding vectors mathematically, however, is more accurate.

Consider the two vectors $\overline{A1} = x \text{ @ } \theta$ and $\overline{B1} = y \text{ @ } \varphi$. Adding vectors $\overline{A1}$ and $\overline{B1}$ begins by converting them to polar notation:

$$\overline{A1} = x\cos\theta + x\sin\theta$$

$$\overline{B1} = y\cos\varphi + y\sin\varphi$$

If $\overline{R1} = \overline{A1} + \overline{B1}$, then

$$\overline{R1} = (x\cos\theta + y\cos\varphi) + (x\sin\theta + y\sin\varphi)$$

For example, consider vectors $\overline{A1} = 7.4 \text{ @ } 211°$ and $\overline{B1} = 5.0 \text{ @ } 188°$. The vector sum $\overline{R1}$ of these two vectors is found by

$$\overline{R1} = (7.4\cos 211° + 5.0\cos 188°) + (7.4\sin 211° + 5.0\sin 188°)$$

$$\overline{R1} \approx -11.294 + \left(-4.507\right)$$

When the vector sum is calculated in this manner, and the resultant is expressed in the form $\overline{R1} = a + b$, the values a and b are actually the (x,y) coordinates of the endpoint of the vector when the vector is centered at the origin. The magnitude of the resultant is found by

$$Magnitude = \sqrt{a^2 + b^2}$$

So the magnitude of our resultant vector equals

$$|\overline{R1}| = \sqrt{\left(-11.294\right)^2 + \left(-4.507\right)^2} = 12.16$$

If the vector is centered at the origin, then the angular direction θ is the angle from the positive x axis to the endpoint of the vector. Therefore

$$\tan\theta = \frac{b}{a} \approx \frac{-4.507}{-11.294} \approx .3991$$

Using the arctan function, $\theta \approx 21.756$. This is not necessarily the value of the vector's angular location. The actual value depends on the sign of a and b, according to the following table.

If a is	and b is	then	to calculate
positive	positive (or 0)	90>θ≥0	leave as is
negative	positive (or 0)	180≥θ>90	add 180
negative	negative	270>θ>180	add 180
positive	negative	360>θ>270	add 360

Since both a and b were negative, we can calculate that θ roughly equals 201.76 degrees.

Finally, you must estimate the amplification factor of the system. For most steam turbines in electrical generation applications, the amplification factor varies between 4 oz./mil and 6 oz./mil. Once you have added a weight and taken a reading, you can calculate the amplification factor very accurately (see box in Section 10 of the System Response chapter in Volume 1). You might check to see if data from previous balancing exists, and use that data to calculate the amplification factor.

If no previous data is available, 5 oz/mil is a good estimate for a steam turbine's amplification factor. Experience enables you to estimate the amplification factor with a degree of certainty.

4.1.1 Solving Static Imbalance

Once you estimate the phase lag and amplification factor of the system, you can solve for the static imbalance. We will use actual vibration readings taken from a turbine with high static imbalance to illustrate the technique.

Vibration readings were taken on a large turbine running below its third-mode resonance. The following readings were taken off bearings 5 and 6 of the turbine:

Bearing #5: 7.4 mils @ 211
Bearing #6: 5.0 mils @ 188

Using Formulas 13-1 and 13-2, the static and couple components were determined to be

Static = 6.08 @ 201.76
Couple = 1.71 @ 245.92

The static vector is much higher than the couple, so a static imbalance solution will be performed.

Locate the Heavy Spot

Section 3.1 of the Phase chapter (Volume 1) outlines the procedure for locating the heavy spot using phase data from the as-is run. You will need to know three things:

1. The phase lag of the turbine (which you have already estimated)
2. The true phase measured by your analyzer
3. The angle, in the direction of rotation, between the vibration and the reference transducer

In most cases a key on the rotor can be used in conjunction with a proximity probe to provide a reference signal. If there are no keys that are easily accessible (or if there are multiple keys), then a piece of reflective tape can be used with a photo-reference pickup.

For our example turbine, the phase lag was estimated at 60 degrees (given the proximity of its operating frequency to the frequency of the third-mode resonance). The angle between the vibration and reference transducers was 0 in this case. Using Formula 5-1 from the Phase chapter:

$$\theta_h = 60 - 201.76 - 0 = -141.76$$

The negative number indicates that the heavy spot is located 141.76 degrees from the reference transducer *opposite the direction of rotation*, or 218.24 degrees in the direction of rotation.

Determine Where to Place Your Weight

Place the balance weight 180 degrees from the heavy spot. Since you are correcting static imbalance, half this weight goes in the same location on either end of the rotor. The weights added at each end should be equal, and they *must* be in the same angular location.

Formulas 6-5 and 6-6 from Section 3.1.2 of the Field Balancing chapter (Volume 1) tell you how to calculate the amount of your trial weight. The authors estimated the amplification factor of the example turbine to be 6 oz./mil. The weight of the turbine was slightly over 1 ton (2,213 lbs., or 35,400 ozs.). Using Formula 6-5 from the Field Balancing chapter, the amount of imbalance equals

$$Ub = 35{,}400 * .0061 = 215.94 \; ounce\text{-}inches$$

Using Formula 6-6 from the Field Balancing chapter, the calculated trial weight equals

$$Tw = \frac{215.94}{6} = 35.99$$

or 36 ounces, split equally on both ends of the rotor. The authors installed an 18-ounce trial weight at both ends, each one 38.24 degrees in the direction of rotation from the reference transducer.

The weight can be bolted, clamped, or tack-welded to the rotor. Make sure the weight is mounted securely and will not come off during the run. Most turbine rotors provide plates at either end of the rotor for attaching balance weights. These plates have drilled and tapped holes spaced at regular intervals around the rotor. Use the hardware weight tables in the Appendix of Volume 1 to determine the bolt length required for the necessary weight.

If there is no mounting hole at the exact location you need, refer to Section 6 of the Field Balancing chapter for information on splitting the weight between two locations.

Take Vibration Readings

Start the turbine and take vibration readings under the same conditions as the as-is reading. The turbine should be running at the same speed under the same load. Perform the static couple imbalance calculation given in Formula 13-1 on the readings from the after-weight run. The authors calculated an after-weight static imbalance of 1.6 @ 310.

Solve for Correction

The vibration vector that you record after adding the weight represents the effect of the weight on the static imbalance. If your calculations were done very carefully, you may find that the vibration is already within tolerance. In this case you can leave your weights where they are, having acceptably balanced the turbine in only one run.

If further correction is required, treat the vibration as a single plane problem according to Section 3.1 of the Field Balancing chapter. Add the necessary weights (or move your existing weights) to cancel the effect of the imbalance. Remember, half the necessary weight must be installed at both ends, in the same angular location.

4.1.2 Solving Couple Imbalance

Solving couple imbalance is very similar to the method used for solving static imbalance. The major difference is the technique used to find the heavy spot.

Locate the Heavy Spot

The magnitude of the couple imbalance vector represents the *total* amount of couple imbalance; half this amount will be located at each end of the rotor. But since the heavy spots are 180 degrees apart at either end, it is important to interpret the angular location of the couple vector correctly.

When you subtract the imbalance readings from each other to determine the couple vector, you are finding the angular location of the heavy spot at the bearing represented by the first term. The other heavy spot will be equal in magnitude and 180 degrees *opposite* this location at the other end of the rotor. For example, the northern-most bearing on a turbine is designated as A, and the southern-most bearing is B. As-is readings $\overline{A1}$ and $\overline{B1}$ are taken at each bearing. If you subtract $\overline{B1}$ from $\overline{A1}$, then the angular location of the resultant vector is the location of the heavy spot at bearing A. Likewise, if you subtract $\overline{A1}$ from $\overline{B1}$, the location of the resultant is the location of the heavy spot at bearing B.

Place Your Weight

The balance weights should be equal, and they must be placed 180 degrees from the heavy spot at each end of the rotor. Since the imbalance vectors are 180 degrees apart at either end, the balance weights will be opposite each other as well.

The weights can be bolted, clamped, or tack welded to the rotor. Make sure that the weight is mounted securely and will not come off during the run. Most turbine rotors provide plates at either end of the rotor for attaching balance weights. These plates have drilled and tapped holes spaced at regular intervals around the rotor. Use the hardware weight tables in the Appendix of Volume 1 to determine the bolt length required for the necessary weight.

If there is no mounting hole at the exact location you need, refer to Section 6 of the Field Balancing chapter in Volume 1 for information on splitting the weight between two locations.

Take Vibration Readings

Start up the turbine and take vibration readings under the same conditions as the as-is reading. The turbine should be running at the same speed under the same load.

Solve for Correction

The vibration vector that you record after adding the weight represents the effect of the weight on the couple imbalance. If the vibration is within tolerance, you can leave your weights where they are, having acceptably balanced the turbine in only one run.

If further correction is required, treat the vibration as a single plane problem according to Section 3.1 of the Field Balancing chapter in Volume 1. Add the necessary weights (or move your existing weights) to cancel the effect of the imbalance. Remember, corrections must be made in both planes so that the correction weights are equal to each other and 180 degrees apart.

5.0 THE INTERMEDIATE SOLUTION METHOD

The static couple method of balancing turbine rotors is not appropriate for all cases of turbine vibration. Don't use the static couple method if:

- The rotors are not symmetrical

- The bearing stiffness is not equal at both ends of the rotor

The static couple method is also ineffective when the static and the couple vectors are equal (or nearly equal) in magnitude. The static couple method treats turbine imbalance by solving the worst case. If the static and couple vectors are equal in magnitude, then they *both* need to be solved.

5.1 Performing Intermediate Solution Balancing

The intermediate solution method is performed just like a multi-plane imbalance correction (discussed in Section 5 of the Field Balancing chapter in Volume 1). Basically, you calculate and add a trial weight in one plane and see if the weight results in acceptable levels in that and other planes. The steps are:

- Identify and catalog correction planes.
- Record as-is readings.
- Place trial weight in the first plane.
- Calculate the single-plane correction. Also calculate the resultant for other planes.
- Repeat as necessary for the remaining planes.

Identify and Catalog the Correction Planes

Choose locations for each of the correction planes; this can be done while the turbine is still rotating. The location of correction planes is very important. For a uniform rotor operating well below the first critical, the correction plane locations are usually a matter of convenience. If the rotor is running above one or more resonant modes, then $N+2$ planes should be used (where N is the number of shaft resonant modes).

If $N+2$ planes cannot be used, the correction planes should be located 25% of the distance between the bearings. For non-uniform rotors, much greater care must be exercised in selecting correction plane locations.

Correction planes should be labeled sequentially in declining order of mass moment of inertia at each plane. The *mass moment of inertia* at the correction plane is proportional to the weight of the rotor concentrated in that plane times the square of the average radius of the rotor. Even though the mass may be the same at two bearings, the mass at the greatest radius has the higher mass moment of inertia. Usually the plane with the highest mass moment of inertia is the one with the greatest imbalance. It is often possible to reduce the number of trial runs by adding weight in the plane with the highest mass moment of inertia.

Record the As-Is Readings

The turbine should be running at its normal operating speed and should be under normal load. Take vibration data in each imbalance plane, and make sure the readings are accurate and reliable. This run results in an amplitude and phase for each plane. Shut down the turbine when the as-is readings have been recorded.

Place Trial Weight in First Plane

Place a trial weight in the first plane (the one with the highest mass moment of inertia). Use Formula 6-6 from the Field Balancing chapter in Volume 1 to calculate the trial weight for this plane. The

weight may be bolted, glued, clamped, or tack-welded to the rotor. Most turbine rotors have a plate at either end with threaded holes for balance weights. Restart the turbine once the weight is securely in place, slow-rolling the rotor as necessary.

Calculate Correction

Calculate a single-plane correction for the first plane, following the single-plane balance procedure discussed in Section 3 of the Field Balancing chapter in Volume 1. At the same time, calculate the resultant vibration from this correction weight for all the remaining planes. If the resultant vibration is acceptable in all planes, then the job is done (and you have balanced the turbine in only one run!).

If excessive vibration is still present, calculate and add a trial weight in the second plane and calculate a two-plane balance solution. If the resultants are acceptable, then the turbine is balanced. Repeat this procedure as often as necessary until the turbine vibration has decreased to an acceptable level.

6.0 GENERATOR ROTORS

Since most steam turbines are used to drive electrical generators, it is not uncommon that the generator rotor may be the cause of vibration in the turbine. Generator rotors create many unique problems:

- Since they are long and narrow, they often run near their third-mode resonance.
- The rotor is a composite, rather than one solid piece. Therefore it may have asymmetrical stiffness values and expansion coefficients.
- The copper coils can move in their slots.
- Copper migration can occur in many generators.
- Shorted field windings in the rotor can create significant thermal vectors.

Thermal vectors created by shorted field windings are the most common problem with generator rotors. This can be detected by plotting the vibration amplitude and phase on polar graph paper as the load and reactive power is changed on the generator. If shorted field windings are present, the vibration vector will change as a function of current.

Since shutting down a generator can cost over a million dollars a day, and the cost of rewinding a rotor is well in excess of a million dollars, generators with shorted field windings are left in service as long as possible. The only factor that determines when they are shut down is the vibration resulting from the thermal vectors; even then, the rotors are usually not rewound. The solution is a compromise: the rotor is balanced unloaded to an amplitude and phase opposite that at normal load. Vibration levels may be high on startup, but the rotor will run smoothly at its normal operating load and speed.

The interior of large generators is filled with hydrogen gas. If vibration weights must be installed on the generator rotor, the hydrogen must be purged from the casing with carbon dioxide and then with air. Once the weight is installed, the casing is purged of air and refilled with hydrogen. This process can take an entire day, which significantly increases the cost of each balance run.

SUMMARY

A turbine is a form of heat engine. Heat energy of steam is transformed into kinetic energy as the steam flows through nozzles. This kinetic energy is transformed into work as the steam reacts with the turbine rotor, causing it to rotate.

A steam turbine has four fundamental parts:

1. Rotor
2. Stator
3. Nozzles
4. Frame

The turbine feet usually rest on soleplates, which are flat metal plates or castings. The turbine is bolted to the soleplate, and the soleplate rests on shims and is bolted (and grouted) to the foundation.

Steam first enters the high pressure (HP) section of the turbine and flows into the nozzles. Control valves are used to carefully time when steam exits the nozzles and flows into the rotor. Once the steam leaves the HP section, it can take two different courses:

1. Return the steam to the boiler for reheating (called **superheating**, since the steam never has a chance to return to water). The superheated steam is returned through one or more reheat sections of the turbine before entering the low pressure (LP) section.
2. Dispense with the reheat sections and duct the steam directly to the LP section from the HP section.

One or more LP sections may be present in the turbine. Rotor blades in the LP sections are at successively larger diameters (just like the HP section) to compensate for the expansion of the steam.

After the steam has passed through the LP section(s), it is now at too low a temperature and pressure to do any useful work. Some turbines, called **condensing** turbines, place the LP section directly above a condenser. In other types of turbines (called **non-condensing**), the steam is piped from the LP section to other parts of the plant, where it may be used for heat or for the manufacturing process.

Turbine vibration is almost always first-order (1*RPM), with imbalance and rotor bow being the two most common sources of vibration. Due to their size, and the fact that they operate at high speeds, turbines usually run above their first-mode resonance (often above the second- and third-mode resonance as well). Coal-fired turbines usually run at 3,600 RPM, above the third critical. Turbines at nuclear power plants usually run at 1,800 rpm, above the second critical. Therefore, a turbine will run through at least one critical during startup and coastdown.

There are two balancing techniques that, in most cases, provide an adequate solution to turbine imbalance without an excessive number of runs:

1. The Static Couple method
2. The Intermediate Solution method

The Static Couple method corrects either the static or the couple imbalance vector (whichever is greatest), leaving the other untouched. This is an acceptable compromise if the two vectors have widely different magnitudes. The Intermediate Solution method is used if both vectors are equal (or nearly equal).

REFERENCES

Nelson Baxter, ABM Technical Services, Plainfield, IN., personal interview.

Sawyer's Turbomachinery Maintenance Handbook, First Edition, Volume II, Copyright 1980 Turbomachinery International Publications, Norwalk, CT.

National Electrical Manufacturers Association (NEMA), Land Based Steam Turbine Generator Sets 0 to 33,000 KW, Standards Publication No. SM 24-1985

National Electrical Manufacturers Association (NEMA), Steam Turbines for Mechanical Drive Service, Standards Publication No. SM 23

Peter A. Phillips, Force-Couple Balancing, Nicolet Scientific Corporation, Northvale, NJ.

Church, Edwin F., Steam Turbines, Copyright 1935, McGraw-Hill Book Co., New York, NY.

Gustaf A. Gaffert, Steam Power Stations, Copyright 1940, McGraw-Hill Book Co., New York, NY.

14 GEARS

INTRODUCTION

This chapter deals primarily with the vibration aspects of gears and gear boxes rather than their theory and design. However, some background on the theory of gearing will help you understand the vibration problems peculiar to these drives.

1.0 GEAR THEORY

When two gears mesh, the teeth that are in contact have a common tangent. A **tangent** is a line that touches a curve at only one point. So when two gear teeth have a common tangent, the surfaces of the two gear teeth touch each other at the same contact point. Figure 72 illustrates two meshing gears and shows their common tangent. Also in Figure 72, a second line (called the **normal**) has been drawn perpendicular to the common tangent.

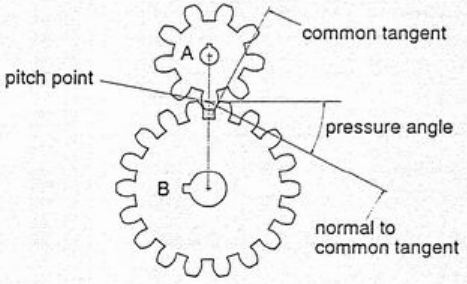


Figure 72 - *two gear teeth in contact*

The centers of the two gears in Figure 72 are located at points A and B. If a line is drawn between the two centers, the normal to the common tangent intersects the center-to-center line at point P, which is called the **pitch point**.

It is very important that both gears in a gear mesh arrangement run at a constant velocity. Otherwise, one of the gears will have to speed up and slow down at each tooth. The loads associated with these acceleration forces are proportional to the inertia times the frequency squared, and result in high vibration levels and early failure.

In order to maintain a constant velocity ratio, the normal to the common tangent at the point of gear tooth contact must always pass through the pitch point, and the pitch point must always lie on the center-to-center line of the two gears. This condition, called **conjugacy**, is shown in Figure 73. Notice that there are two contact points. Since these gears are conjugate, a single line is normal to the common tangent at both contact points, and this normal line passes through the pitch point.

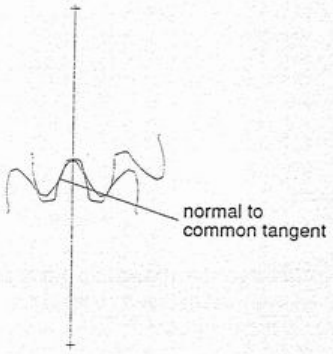


Figure 73 - *conjugacy*

Notice that the pitch point is roughly in the middle of the tooth. If we were to rotate both gears until the next two teeth came in contact, and construct another normal to the common tangent, we would see that the new pitch point is roughly in the center of the next tooth. If we continued completely around the gear, the pitch point would be located at the same point in the center of every tooth, as shown in Figure 74.

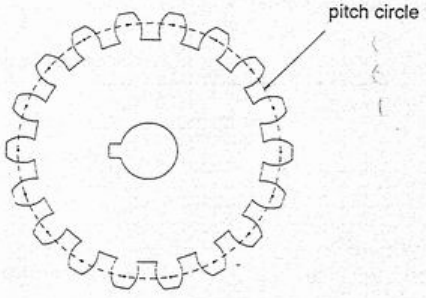


Figure 74 - *the Pitch Circle*

Each of these pitch points is the same distance from the center of the gear. A circle that connects the pitch points is called the **pitch circle**, and is an important term used in defining gears. Some of the gear descriptions that are based on the pitch circle are

- **Pitch Circle Diameter** (Pc) - The diameter of the pitch circle
- **Diametral Pitch** (Pd) - The number of gear teeth divided by the pitch circle diameter
- **Circular Pitch** (Cp) - The distance between teeth on the circumference of the pitch circle

There are an infinite number of shapes that meet the two conditions for conjugacy, but most of them are unsatisfactory for modern gearing. The most common shape in general use is the **involute**. The involute shape is the curve traced by the end of a tight string as it is unwound from the circumference of a circle, as shown in Figure 75.

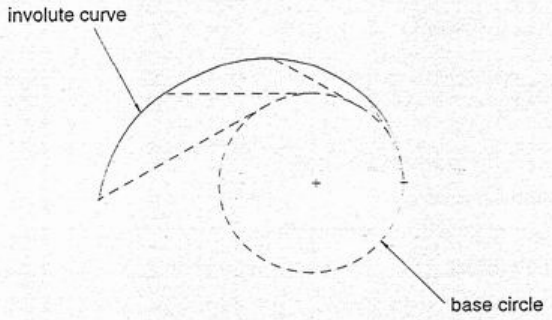


Figure 75 - *tracing an involute curve*

The advantages of the involute gear shape are as follows:

- Small errors in center to center distance do not violate the meshing action.
- The common normal and the force always act in the same place and in the same direction. Therefore, involute gears run at the lowest noise and vibration levels.

The angle between the normal to the common tangent and a line perpendicular to the center-to-center line through the pitch point is called the **pressure angle**, shown in Figure 72, and is nominally 20 degrees in most modern gear designs.

Some other terms used to define gears are

- **Normal Pitch** (Np) - The distance along the normal to the common tangent between successive involute tooth surfaces; $Np = Cp * cos(A)$
- **Base Circle Diameter** (Bd) - The diameter of the circle from which the involute curve is generated (also called the **root circle**)
- **Gear Ratio** (Ng) - The ratio of the number of teeth on each gear
- **Base Helix Angle** - The line inclination from a line parallel to the axis of rotation
- **Lead** - The distance of travel axially along a helical gear for one tooth to rotate through 360 degrees
- **Line of Action** - The distance along the normal to the common tangent during which one tooth is in contact with one tooth on the other gear
- **Backlash** - The clearance between the adjacent teeth when two teeth are in contact

- **Working Depth** - The radial distance from the point of tip contact on one tooth to the tip of the contacted tooth
- **Contact Ratio** - The average number of pairs of teeth that are theoretically in contact; this is approximately equal to the length of the line of action divided by the normal pitch
- **Addendum** - The difference between the pitch circle radius and the radius of the outside diameter
- **Dedendum** - The difference between the radius of the root circle and the radius of the pitch circle

The primary reasons for the use of a gear box rather than some other drive are as follows:

- Very high power-to-size ratio
- Rigid
- Drive without slip
- Accuracy
- Less expensive for the torque transmitted
- They can run at high speed

The main disadvantages of gear boxes are as follows:

- They require lubrication.
- They require precise alignment.
- They can be quite noisy.

The major gear types used in most gear box applications are listed in the following sections.

1.1 Spur Gears

Spur gears are used in parallel shaft boxes. They have straight teeth that are parallel to the axis of the gear, and the tooth contact occurs across the whole length of the tooth. Figure 76 illustrates two meshing spur gears.

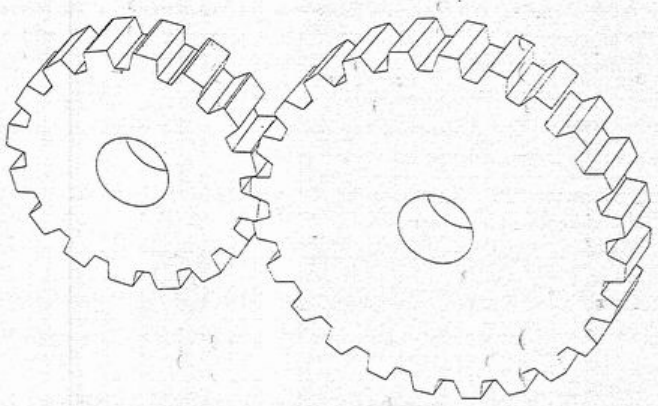


Figure 76 - *spur gears*

In most of these drives, two pairs of teeth are in contact for half the time and one pair is in contact for the rest of the time. This means that the maximum stress capability of the drive is limited to the stress capability of only one tooth.

The longer a pair of teeth are in contact, the lower the level of vibration and noise generated by the drive.

Any variation in the tooth profile that may be caused by design, manufacturing, or tooth deflection occurs across the whole tooth. This can cause a very high vibration at the tooth mesh frequency and can increase the stress load on the gear teeth.

1.2 Helical Gears

Helical gears are cylindrical gears with helical (spiral) teeth, which are usually cut on parallel axes so that the line of contact is a slanting line with contact starting at one end and traveling to the other. Figure 77 shows a helical gear.

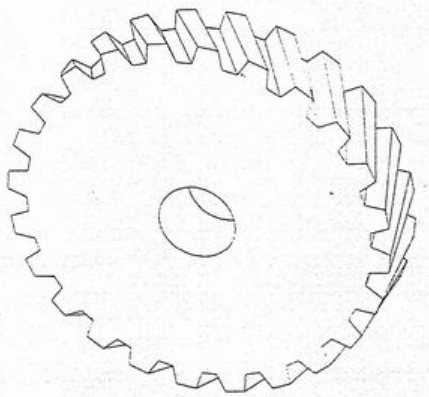


Figure 77 - *a helical gear*

This design has an averaging effect on tooth profile errors so that the drive runs much smoother than the spur gear arrangement. In addition, the contact ratio is approximately 1.5 rather than 1, so tooth stress considerations can be based on a face width of 1.5 times the actual face width.

The major disadvantage of the helical design is the axial force due to the slanting line of contact. This axial force can result in high axial vibration, which is very rarely seen in spur gear designs. However, the lower radial vibration and the increased load capacity of the drive is usually more important than the possibility of axial vibration.

In some cases a double helix is used, called a **herring bone** pattern. The axial location of these gears must be carefully controlled so that the axial thrust cancels itself out.

1.3 Crossed Helical

Crossed helical gears are helical gears that operate on non-parallel axes. Usually there is quite a bit of end-to-end sliding and only point contact, which limits the load capacity.

The crossed axes add high relative side-to-side velocities at the contact point and parallel sliding velocities, which increases the possibility of lubrication breakdown, and pitting and removal of metal on the gear tooth (called **spalling**).

1.4 Worm Gears

Worm gears have teeth similar to screw threads. If the gear is fully conjugate, there is line contact; on the other hand, when spur or helical gears are used, there is only point contact and quite a bit of endwise sliding. Figure 78 shows a worm gear arrangement.

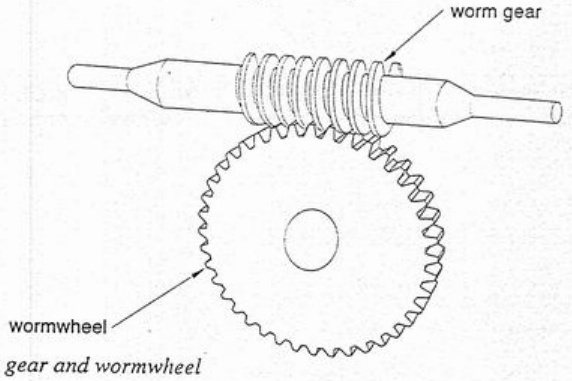


Figure 78 - *worm gear and wormwheel*

These gears are used for right-angle and high-ratio drives where transmission of power between non-intersecting shafts (usually at right angles).

1.5 Bevel Gears

Bevel gears are conical and work with intersecting axes. They are the simplest form of low-ratio right-angle drives. Straight bevel gears have straight, axially arranged teeth. Figure 79 shows a straight bevel gear.

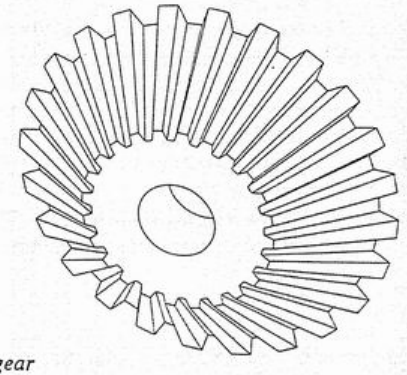


Figure 79 - *a straight bevel gear*

These gears are similar to spur gears in their operation but are much more sensitive to assembly dimensions. They should not be used for high-speed drives or where high accuracy is required.

Another type of bevel gear is the **spiral bevel**, which has curved teeth that are obliquely disposed to the axial section of the gear and are the conical equivalent of the helical gear. Straight bevel gears result in a smoother drive than the spiral bevel gear but still do not run as smooth as the parallel shaft boxes.

The **Hypoid** gear is similar to the spiral bevel gear, except the axes do not intersect. These gear designs are very complex and have high sliding velocities.

2.0 GEAR MESHING AND VIBRATION

The mechanics of gear meshing produce a gear mesh frequency caused by the successive contact of the gear teeth. Errors in manufacturing and incorrect meshing can produce additional pulses with specific frequencies.

2.1 Load Pulses

As we have seen, ideal gears would transmit motion with a uniform pitch line velocity equal to the radius of the gear pitch line diameter times the frequency of the gear. For example, for two gears of pitch line radius *R1* and *R2*, and shaft frequencies of *F1* and *F2* respectively, the pitch line velocity *Vp* is

$$Vp = R1 * F1 = R2 * F2$$

Since the best gears have adjacent pitch errors in the order of 0.0002 inches and teeth whose bending stiffness is about 2,000,000 lbs./inch per inch of tooth width, the relative tooth velocities can be as high, or higher than, 180 feet per second. The resultant stress wave can be very high, even taking the oil film into account, and it excites the tooth at its resonant frequency.

The load pulse is roughly trapezoidal and, unless the tooth is resonant, the rise and decay of the pulse are dependent on the relative velocity between the gears. Thus the load carrying capacity of a pair of gears is proportional to the square root of the pitch line velocity.

The major components of this pulse are

- the shock wave (depending upon the velocity with which the teeth come together)
- the load pulse (whose shape depends on the tooth load, the operating frequency)
- the gear geometry

As the ratio of tooth contact frequency to tooth resonant frequency approaches 1, the double stress reversal increases dramatically. and early tooth failure will result.

Since the gear teeth roll and slide upon one another, almost all spur and helical gears are designed with an involute profile, so that the tooth contact lies on a straight line of action that passes through the pitch point.

2.2 Transmission Error

As we have already seen, the pitch point is a point on the center line of the gear, and the pinion through which the normal to tooth contact must pass in order to have conjugacy. That is, the surfaces must act upon one another with smooth driving action and constant normals to the profiles of mating

teeth, so that at all points of contact they pass through the pitch point. Thus the relative normal velocity is zero, and the sliding velocity is the tangential velocity.

Since perfect gears cannot be made, there is always transmission error. The **transmission error** is, in fact, a measure of the non-conjugate action that results from manufacturing errors, tooth deflections, and bearing and casing deflections. The error is usually measured at constant torque and is defined as the difference between where the output shaft would be if the gears were perfect and the shaft's actual position.

Transmission error usually results in a vibration at gear mesh frequency that may be modulated by a once-per-revolution signal caused by pitch errors in one or both of the gears. Synchronous time averaging can usually be used to identify the bad gear. Helical gear boxes usually have much lower gear mesh frequency levels than spur gear designs, although the axial vibration at gear mesh may be much higher.

Helix angle errors also result in transmission error and are equivalent to added load at the tooth. These errors also increase the level of the gear mesh frequency.

AGMA, ANSI, and ISO have standardized on several systems of gearing; one current standard for spur gearing is either a 20 or a 25 degree pressure angle with full depth addendums.

Some of the obsolete profiles are

- 14 ½-degree full-depth involute
- 14 ½-degree composite system
- 20-degree stub tooth involute
- Cycloidal tooth profiles

2.3 Deflections

The involute profile only performs as an involute when the load is zero. Once load is applied, the teeth deflect and change the contact locations.

In order to compensate for these deflections and the manufacturing errors, one or more of the following modifications are often made on the involute profile:

1. **Tip and/or root relief** - Few gears are designed with correct tip relief. The minimum amount should be such that, if two gears that had the worst possible tooth errors were to come in contact, no interference would occur.

   This amount is usually estimated; make sure it is at least equal to the sum of the tooth deflection, the base pitch error, and the variation in profile from tooth to tooth. Incorrect tip or root relief results in noise and vibration that is similar to involute error. The tip relief for uniform load transfer can only be correct for one design load. At any other load, first one pair of teeth and then two will carry it. The result is a transmission error that looks like involute error but that dynamically can also give rise to vibration at half tooth frequency and half-order harmonics.

2. **Other root modifications** - Again, these modifications can only be correct for one load and one speed.

3. **Crowning** in the axial direction.

3.0 VIBRATIONS IN GEARING

The principal vibration frequencies in the gearing are

1. Those associated with the bearings used on the pinion and gear shafts. These are the normal bearing fault frequencies and their harmonics for the bearing type.
2. The primary frequency of the pinion shaft and its harmonics.
3. The primary frequency of the gear shaft and its harmonics.
4. The gear mesh frequency (the number of teeth on the gear times its shaft frequency).
5. Harmonics of the gear mesh frequency.
6. Side bands of the gear mesh or gear mesh harmonics. These side bands are the gear mesh or the gear mesh harmonic plus and minus the pinion shaft or the gear shaft frequency, or both.
7. The wobble (disc resonance) of the gear or gears.
8. The tooth resonance of one or more of the gears.
9. Frequencies from the gear hobbing or cutting machine, or from the cutters. These are usually identified as ghost errors in the spectra.
10. Frequencies from bolt holes in one or more of the gears.
11. Shaft deflections and shaft resonances.
12. Bearing deflections due to tooth load.

One or more of these forcing functions may excite the gear box panels so that the gear box is considered to be noisy. One common misconception is that a noisy gear box is one that will fail or is about to fail. Although it is possible that such a gear train is in poor condition and should be repaired, in general it is not possible to equate noise with gear life.

Many times the gear mesh frequency is amplified by the gear box case. Most manufactures try to design the case so that vibration from the gear mesh and harmonics are not fed to the panels. In some cases, the main framework that carries the static loads and the vibration is covered with a highly damped outer shell, which is isolated from the frame.

Since a gear box is basically a torque translator, almost all of the possible faults are related to transmission error. These faults can generally be identified more readily by measuring the torsional vibration as well as the linear vibration and the noise spectra.

The torsional vibration can be measured on both the input and the output shafts by using a rotational accelerometer or a torque transducer. Use a system that measures torsional frequencies that are at least equal to the gear mesh frequency. The tooth variations are usually much clearer in the signal from the rotational accelerometer than from a linear accelerometer.

A microphone can also be used to detect tooth faults as well as the location of high-noise sources on the gear box case.

Eccentricity of the base circle, sometimes called **base circle runout**, results in transmission error due to the adjacent pitch error. This causes all harmonics of the once-per-revolution vibration of the gear to appear in the spectra, along with all harmonics of the gear mesh sidebanded by the once-per-revolution signal.

Adjacent pitch errors normally are statistically distributed, except when there is a major defect in the manufacturing process. If the errors are, in fact, Gaussian, then all harmonics of the shaft frequency will be present in the spectra, with the exception of the gear mesh frequency.

Dropped-tooth and swipe-mark errors can occur on shaped gears, and both appear as a once-per-revolution signal. The dropped-tooth signal is similar to an adjacent pitch error, and the swipe-mark signal looks like an involute error.

Both of these errors result in impacts at once-per-revolution and can be very noticeable if a pair of gears with non-integral tooth ratios each have a bad tooth. The signal from the gears is a beat frequency, and the levels can be quite high.

If the backlash is too small, oil can be trapped in the tooth roots, which usually causes very high noise levels at gear mesh frequency. This is much more apparent when the relative tooth velocities are high. Since these forces can be very significant, they can lead to early bearing failure and can induce significant shaft deflection.

When the relative tooth velocity is very high, the oil squirting from the root of the meshing teeth may impact the casing of the gear box with sufficient force to excite case resonances that amplify the level of the gear mesh frequency.

If the torsional dynamics of the system cause the gear teeth to lose contact, the noise level and the tooth impact load are increased significantly

The following are general rules for gearbox noise:

1. Doubling the transmission error increases the noise from the gear box by 6 dB.
2. Doubling the frequency of operation increases the noise by 6 dB.
3. Doubling the tooth load increases the gear box noise by 3 dB. The increased tooth load also changes the transmission error so that the cumulative effect may be an increase or a decrease in the noise.
4. Doubling the number of gears increases the noise by 6 dB.

As we pointed out before, the noise may or may not indicate a faulty gear box.

SUMMARY

Both gears in a gear mesh arrangement must run at a constant velocity; otherwise, one of the gears will have to speed up and slow down at each tooth. The loads associated with these acceleration forces are proportional to the inertia times the frequency squared, and they result in high vibration levels and early failure.

When two gears mesh, the teeth that are in contact have a common tangent. The surfaces of two meshing gear teeth touch each other at the same contact point. If a line is drawn between the centers of the two gears, the normal to the common tangent intersects the center-to-center line at point P, which is called the **pitch point.**

In order to maintain a constant velocity ratio, the normal to the common tangent at the point of gear tooth contact must always pass through the pitch point, and the pitch point must always lie on the center-to-center line of the two gears. This condition is called **conjugacy**. The angle between the normal to the common tangent and a line perpendicular to the center-to-center line through the pitch point is called the **pressure angle**. The pitch point on each tooth is the same distance from the center of the gear. A circle that connects the pitch points is called the **pitch circle**; this is an important term used in defining gears.

There are an infinite number of shapes that meet the two conditions for conjugacy, but most of them are unsatisfactory for modern gearing. The most common shape is the **involute**. The involute shape is the curve traced by the end of a tight string as it is unwound from the circumference of a circle.

The major gear types used in most gear box applications are

1. Spur Gears
2. Helical Gears
3. Crossed Helical
4. Worm Gears
5. Bevel Gears

REFERENCES

Candee, Alan H., "Information Sheet - Basic Gear Geometry," AGMA Standard 115.01-1959 (1974).

"National Standard - Nomenclature of Gear Tooth Failure Modes," AGMA Standard 110.04-1980.

Smith, James Derek, Gears and their vibration: a basic approach to understanding gear noise, Copyright 1983, Marcel Dekker, Inc., New York, NY.

Broersma, G., Design of gears, Copyright 1967, Culemborg, H. Stam,

Dale, A.K., "Gear Noise and the Sideband Phenomenon," ASME Paper 84-DET-1/4, Cambridge, MA, 1984.

Mark, W.D., "Analysis of the vibratory excitation of gear systems: Basic theory," Journal of the Acoustical Society of America, Volume 63, No. 5, May, 1978, pp. 1409-1430.

Gu, A.L. and Badgley, R.H., "Prediction of vibration sidebands in gear meshes," ASME Paper 74-DET095, October, 1974, pp. 1-9.

Buckingham, Earle and Eliot K., "How design errors influence the functioning of gear trains," Machinery, August, 1964, pp. 108-120.

Rieger, N.F., "Vibration in geared systems," Machine Design, September 16, 1985, pp. 164-173.

Welbourn, D.B., "Gear noise spectra - a rational explanation," ASME Paper 77-DET-38.

Welbourn, D.B., "Gear errors and their resultant noise spectra," Proceedings of the Institute for Mechanical Engineers, Volume 184, Pt. 30, 1969-70, pp. 131-139.

15 POWER TRANSMISSION COMPONENTS

INTRODUCTION

The transmission of power is as basic to industry as electricity. Components involved in power transmission are subjected to rotary or reciprocating motion, and their dynamic behavior presents unique problems to the vibration specialist. We have devoted an entire chapters to gears, one of the more common components involved in power transmission. In this chapter we will discuss four other power transmission components:

1. Belts
2. Chains
3. Universal joints
4. Flexible couplings

1.0 BELTS AND BELT DRIVES

Belt drives are the most common and the lowest cost way to transmit power from one shaft to another. Although they typically are not as durable nor capable of transmitting as much power as gear drives, they are much less expensive, can run quieter, and can absorb appreciable shock. Improved reinforcing materials are making it possible to use belts for higher power transmission.

1.1 Types of Belts

There are four major types of belts:

1. **Flat belts** - These are flat belts made of either leather, rubber, or rubber-impregnated fabric. Pulleys for flat belts are not grooved; they are flat, with a central crown to help hold the belt. Flat belts are strong, easy to make, and are resistant to extreme dust conditions. Endlessly-woven flat belts (such as the Habasit belt) can transmit up to 100 Hp per inch, and are used in high-quality, high-speed machinery. These belts are vastly superior to V-belts or regular leather flat belts in terms of vibration (this is explained later). However, flat belts are not very flexible and they take up a great deal of room due to the necessary size of the pulleys.
2. **Round belts** - These are made of rubber, sometimes with cords for additional strength. Round belts are usually used for light loads, such as movie projectors and sewing machines.
3. **Timing belts** - These are flat belts with equally-spaced teeth that mesh with notches on the pulley. The belt teeth are either trapezoidal for medium duty or rounded for heavy duty service. Timing belts do not slip on the pulley (unlike all the other types of belts); consequently they are used where constant velocity and strict timing is crucial. Belt speed can be as high as 16,000 feet per minute. A variation on these belts is called **synchronous both sides**, which have teeth on both the inside and outside surface. These belts are used in serpentine drives where timing is required.
4. **V-belts** - A V-belt is the most common belt in use today. V-belts are essentially V-shaped in cross-section, so that the belt becomes wedged against the sides of the sheave. This wedging action enables a V-belt to be run as fast as 6,000 feet/minute, with power transmission efficiencies as high as 95%.

1.2 V-Belts

The V-belt is the basic style used for power transmission in most belt drives. V-belts are normally considered to be the best compromise for most belt drive systems because of their relatively low cost, high tractive force, reasonable operating speed range, and service life. They are the most popular belts for transmitting high loads between sheaves with short center distances. Figure 80 illustrates how a V-belt wedges into a sheave groove.

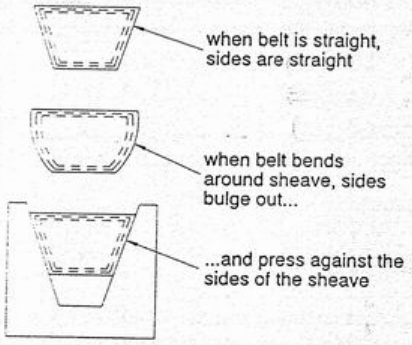


Figure 80 - *how a V-belt wedges against the sheave*

The walls of a V-belt are straight when the belt is outside a sheave. Bending the belt around the sheave puts the top (outer) surface of the belt in tension, while the bottom (inner) surface is put into

compression. This causes the sides of the belt to bulge out and press against the sides of the sheave, wedging it tight. The sheave groove is machined so that the bottom of the belt never rests against the bottom of the groove. If the base of the belt did touch the bottom of the groove, then the belt would be pushed up and the grip against the sheave would be lost.

Table 3 is a listing of the standard V-belt sizes established by both the Rubber Manufacturers Association and the Mechanical Power Transmission Association. These belts, as well as many other types, are of endless construction so that provision must be made to allow for belt replacement.

BELT SIZE	HEIGHT (inches)	MAXIMUM WIDTH (inches)	BELT SIZE	HEIGHT (inches)	MAXIMUM WIDTH (inches)
2L	5/32	1/4	8V	29/32	1
3L	7/32	3/8	A	5/16	1/2
4L	5/16	1/2	B	13/32	21/32
5L	3/8	21/32	C	17/32	7/8
3V	5/16	3/8	D	3/4	1 1/4
5V	17/32	5/8	E	29/32	1 1/2

Table 3 - *standard belt sizes*

The capability of the V-belt to carry load lies in the reinforcing cords inside the belt. These cords are normally made of rayon, nylon, polymers, steel, or glass and are located in the cushion section of the belt. This section is surrounded by a harder rubber which is usually enclosed in an abrasion-resisting jacket. A typical V-belt cross-section is shown in Figure 81.

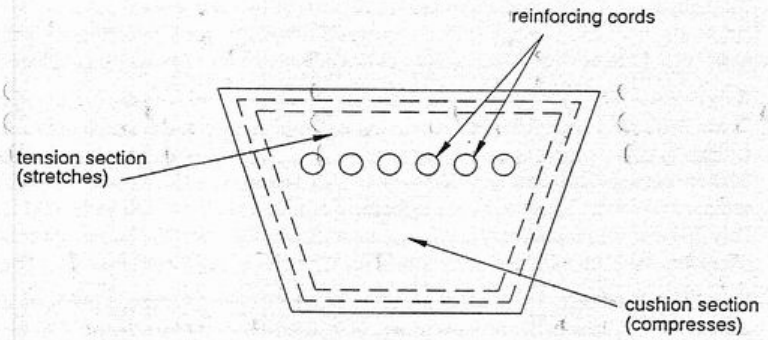


Figure 81 - *typical V-belt cross section*

During operation the belt must bend as it passes around the sheave. The neutral axis (the center) is the only part of the belt that does not change length while it is bent around the sheave; the outer surface must increase in length and the inner surface must decrease in length at least twice for each complete belt cycle. The centrifugal force at the bend increases by the square of the operating speed and adds to the energy being absorbed by the belt.

1.2.1 Types of V-belts

The normal operating speed range of most V-belts is between 1,500 and 6,600 feet per minute. The optimum range for any given belt drive depends on the type and section of the belts. Special V-belts and belts of other design, such as the endless woven flat belt, can operate at speeds as high as 15,000 feet per minute.

Open end belt grips can be used in cases where it is not possible to disassemble the machine in order to replace the belt or belts. These grips are made for most of the standard V-belt sizes. When open end V-belts are used, the load transmission and operating speed capacity must be reduced by approximately 50%.

The **link type V-belt** is a much better solution to the captive sheave problem. These belts are made of individual fabric links, held together either by metal rivets or by tabs made of the same fabric material. Link belting is made for normal or high-power applications and can be stocked by the roll. The belts can be made to size from the roll, and machine disassembly is not necessary for belt replacement.

The link construction greatly reduces the possibility of density and stiffness variation in the belting so that this type of belt, especially the type without the rivets, runs at lower vibration levels than the standard V-belt. Load capacity is only slightly reduced, and there is no reduction in the operating speed range. However, the belts do tend to elongate until all the links are properly seated.

In addition to the standard V-belts, open end belts, and link belts, some of the more common types are listed below:

- The **double V** is made in almost all of the standard V-belt sizes and is essentially two standard belts molded back-to-back. Sizes are AA, BB, and so on. These belts are used on drives with reverse bends where power is transmitted by both sides of the belt.
- The **narrow wedge** is similar to the standard V-belt, but the narrow wedge shape allows the belt to transmit much more power at any given speed.
- The **multiple-V** (also called **banded-V**) consists of two or more standard or narrow wedge belts with a common backing. Belt turnover is eliminated, and belt slippage is the same for each belt. Lateral vibration is greatly reduced, but sheave wear may be increased.
- **Poly-V** (or **V-ribbed**) belts are more flexible than V-belts and usually require more tension. These belts are a single piece of rubber molded into many V-belts attached to a flat surface. Unlike V-belts, which use wedging to transmit power, V-ribbed belts depend entirely on friction between the belt and the sheave. Therefore, tension must be higher than with ordinary V-belts. These belts are recommended for speeds as high as 10,000 feet/minute. Poly-V-belts are generally more efficient than standard V-belts, can be used on smaller pulley diameters, and run with less vibration. This type belt is often used for serpentine drives.
- **Variable speed** belts are made especially for drives where the speed is adjusted while the unit is running. The belts are made with a wide, thin section and are very stiff in the crosswise direction. They are flexible enough to run over small pulley diameters.
- The **wide angle** belt has a large V angle that reduces the wedging action but increases the effect of the tensile cords. These belts are made of material with high frictional properties such as polyurethane. They can run over very small diameter pulleys and at speeds in excess of 10,000 feet per minute.

Figure 82 illustrates these various belt types.

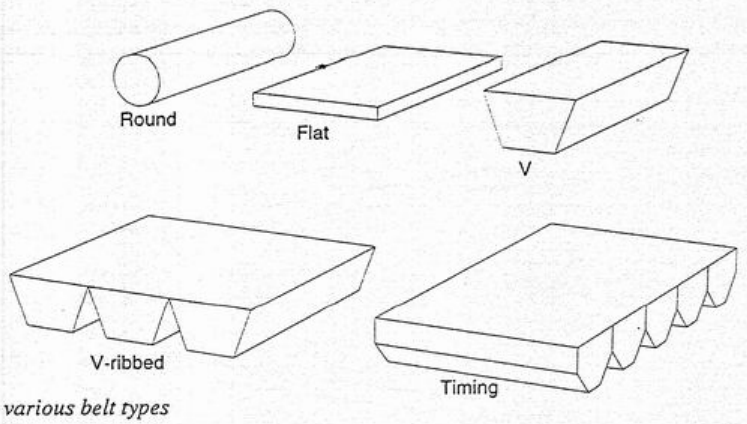


Figure 82 - *various belt types*

2.0 BASIC THEORY OF BELT DRIVES

Belts are friction drives; they depend on friction between the belt and the sheave to transmit power between drive and driven sheaves.

The ability of a belt to transmit power depends on

1. Belt tension (tension on the belt holds it tightly against the sheave)
2. Friction between the belt and the sheave
3. The arc of contact between the belt and the sheave (called **wrap**)
4. The speed of the belt, usually measured in feet/minute

When compared with chains, belts have several advantages:

- Belts can run at much higher surface velocity (ft/min) than chains.
- Belts require no lubrication.
- Belt drives are usually quieter than chain drives.
- Single belt drives accommodate more misalignment than chains.
- Flat belts can be used over very long distances, where chains could not.

However, belts are easily damaged by heat, oil, and grease. And since belts slip within the sheaves, they cannot be used where exact speed is required (except with a timing belt). Although there are many exceptions to the rule, belts are not the best means of transmitting power at low speeds and for large drive ratios. In these cases the efficiency, normally above 95%, may drop to under 90%, and the drive may not be the most cost-effective solution.

2.1 A Belt in a Belt Drive

Figure 83 shows a sample belt connecting two sheaves, and illustrates the **arc of contact** (or **wrap**): that portion of the sheave circumference where the belt is in contact with the sheave.

When the sheaves are at rest, the tension throughout the belt is constant (determined by the center-to-center distance between the sheaves). When the driving pulley rotates, however, the driving

side of the belt (the side coming into the driver pulley) is put under greater tension. The following side of the belt (the side leaving the driver pulley) becomes somewhat looser than the driving side. The driving side of a belt is often called the **tension side**, while the following side is called the **slack side**.

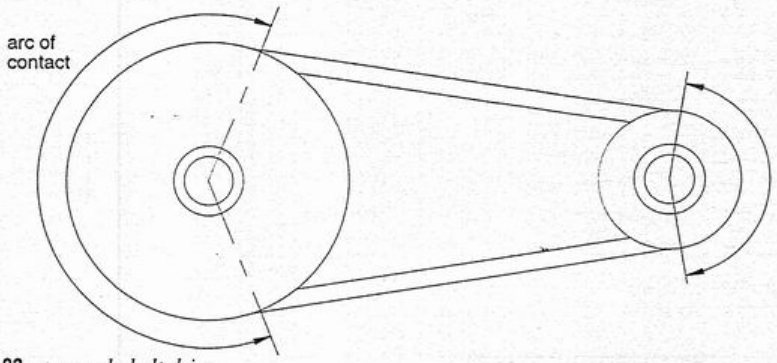


Figure 83 - *a sample belt drive*

2.2 Belt Slip and Belt Creep

If the load on the belt becomes great, the difference between the tension and slack sides can overcome the friction between the belt and the sheave. This condition is called **belt slip**, since the belt slips away from the grasp of the sheave and slides along it. Belt slip occurs for all belts except timing belts, since the teeth on the timing belt prevent slip motion.

When a belt slips, the friction between the belt and the sheave surface creates tremendous heat. This high heat causes **slip burn**, which can lead to premature breakage. Slipping also wears the sheave surface, making it harder for the belt to grip. Sheave wear also leads to differential driving, which is discussed in a moment.

The power transmission efficiency of a belt drive increases with velocity, meaning that for maximum efficiency the belt drive should be run as fast as possible. However, centrifugal force is proportional to the square of the velocity; speeding up a belt to increase efficiency also increases centrifugal force. Centrifugal forces acting on a belt tend to pull it out of the sheave groove, which increases the possibility of belt slip.

The higher tension on the tension side of a belt causes the belt to stretch somewhat. As a result, the driving pulley receives a greater length of belt than it gives out (the reverse is true for the driven pulley). This disparity creates relative motion between the belt and the sheave called **belt creep**.

The amount of belt creep depends on the internal banding used to reinforce the belt. Belts with steel bands (such as V-belts) don't stretch enough to be measurable, so belt creep is insignificant. Creep can be more noticeable in belts with fiberglass banding, since fiberglass stretches more than steel.

Even under the best of conditions, belts slip. In multiple-belt applications, the individual belts not only slip, but slip or creep at different rates. The slippage adds both to the internal heat generated within the belt itself and to the wear on the sheave and makes it impossible to use belts for synchronous drive applications (except for timing belts).

2.3 Drive Ratios

The **drive ratio** is the ratio of the sheave diameters in a belt drive. If both sheaves have the same diameter, then this drive has a 1:1 drive ratio. Proper drive ratios are essential for efficient power transmission and for reducing belt slip. The arc of contact is reduced when drive ratios become excessively high. Not only can a small arc of contact cause belt slip, it also places tremendous strain on the belt material as the belt is forced to bend around the small radius. Figure 84 illustrates belt drive ratio.

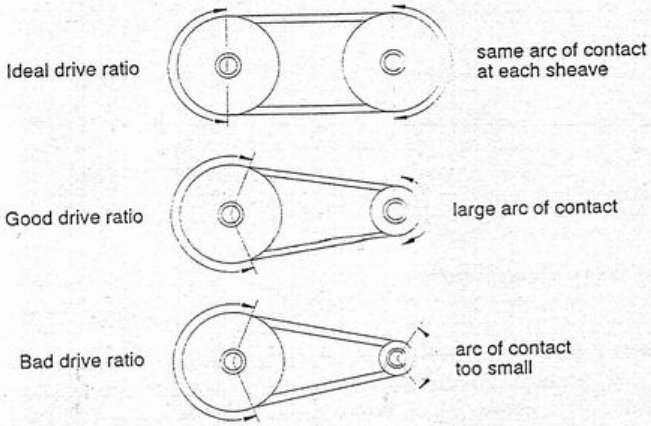


Figure 84 - *belt drive ratio*

An ideal belt drive would have a 1:1 drive ratio, creating an equal arc of contact on both sheaves. Drive ratios of 3:1 or less are acceptable, since even a 3:1 ratio still provides sufficient arc of contact to prevent slip. Ratios above 3:1 should be avoided. If a high drive ratio is necessary, the drive should be constructed in two steps, using an intermediate sheave to provide two gradual steps. Note that, while a 1:1 drive ratio is ideal for belt life and power transfer, it is *not* ideal from a vibration standpoint, since both sheaves would be operating close to the same frequency.

Drive ratios are most commonly expressed as the ratio of the drive sheave diameter to the driven sheave diameter. Using this convention, a ratio of 2:1 means that the drive sheave is twice the diameter of the driven sheave; a ratio of 1:2 means that the driven sheave is twice the diameter of the driver.

In order to prevent confusion, the terms **up ratio** and **down ratio** are often used to refer to the effect of the sheave sizes on the driven speed. An up ratio means that the driven sheave is sped up by the belt drive, while a down ratio means that the driven sheave is slowed down.

2.3.1 Drive Ratio and Output Speed

The drive ratio also affects the output speed of the driven shaft. This relationship is shown in Figure 85. When two sheaves are connected with a 1:1 drive ratio, they both turn at the same RPM (one revolution of the driver shaft causes one revolution of the driven shaft). Figure 85(a) illustrates this condition, with both sheaves having a one-foot diameter.

If we change the drive to an up ratio of 2:1 by decreasing the size of the driven sheave, then the driven pulley will rotate twice as fast. Figure 85(b) shows a one-foot diameter drive sheave and a

½-foot diameter driven sheave. If the driver in Figure 85(b) rotates at 1,000 RPM, the driven sheave will rotate at 2,000 RPM, since each revolution of the driver will cause the driven to rotate twice.

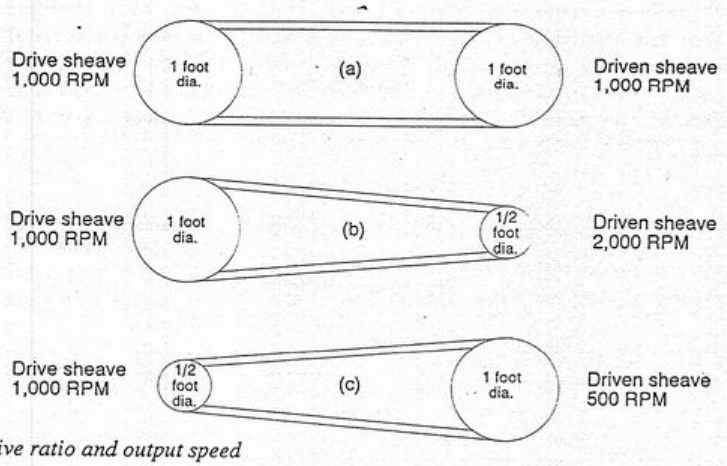


Figure 85 - *drive ratio and output speed*

The example in Figure 85(c) shows a down ratio of 2:1, realized by reducing the drive sheave to ½ foot in diameter, while the driven sheave is one foot in diameter. Although the drive ratio has remained at 2:1, the output speed is now *slower* than the drive sheave; in this case, twice as slow. If the drive sheave rotates at 1,000 RPM, the driven sheave will rotate at 500 RPM.

2.3.2 Variable-Pitch Sheaves

One of the advantages of the V-belt drive is that drive ratios can easily be changed by using a variable-pitch sheave. The pitch diameter of the sheave is varied by moving elements in the sheave closer together or farther apart so that the belt rides deeper down or farther up in the groove.

Variable-pitch sheaves are used to adjust a system for optimum drive ratio; for example, balancing air flow to building requirements. However, the variable pitch sheave should never be the permanent drive sheave. Once the system has been optimized, the variable sheave should be replaced with a fixed sheave of the same pitch diameter.

2.4 Belt Tension and Alignment

Correct belt tension and alignment are very important for all belt drives. Both should be checked on a regular basis to prevent wear on both the belts and the sheaves. Tension and alignment have a direct bearing on

1. The life of the belts and the pulleys
2. The power handling capability of the drive
3. The vibration level of the system
4. The noise level of the drive
5. The maintenance cost of the drive

Belt idlers should not be used to maintain belt tension. Idler pulleys add another spring-mass system to an already complicated machine and almost always make things worse. They not only do not work well, but they also reduce the life of the belts.

The constant working of the belt material, coupled with the slippage inherent in all non-synchronous drives, adds to the wear and not only limits the life of the belt and sheave, but makes it necessary to constantly correct for tension and alignment.

Notched V-belts may be used to reduce the stiffness of the belt. These belts are more flexible than the standard belts and will operate on smaller diameter sheaves, but they may be noisier at higher speeds.

3.0 NOISE IN BELT DRIVES

Noise in a belt drive is often interpreted as a sign of belt slip. Consequently, belts may be tightened beyond proper limits, which can damage both the belts and the sheave bearings. Belt dressings are often used instead of tightening the belt. These are lubricants intended to reduce belt noise, but they can often soften and degrade the belt.

Belt squeak is a high-pitched chirping that often occurs at idle, or when the drive is cold, damp, or exposed to excessive dust. Squeak is harmless except when it is mistaken for belt slip, leading to excessive tightening of the belt.

Belt squeal is load-dependent. It is a high-pitched howl that occurs during acceleration or when the belt drive is heavily loaded. Squeal *is indeed* caused by belt slip; if it is present, the belt tension should be checked immediately.

4.0 BELT AND SHEAVE WEAR

Belts are susceptible to heat, grease and oil, and slip burn. The internal cords may break if the belt is forced over a sheave during installation. Belts can tear if they come in contact with part of the machine, and multi V-belts can be cut if they jump a sheave groove.

4.1 Sheave Wear

Sheave wear is a major source of belt drive vibration, and it leads to premature belt drive failure. If two sheaves are misaligned, or a sheave has not been mounted squarely on the shaft, the belt does not ride correctly in the sheave groove. This leads to uneven wear or a "dishing out" of the groove walls, both of which increase the possibility of belt slip. Also, the high axial vibrations generated by uneven sheave wear can lead to early bearing failure.

A shiny sheave groove bottom means that the sheave (or the belt) has worn so much that the belt is riding against the bottom of the sheave. When a belt rubs the bottom it loses its grip on the side, causing slip and loss of power. The constant friction between the belt and the sheave bottom can also lead to slip burn. This condition usually appears first on the smaller sheave in the drive.

Many belt drives use multi-grooved sheaves to distribute high drive loads among several belts. It is important to ensure that all the belts in multi-belt drives are the same length. If one belt is longer than the others, it will not grip as tightly and will therefore do less work. The high load must then be shared among the remaining belts, which may overload them.

Multi-grooved sheaves may wear unevenly, creating a condition called **differential driving**: the belts ride lower in the sheave grooves that are excessively worn. The lower-riding belts then operate at a different speed ratio than the other belts. A great deal of power transmission efficiency is lost,

since some of the belts try to run faster than the others. Extreme cases of differential driving can cause torsional excitation of the shafts.

Ideally, you should replace all the sheaves in the belt drive every time you replace the belts. However, this may be uneconomical. Replacing the sheaves every second set of new belts ensures that your system will operate at best efficiency and at the lowest vibration.

5.0 BELT DRIVE VIBRATION

Belt drives are one of the most common (and the most significant) vibration forcing functions in industrial machinery. However, they are usually the last component to be suspected as the source of the vibration.

5.1 Sheave Alignment

The belt sheaves should be carefully checked for both parallel and angular alignment, for uneven wear, for pitch line runout, and for balance. In several cases a source of vibration was found to be from a pulley that, although keyed, was loose on the shaft.

5.2 Belt Tension

Belt tension is very important in all types of belt drives, not only from the standpoint of wear and power transmission, but vibration and bearing load as well. If the tension is too light, slippage increases, wear increases, and standing waves on the belt are more pronounced. On the other hand, too much tension also increases belt and sheave wear, adds to the bearing radial load, and increases the vibration level from the belt itself.

Standing waves in the belt or belts result in a cyclic change in belt length so that high vibration levels are seen at the sheave bearings, usually higher in the direction of the belt. If this frequency is close to a system resonance, the vibration level may be excessive.

5.3 Primary Belt Frequency

The **primary belt frequency** is the number of times per minute that the belt makes one complete circuit. This frequency is found by

$$Fb = \frac{\pi * Sd * RPM}{Bl} \qquad \text{(Formula 15-1)}$$

where

Fb = primary belt frequency (CPM)
Sd = sheave diameter (in. or mm)
RPM = the speed of that sheave (CPM)
Bl = the belt length (in. or mm)

For example, the belt frequency Fb for a 150-inch long belt over a 20-inch diameter pulley spinning at 1,800 RPM is equal to

$Fb = (\pi * 20 * 1{,}800)/150$
$= 113{,}097.34/150$
$= 753.98$ CPM

A belt rotates around at least two sheaves; serpentine drives have more than two sheaves. It does not matter which sheave you use in this formula, since the peripheral velocity of all the sheaves within the drive must be equal (if they weren't, the sheaves would slip beneath the belt).

The predominant belt frequency is very seldom the primary belt frequency, but rather some multiple of the number of sheaves over which the belt passes and the number of belts in the drive times the primary belt frequency. If a system resonance is excited by one of these frequencies, the resonant amplification makes that component appear as the dominant belt vibration.

Belt frequencies are most noticeable in V-belt drives. A V-belt has a great deal of mass in comparison to other belts: central cords, a hard rubber interior, and a tough jacket. These masses change direction as the belt travels around the sheaves, creating a noticeable belt frequency. Endlessly woven flat belts (such as Habasit belts), on the other hand, have very little mass. Consequently, belt frequency amplitudes are so low in flat belt drives that they are often difficult to detect.

5.3.1 Finding Belt Length

If you do not know the length of a belt between two pulleys, and cannot remove the belt in order to measure it, you can calculate its length if you know the radius of each pulley and the distance between their centers. Figure 86 explains this process.

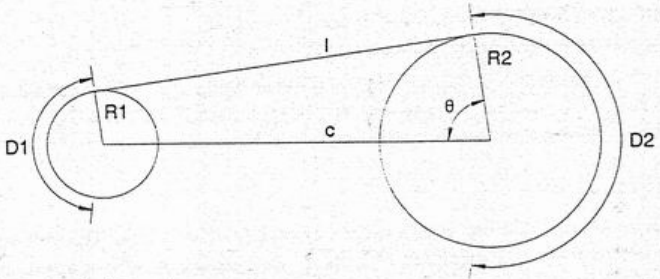


Figure 86 - *determining belt length*

First, we find the length l by the formula

$$l = \sqrt{c^2 - (R2-R1)^2} \qquad \text{(Formula 15-2)}$$

where

l = belt length (in. or mm)
c = the center-to-center distance between the two pulleys (in. or mm)
$R1$ = the radius of one pulley (in. or mm)
$R2$ = the radius of the other pulley (in. or mm)

To find the length of arcs $D1$ and $D2$, first calculate the angle θ by

$$\theta = \arcsin\left(\frac{l}{c}\right) \qquad \text{(Formula 15-3)}$$

The operation "arc sin" means inverse sine, or "the angle whose sine is" (often written in math textbooks as $\sin^{-1}$). Now the arcs *D1* and *D2* can be found by

$$D1=\pi*R1*\hat{\left(\frac{\theta}{90}\right)}$$

$$D2 = 2\pi * R2 * \left(1 - \frac{\theta}{180}\right)$$

Finally the belt length *Bl* is equal to

$$Bl = 2 * l + D1 + D2 \qquad \text{(Formula 15-4)}$$

A popular formula for quickly determining the belt length for best drive efficiency is:

$$L = 1.6 * (3.5 * D + d) \qquad \text{(Formula 15-5)}$$

where

L = engineered belt length (inches or mm)
D = diameter of large sheave (inches or mm)
d = diameter of small sheave (inches or mm)

Unfortunately, engineering belt length calculations like Formula 15-5 do not consider what effect the resulting belt frequency has on the system. Refer to section 5.4.2 for an actual example of engineered belt length having a detrimental effect on a belt drive.

5.4 Belt Strand Frequency

It is very important to check the belt strand frequency on belt-driven fans. The **belt strand** is that portion of the belt that is not in direct contact with one of the sheaves, as shown in Figure 87.

A belt strand has a resonant frequency for each of its mode shapes; usually, the first mode shape (the belt strand's first degree of freedom, as shown in Figure 88) is the one of greatest concern.

V-belt drives are particularly susceptible to excitation of belt strand resonance. As a V-belt enters a sheave, the belt is held tight inside the sheave groove. When the belt exits the groove, it is held by the tight grip of the sheave as long as possible, until it suddenly releases and snaps straight. This snap acts as an impact and excites the belt strand into one of its resonant modes. Since the belt exits each sheave in the drive, both the tension and the slack strands are excited.

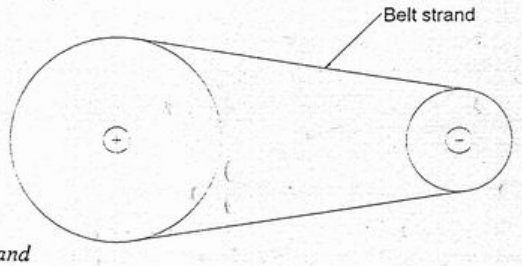


Figure 87 - *the belt strand*

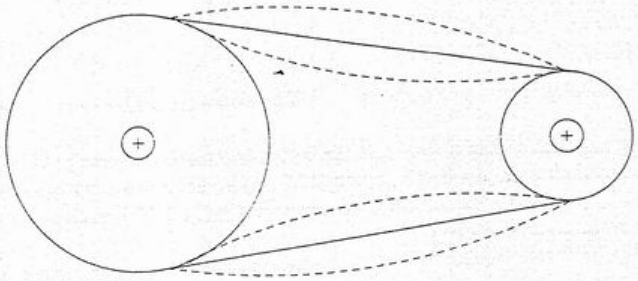


Figure 88 - *belt strand resonance*

Endlessly-woven flat belts, on the other hand, rarely experience excitation of the strand resonance. Since the flat belt is held in place by a crown on the sheave, it does not snap as it exits the sheave. Consequently, there is no periodic impacting to excite the belt strand resonance. The resonant frequency of an endlessly-woven belt strand is highly damped, due to the high-friction material used to make the belt.

5.4.1 Measuring belt strand resonance

Mount an accelerometer on the strand of the belt; an easy method is to set the accelerometer on the top of the belt strand and wrap several layers of electrical tape around the transducer and the belt. Tap the belt strand lightly to determine the first-mode resonance of the belt strand. When the motor is not running, both belt strands will have the same tension. Therefore, the resonant frequency found on one belt strand is the same on the other.

The tension side and slack side that result when the belt drive is under load affect the resonant frequency of each belt strand. The resonant frequency of the belt strand under tension will increase above the static belt strand frequency. Likewise, the resonant frequency of the slack belt strand will decrease below the static belt strand frequency.

When examining dynamic vibration data for belt strand resonance, look for a spread of 2-3 Hz above and below the static belt strand frequency. A vibration at belt strand frequency may have a broad skirt, since the two belt strand resonances are so close together.

5.4.2 An example of belt strand resonance

The authors encountered a large air handler driven by five D-sized belts. This is a case of belt strand resonance exciting another resonance in the system, with disastrous results.

Five D-size belts ran between two sheaves. The large sheave, *D*, had a diameter of 16 inches, while the small sheave *d* had a diameter of 12 inches. The large sheave *D* had a rotational frequency of 22.5 Hz (1,350 RPM), and the small sheave *d* had a rotational frequency of 30 Hz (1,800 RPM). The small sheave *d* was the drive sheave, mounted on an 1,800 RPM electric motor.

Using Formula 15-5, the belt length for best efficiency was calculated to be 108.8 inches:

$$L = 1.6 * (3.5 * 16 + 12)$$
$$= 108.8 \text{ inches}$$

109-inch belts were actually used.

Given a belt length of 109 inches, the primary belt frequency was calculated using Formula 15-1:

$$Fb = \frac{(16\pi * 1{,}350)}{109}$$
$$= 10.4 \text{ Hz}$$

Note that you would obtain the same frequency *Fb* if you used the diameter (12 inches) and RPM (1,800) of *d* instead.

Finally, using a transducer and an impact test, the belt strand resonance was measured at 28.5 Hz.

Notice that the second order of belt frequency *Fb* (20.8 Hz) is very close to the operating frequency of the large sheave *D* (22.5 Hz). Also, the third order of *Fb* (31.2 Hz) is close to the operating frequency of *d*, the small sheave (30 Hz).

It is never good to have two forcing functions so close to each other. One reason is that the two frequencies can beat together at a low frequency. When the two frequencies are in phase, they can cause significant vibration if their amplitudes are high. Another reason is that it is very easy to misinterpret the cause of the vibration. If the amplitude of the third order of *Fb* is very high, you might mistakenly interpret this as a bad motor, since the frequency is so close to the operating frequency of the fan.

As it happens, the motor support also had a resonance at 28.5 Hz. When the air handler was turned on, it vibrated so badly that the unit eventually destroyed itself! The following time domain plot was taken on the motor support, showing how the maximum support vibration occurred at 28.5 Hz, excited by the belt strand resonance.

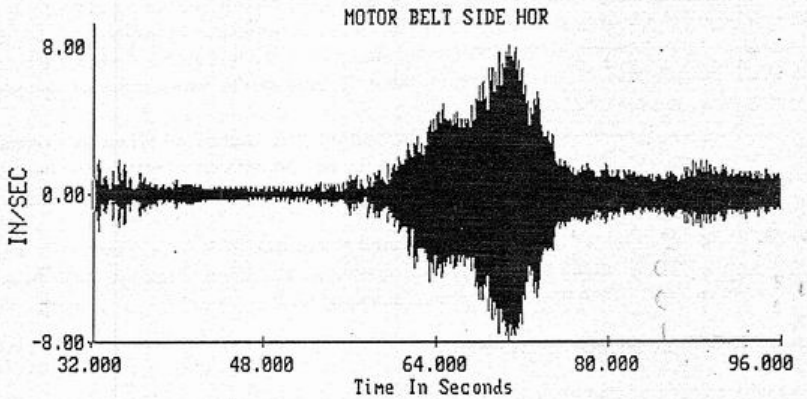


Figure 89 - *time domain plot of support resonance excited by belt strand*

5.5 Belt Types and Vibration Severity

If the belts are identified as the vibration forcing function, and the level is excessive, sheave alignment, groove wear, and belt condition should be carefully checked. If the pulleys are in good condition and replacing the belts does not reduce the vibration level, it may be necessary to change the belt type.

The following table rates belts from the standpoint of vibration alone, assuming that power transmission, belt tension, belt condition, sheave alignment, and sheave balance are all correct for the type.

Table 4 rates belt types from 0 to 100, with the type having the lowest vibration rated as 100, and those types having the highest vibration rated as 0.

Belt Type	Severity
Endless woven	100
Poly-V	60
Link (without rivets)	50
Link (with rivets)	40
Wide angle	30
Timing	25
Multiple V	12
Standard V	10
Open end	5

Table 4 - *vibration severity for various belt types*

6.0 CHAINS AND CHAIN DRIVES

Chain drives are very much like flexible gears: the chain is similar to an endless gear rack, while the sprockets resemble drive, driven, and pinion gears. Chain drives are less flexible than belt drives, but chains can deliver more power than a belt drive. Also, chains are not susceptible to slip or creep, so they can provide constant-speed drive or can be used for timing applications.

6.1 Types of Chains

There are three major types of chains:

1. **Roller** - A roller chain consists of metal rollers held between two side pieces (called **sidebars**) by bushings inside the rollers. The rollers are free to move around the bushing, helping the chain engage the sprocket. Bicycle chains are the most common example of roller chains. A variation on the roller chain is the **rollerless** chain, which only has solid pins. Both types of chains are relatively quiet, can operate up to 4,500 feet/minute, and are capable of moderate to heavy loads. Figure 90 illustrates the basic design of a roller chain.
2. **Block** - A block chain is similar in design to a roller chain, except that cast blocks are used instead of pins and rollers. These chains are used mostly in conveyor applications. Block chains are not usually used for power transmission due to their excessive noise. Block chains are shown in Figure 90.
3. **Silent** (or **inverted tooth**) - Silent chains are not exactly quiet; however, they do make much less noise than roller or block chains. A silent chain is made from a series of flat metal pins, shown in Figure 90. Each pin has a straight side, which mates with a straight-sided sprocket

tooth. The pins tend to roll along the tooth as the sprocket rotates, significantly reducing vibration and noise. Silent chains are most often used as timing chains in automobile engines.

Other popular chain designs include

- **Ewart** - designed for easy assembly and disassembly. They may have either an **open link** (where one end is hooked to hold the links together) or a **closed link** (also called a **pintle link**), where the links are held together by a pin and bushing. Ewart chains are used mostly for conveyor applications and for lifting, although they may be used for very light loads at low speeds.
- **Offset link** - Another variation on the roller chain, where each link has both a roller and a pin. The sidebars of these chain links are *offset* (bent) to allow for the greater length of the pin. These links are often used to make chains with an odd number of links. Offset links wear much faster than normal roller or block links, so they are not used as often as roller links.

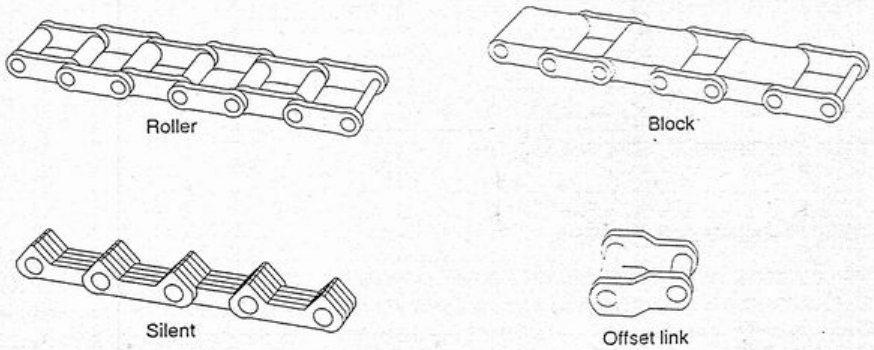


Figure 90 - *common chain types*

When compared with belts, chains have several advantages:

- Chains do not slip or creep. Therefore they transfer shaft angular velocity at a constant speed ratio.
- Chains are more efficient, since no power is lost due to slip.
- Chains can operate at higher temperatures than belts.
- Chains are more practical at low speeds.
- Age, oil and grease, and sunlight do not deteriorate chains.
- Chain drives take up less room. The chains themselves are usually smaller than belts, and the sprockets are usually smaller than belt sheaves.

However, chain drives require frequent lubrication; some chain drives operate with part of the chain running through a pool of oil. Chains can only tolerate very little misalignment, much less than belt drives. Also, chain drives are more noisy than belt drives.

7.0 PRINCIPLES OF CHAIN DRIVES

Figure 91 shows a chain engaging a drive socket, and illustrates some of the terminology used to describe chains and chain drives.

- **Sprocket** - The toothed wheel that engages the chain. The drive socket is often called either the **drive gear** (although strictly speaking they are not gears, since the teeth do not have an involute profile) or the **pinion**.
- **Arc of contact** - The angle through which the chain is in contact with the sprocket (also called **wrap**). Drive sprockets should have a minimum wrap of 90 degrees; 120 degrees is desirable.
- **Pitch** - The distance between two successive roller centers. The chain shown in Figure 91 is called a **single pitch** chain; that is, a roller makes contact with each sprocket groove.

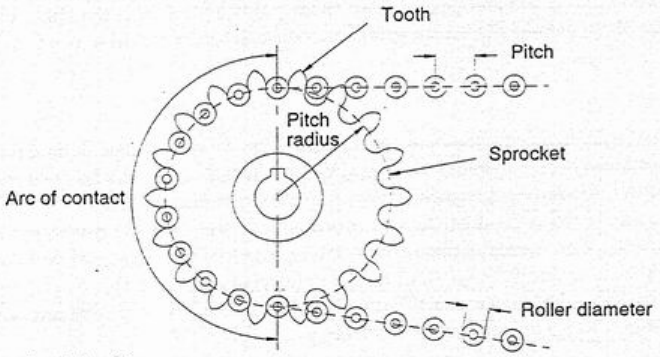


Figure 91 - *a basic chain drive*

Double pitch chains have sidebars that are twice as long as a single pitch chain of the same width. These chains are used with a **hunting tooth** sprocket as shown in Figure 92.

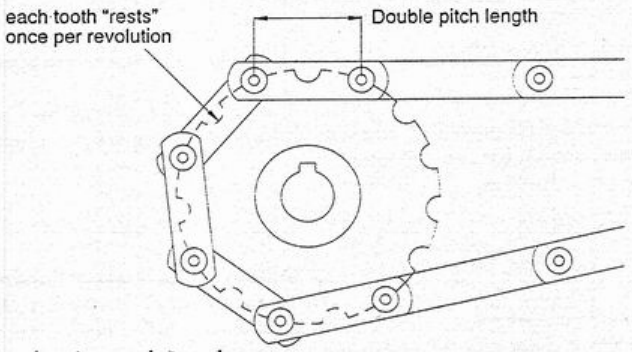


Figure 92 - *a hunting tooth sprocket*

The hunting tooth design lessens sprocket wear. Due to the double pitch length of the chain, each tooth on the sprocket contacts the chain *every other* revolution. This allows each tooth to "rest," minimizing wear. Teeth on a hunting tooth sprocket must be extra strong, since fewer teeth are in contact with the chain throughout its wrap.

- **Pitch radius** - The distance from the center of the sprocket to the center of a link joint when the link lies in a sprocket groove. The **pitch circle** is the circle whose radius is equal to the pitch radius. The **pitch diameter** is equal to two times the pitch radius.
- **Chain width** - The distance between the inside edges of the sidebars.
- **Roller diameter** - the diameter of each roller in a roller chain.

As the chain enters the drive sprocket, the rollers lay inside the sprocket grooves. The sprocket teeth enter in between the rollers and push against them. This action pulls the chain into the drive sprocket. Since the chain cannot slip on the sprocket, a constant velocity drive is achieved.

7.1 Drive Ratio

The **drive ratio** of a chain drive is the ratio of the speed of the drive sprocket to the driven sprocket. This ratio depends on the relative diameters of the sprockets. If both sprockets are the same diameter, then each one rotates at the same number of RPMs; this is a drive ratio of 1:1.

If the drive sprocket is twice the diameter of the driven sprocket, then the driven rotates twice as fast (a drive ratio of 2:1). Conversely, if the driven sprocket is twice the diameter of the drive sprocket, then the driven rotates at one-half the speed of the drive (a drive ratio of 1:2).

Drive ratios exceeding 7:1 are not recommended for roller chains. Silent chains can accommodate drive ratios as great as 12:1; ratios of 8:1 are common.

7.2 Number of Sprocket Teeth

The minimum number of teeth on the drive sprocket depends on the chain speed. Drive sprockets on roller chain drives should have 17 or more teeth for moderate speeds and 21 or more teeth for high speeds. Generally, roller chain sprockets should have a minimum of 12 teeth to avoid excessive wear and vibration.

Silent chain sprockets should have a minimum of 17 teeth, while as many as 25 teeth should be used for high-speed applications.

As a general rule, if a chain has an even number of pitches, then the sprockets should have an odd number of teeth (and vice versa). This prevents a single tooth from contacting the same link each revolution, which would cause uneven wear and vibration.

7.3 Center-to-Center Distance

The distance between sprocket centers in a chain drive must be greater than the sum of the sprocket outside radii; otherwise the sprockets would hit each other as they rotated. For roller chains, center-to-center distances should range between 30 and 50 times the chain pitch, with 40 times being normal. A distance of 80 times chain pitch is considered a maximum. For silent chains, good center-to-center distances should equal or exceed .8 times the difference in diameters of the two sprockets.

Proper center-to-center distances can be calculated without knowing the pitch of the chain. For chain drives with drive ratios between 2:1 and 7:1, the recommended center to center distance equals the diameter of the large sprocket plus the radius of the smaller sprocket. This rule ensures that there will be sufficient chain wrap on the smaller sprocket.

7.4 Chain Slack

When a chain engages the drive sprocket, one side of the chain (the side being pulled into the sprocket) is placed in tension; the other side becomes slack. Sprocket rotation on horizontal chain drives should be designed so that the slack strand is on the bottom and falls away from the tight strand, as shown in Figure 93(a).

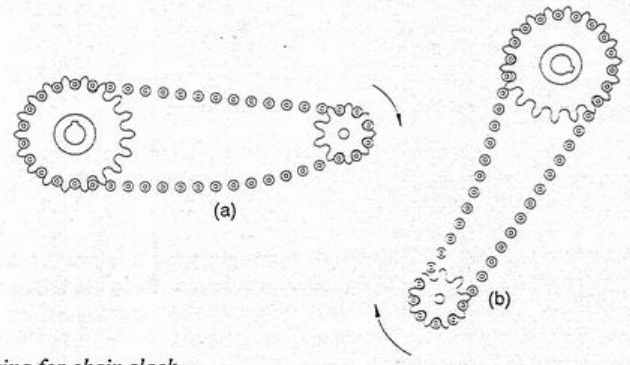


Figure 93 - *allowing for chain slack*

Vertical chain drives should be avoided. If two sprockets must be mounted vertically, they should be at an incline so that the slack strand falls into the *driven* sprocket, as shown in Figure 93(b).

7.5 Chain Wear and Tension

Chains "stretch" during their life due to bushing and pin wear. This wear causes the length of the chain to increase, which increases slack. If the tension on a chain is correct, then the chain flexes only twice as it travels around its circuit: once when it enters the drive sprocket and once when it exits the driven sprocket. A loose chain, however, will flex *four* times around its circuit: once when it enters and exits *each* sprocket. This increases wear on both the chain and the sprocket.

Single pitch chains should be replaced when wear elongation equals 3% of the original chain length, while double pitch chains should be replaced at 1 ½% elongation. Chains may be operated past this point, but they must constantly be checked for excessive slack and tightened when necessary. The few extra hours of run time are usually not offset by the extra time spent checking and retightening the chain.

Chains ride higher in the sprocket groove as they wear. Excessively worn chains ride so close to the top of the tooth that the chain may jump a tooth, which causes extremely high loads on the chain link and the shaft.

7.6 Chordal Action

Chordal action is a rise and fall of each chain pitch as it engages a sprocket tooth. As the chain engages the sprocket, the centers of the chain link joints lie on the sprocket's pitch circle (at radius *Pr*) as shown in Figure 94(a). The line between joint centers, however, forms a chord to the pitch circle. The midpoint of this chordal line is at distance *r* from the sprocket center.

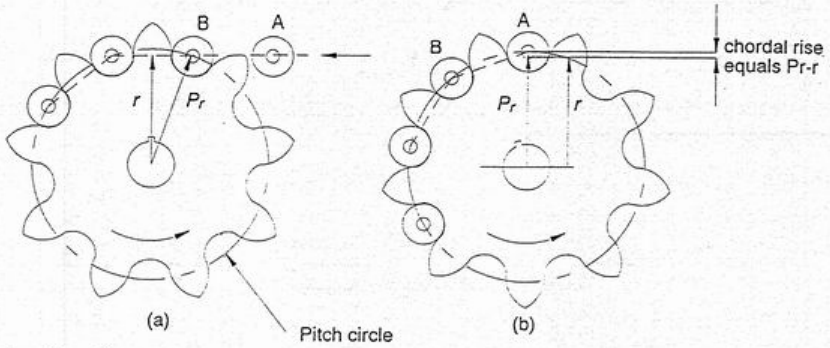


Figure 94 - *chordal action*

As roller A engages the sprocket in Figure 94(a), its center begins to follow the chordal line to the position of roller B. However, as the sprocket rotates, roller A is forced to follow the pitch circle instead of the chordal line. This is shown in Figure 94(b). As the roller follows the pitch circle, its center is raised up by the distance $Pr - r$. This raising up causes the center line of the following links to rise momentarily, only to fall as soon as the next roller first contacts the sprocket.

The amount of chordal action (the magnitude of the rise) depends on the number of teeth in the sprocket: the more teeth, the less the chordal rise.

The rise and fall of chordal action causes periodic fluctuations in the tension strand, which in turn causes angular velocity fluctuations in the driven sprocket. In most cases, however, the chain is able to absorb most of the speed variations.

7.7 Determining Chain Length

The length of a chain can be calculated if the following are known:

- *P* - The pitch of the chain
- *R* and *r* - The pitch radii of both sprockets
- *N* and *n* - The number of teeth on both sprockets
- *D* - The center to center distance between the sprockets

Figure 95 illustrates the process.
First, you must find the angle *a* and the tangent length *A* by using the following formulas:

$$a = arcsin\left[\frac{(R-r)}{D}\right] \qquad \text{(Formula 15-6)}$$

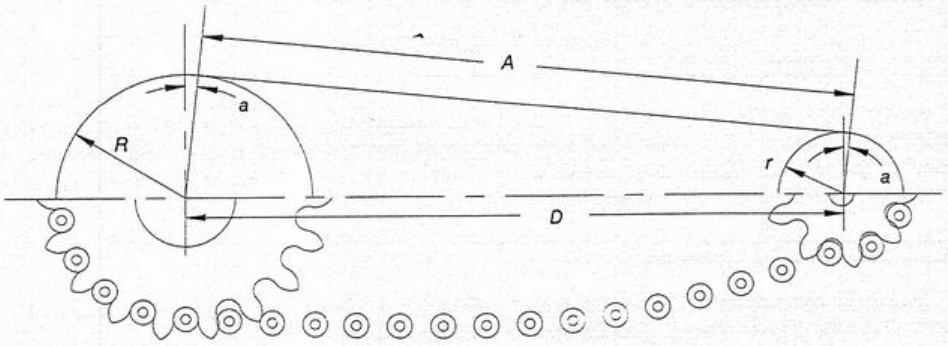


Figure 95 - *determining chain length*

$$A = D \cos a \qquad \text{(Formula 15-7)}$$

where

R = pitch radius of larger sprocket (inches)
r = pitch radius of smaller sprocket (inches)
D = center-to-center distance (inches)

The operation "arcsin" means inverse sine, or "the angle whose sine is" (often written in math textbooks as $\sin^{-1}$). Now the chain length L can be found by

$$L = \frac{NP\,(180 + 2a)}{360} + \frac{nP\,(180 - 2a)}{360} + 2D \cos a \qquad \text{(Formula 15-8)}$$

where

L = length of chain (inches)
N = number of teeth on large sprocket
P = chain pitch length (inches)
n = number of teeth on small sprocket
D = center to center distance (inches)

Chain length should be an even number of pitches.

If you know the center-to-center distance in chain pitches, you can find the length of the chain in pitches by the following formula:

$$Lp = \frac{(N + n)}{2} + \frac{a(N - n)}{180} + 2Dp \cos a \qquad \text{(Formula 15-9)}$$

where

Lp = length of chain in pitches
N = number of teeth on large sprocket

n = number of teeth on small sprocket
Dp = center-to-center distance in pitches

8.0 CHAIN DRIVE VIBRATION

Unlike belts, chains do not have a fundamental frequency that produces prominent vibratory amplitudes in chain drives. And, since both chain strands are made up of many flexible elements, chain strand resonance is not large enough to be of any consequence. There are, however, certain vibratory frequencies that can be prominent in chain drives.

8.1 Tooth-Induced Vibration

Due to the constant impact between sprocket teeth and chain, a tooth-induced vibration is present in chain drives. The frequency of tooth-induced vibration equals

$$Ft = Td * Fd$$

where

Ft = tooth-induced vibration frequency (Hz)
Td = number of teeth on the drive sprocket
Fd = rotational frequency of drive sprocket (Hz)

Or, you can calculate the tooth-induced vibration frequency Ft by multiplying the number of teeth on the driven sprocket times the driven sprocket frequency. Either method yields the same result.

Multiplying the number of teeth by the sprocket's rotational frequency gives the frequency in Hz (cycles per second); multiply this number by 60 to get the frequency in cycles per minute. If this frequency (or any harmonics) coincide with the resonant frequency of some element in the system, this frequency will be amplified to potentially dangerous levels.

8.2 Other Sources

You should never insert a new link in a wear-lengthened chain. The new link has a shorter pitch than any of the other links, which causes a shock each time the new link engages a sprocket.

Excess or insufficient tension on the chain can lead to vibration. Too much tension puts too great a load on the shaft bearings, which can lead to bearing failure. Too little tension causes pulsations and whipping in the chain, which creates an irregular sprocket speed. These jumps in sprocket speed can lead to excessive torsional vibration.

Chains that are excessively worn can jump a tooth, which causes high shock and torsional vibration.

9.0 UNIVERSAL JOINTS

A universal joint (also called a u-joint) is a device for transmitting torque and/or rotational motion from one shaft to another when the shaft axes are at fixed or varying angles of intersection. The shaft that is connected to the power source (such as a motor or an internal combustion engine) is called the *driver*, while the other shaft connected to the driven equipment is called the *driven* shaft.

Common applications of universal joints are in automotive drive trains.

Universal joints are classified according to their basic design. The main interest in any universal joint is in how it transmits motion. There are two motion transmission characteristics:

1. Non-uniform
2. Uniform

9.1 Non-Uniform Joints

A non-uniform (or non-constant velocity) universal joint transmits motion with varying ratios of instantaneous angular velocity when the angle between the shafts is greater than zero. The average angular velocity ratio between the driver and driven shafts is one; at any instant, however, the ratio may be more or less than one.

9.2 Uniform Joints

A uniform (or constant velocity) universal joint transmits motion between the driver and driven shafts with an angular velocity ratio of one.

There is also a category of universal joints called *near constant-velocity*, where motion is transmitted with an angular velocity ratio of one when the joint is operated at its design angle or at zero. At other angles, the angular velocity is other than one. These deviations from a ratio of one are minor compared to non-uniform joints, however, so these joints are usually included in the same category as constant velocity joints.

9.3 Other Classifications

Universal joints may also be classified by other elements of their design:

- **Support** - A *self-supporting* joint is supported internally and is installed on only one shaft, either the driver or the driven. A *non-self-supporting* joint must be supported by both the driver and driven shafts in order to maintain proper joint alignment.
- **Axial movement** - A *fixed center* joint keeps the center of the joint at a fixed location, resisting axial movement. An *end motion* joint allows axial movement between the driver and driven, and therefore has a variable joint center.
- **Joint angle** - A *high angle* universal joint accommodates a large angle between driver and driven shaft axes, and is usually of the fixed center type. A *low angle* joint allows only a small angle between the shafts, and is usually an end motion type.
- **Direction of power transmission** - Most universal joints are bi-directional, transmitting torque from either shaft to the other with equal capacity.

The universal joint designs that are in greatest use today are

1. the Cardan (or Hooke) joint
2. the Ball and trunnion joint
3. the Roller and trunnion joint
4. the Sliding block and trunnion joint
5. the Rubber coupling

10.0 CARDAN UNIVERSAL JOINTS

The most common design of universal joint is like that found in automotive drive trains. This design is referred to as either the *Cardan* or *Hooke* universal joint. This is a very old design; references to this type of joint can be found in architectural manuscripts dating from the thirteenth century.

A sixteenth-century Italian mathematician named Girolamo Cardano used this design to suspend a ship's compass so that it maintained its horizontal position regardless of the motion of the ship. This design came to be called *Cardan Suspension* in honor of its inventor. Cardano, a prolific writer, was also the author of the world's first mathematical text on probability theory, and was the inventor of the "autokey" system of cryptography.

Later, a British scientist named Robert Hooke was granted a patent on the universal joint in the seventeenth century. Hooke's design was an extension of Cardano's suspension techniques.

The Cardan joint is a non-constant velocity joint consisting of two yokes connected by a cross through four bearings. Figure 96 illustrates this design.

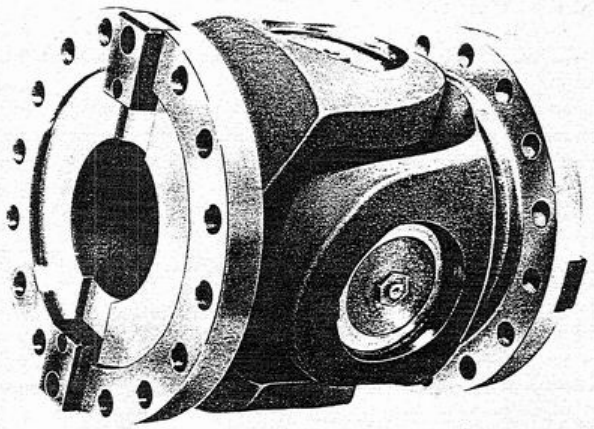


Figure 96 - *the Cardan joint* Universal joint courtesy of Zurn Industries, Inc., Erie, PA

Other types of Cardan joints may use pins and blocks or a grooved sphere instead of the cross.

The Cardan joint is relatively inexpensive to manufacture, and the rugged design provides long life and easy servicing. Also, Cardan joints can withstand high axial forces such as those produced by a sliding spline as shaft lengths change due to vehicle suspension movements.

10.1 Non-Uniform Motion

When a Cardan joint operates at an angle, the angular velocity transferred from the driver to the driven shaft is not equal. That is why this type of joint is called a non-uniform joint.

To understand the source of this non-constant velocity, consider the motion of a single point on the driver yoke. If you were to look down the axis of the driver shaft, toward the yoke, the path of the point during one revolution would be a circle. This is illustrated in Figure 97.

Each arm of the cross member acts as a lever, transmitting torque from the driver shaft to the driven shaft. If we assume that the Cardan joint is operating at zero angle (the shaft axes are in-line), then

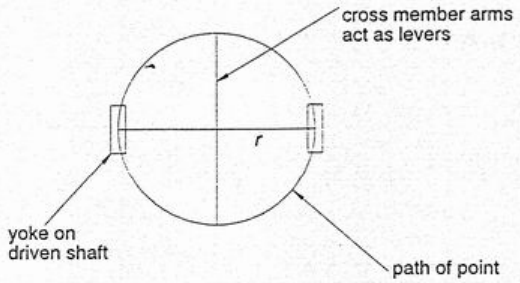


Figure 97 - *motion of a point at zero joint angle*

the path of the point would be a circle when viewed from either direction. Also, the effective length of the lever arms is the same when viewed from either shaft axis.

Now, if the driven shaft is placed at an angle to the driver shaft, then the path of the point is no longer a circle when viewed along the axis of the driven shaft. When viewed from the driven shaft, the point on the driver yoke follows an elliptical path, as shown in Figure 98.

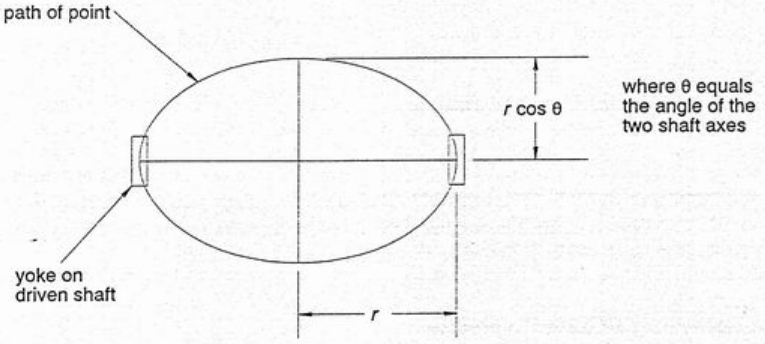


Figure 98 - *motion of a point at an angle*

This is actually a very familiar concept; artists will draw a circular shaft as an ellipse to indicate that our view of the shaft is from an angle.

In the case of Figure 98, the effective length of the lever arms is no longer equal. These unequal lengths affect the torque transmission, and cause the angular velocity of the driven shaft to vary continually during one revolution. In fact, there are two complete speed variations during one revolution, as shown in Figure 99.

Figure 99 is a plot of angular velocity vs. the rotation of the driving yoke for a Cardan joint operating at a 4° 30′ angle with the driver shaft operating at 3600 RPM. This plot assumes that the angular velocity of the driver shaft remains constant throughout the rotation.

Figure 100 is a polar diagram of the angular velocity of both the driver and driven shafts through one revolution.

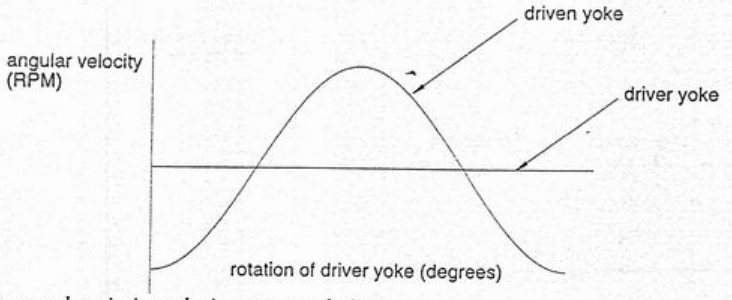


Figure 99 - *speed variations during one revolution*

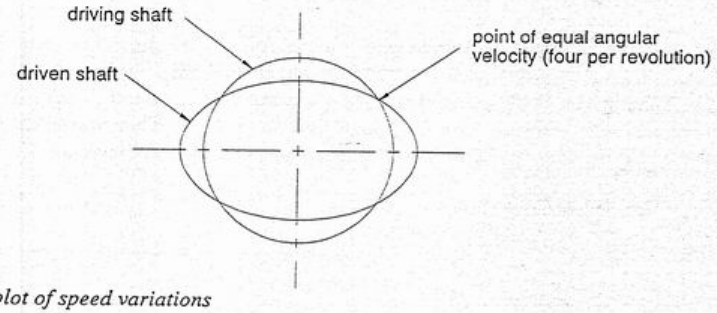


Figure 100 - *polar plot of speed variations*

The angular velocity of the driver shaft is shown as a circle, since it is assumed that its angular velocity remains constant. The elliptical plot of the driven shaft's angular velocity intersects the circle in four places; at these points the angular velocity of both shafts are equal. During the remaining time, the driven shaft rotates either faster or slower than the driver shaft.

10.2 Achieving Constant Velocity

If a Cardan joint must be operated at an angle, then the driven shaft will not rotate at a constant velocity. To achieve constant velocity across the shafts, it is necessary to use two Cardan joints operating at equal angles.

Using two Cardan joints of equal angles is called **joint phasing**. The technique can be used for shafts whose axes are parallel or intersecting, as shown in Figure 101.

Figure 101(a) shows two shafts whose axes are parallel to each other, connected by two Cardan joints. Figure 101(b) shows two Cardan joints used to connect two shafts whose axes intersect. In both cases the angle $\theta 1$ represents the angle between the driver shaft and the connecting shaft. Likewise, angle $\theta 2$ is the angle between the connecting shaft and the driven shaft.

If $\theta 1 = \theta 2$, then there is no residual joint angle through the system and constant velocity is transmitted. It should be noted that, in addition to maintaining equal angles, the yokes of each joint must be phased appropriately as well. The method of yoke phasing depends on the type of shaft configuration and is beyond the scope of this chapter.

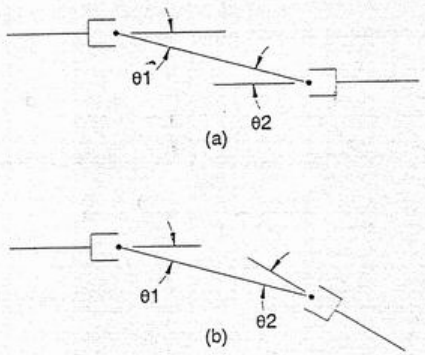


Figure 101 - *joint phasing*

11.0 BALL AND TRUNNION UNIVERSAL JOINT

A ball and trunnion universal joint is a non-constant velocity joint using a ball-headed shaft and two toroidally-shaped balls held opposite to each other by a pin. Also called a **bipot** joint, the ball and trunnion universal joint was developed in 1914, and was used throughout the early 1920s in small American passenger car driveshafts.

A **trunnion** is a pin or a cylindrical projection that is used as a pivot axis. For example, the two cast projections that extend from either side of a cannon barrel and allow the barrel to pivot in the frame are trunnions. The basic design is shown in Figure 102.

In a ball and trunnion universal joint, a ball-headed shaft is attached to the end of one of the shafts (either the driver or the driven). A pin is placed through the ball head, providing a trunnion on either side for needle bearings and balls. The ball and trunnion assembly then slides into a housing.

The housing has two partially cylindrical bores that accept the ball and trunnion assembly. This housing is mounted to the other shaft, and torque is transferred from one shaft to the other by the tight clearance between the balls and the housing.

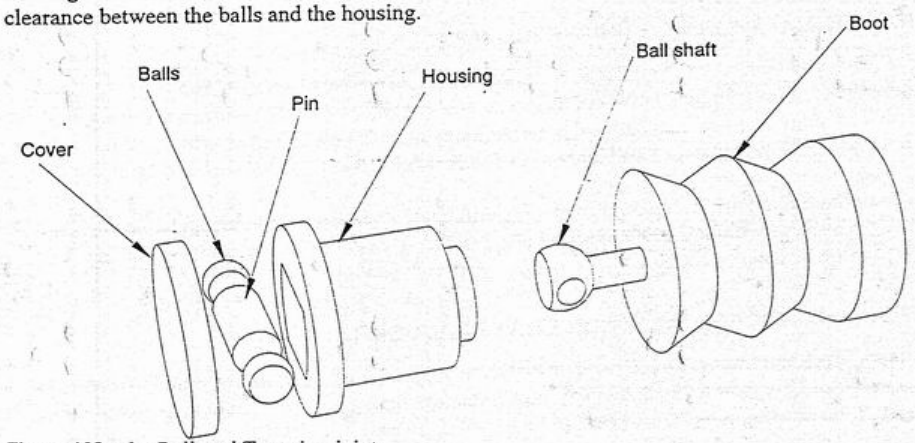


Figure 102 - *the Ball and Trunnion joint*

When the joint is operated at an angle, the balls rotate against the housing. The axial position of the balls within the housing determines the maximum operating angle of the joint. This is shown in Figure 103.

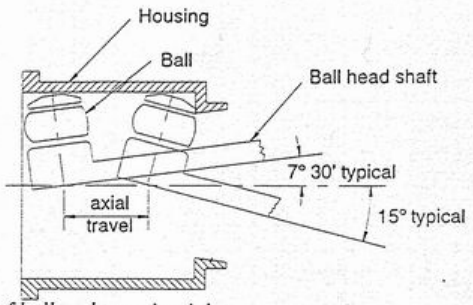


Figure 103 - *maximum angle of ball and trunnion joint*

The balls are often toroidal in shape instead of spherical, so that the contact area between the ball and the housing is kept at a maximum under high torque loads.

Ball and trunnion joints allow a great deal of axial movement with very little axial force. This ease of axial movement minimizes thrust forces on drivetrain supports and eliminates spline noise. However, these joints have a much smaller allowable joint angle than Cardan joints, since large angles severely limit axial movement. Therefore, ball and trunnion joints are usually used in low-angle installations so that maximum axial movement is available.

As with the Cardan joints, ball and trunnion joints do not transfer angular velocity at a constant rate when operating at an angle. There are two complete velocity shifts during one rotation, causing torsional excitation of the surrounding structure.

For constant velocity applications, two ball and trunnion joints can be phased. The phasing technique depends on whether the housing or the ball and trunnion is doing the driving.

12.0 ROLLER AND TRUNNION UNIVERSAL JOINT

This joint, first patented by Lipe in 1909, is similar to the ball and trunnion universal joint. Instead of mounting balls to the trunnion, however, this joint uses a needle roller. The advantages of a roller and trunnion joint are identical to a ball and trunnion joint, except that a slightly higher axial force is needed to create axial movement. This higher force is still much lower than that required for a sliding spline.

A roller and trunnion joint is a non-constant joint, with the driven shaft experiencing two complete speed changes in one revolution when operated at an angle. Two roller and trunnion joints may be phased to provide constant velocity.

13.0 SLIDING BLOCK AND TRUNNION UNIVERSAL JOINT

The sliding block and trunnion joint uses block-shaped bearings on the ends of the trunnion instead of balls or rollers. This joint is much simpler in construction than a ball and trunnion or roller and trunnion joint; the housing is machined flat, and needle rollers are not needed in the bearing blocks (since the blocks slide in the housing slots).

Sliding block and trunnion joints have the same advantages and disadvantages as ball and trunnion and roller and trunnion joints. The axial thrust force necessary for axial movement, however, is nearly equal to that of a sliding spline. The maximum joint angle of a sliding block and trunnion joint is somewhat lower than a ball or roller and trunnion joint.

The procedure for phasing a sliding block and trunnion joint to obtain constant velocity is the same as for the ball and trunnion joint.

14.0 RUBBER COUPLINGS

Rubber couplings have an elastic intermediate material (usually natural rubber) that connects two yokes or forked shafts. The flexible intermediate material absorbs shock and torsional vibration, dampens drivetrain disturbances, and allows limited axial movement. These couplings are considered universal joints because the flexible intermediate material allows the joint to be operated at an angle.

The biggest advantage of rubber couplings is that they are maintenance-free, relatively unaffected by dirt and moisture, and require no lubrication unless a centering device is used. A centering device is a cylindrical projection that extends from one shaft and fits inside a cylindrical hollow extending from the other shaft. The tight fit between the two cylinders maintains a zero angle, but lubrication is required.

Rubber couplings must be limited to low-torque installations if any appreciable life is to be expected. Although rubber couplings can tolerate intermittent joint angles of up to 15 degrees, they generally cannot tolerate continued operation at angles above 3 degrees.

As with all universal joints, a rubber coupling is a non-constant velocity joint when operated at an angle. Two couplings must be phased to provide constant velocity.

15.0 VIBRATION IN UNIVERSAL JOINTS

The non-uniform angular velocity characteristics of a universal joint produce varying speeds and torques, causing vibration excitation. Vibration in a universal joint can be broken into three categories:

1. Torsional vibration
2. Inertial vibration
3. Secondary couple vibration

15.1 Torsional Vibration

It is practically impossible to maintain equal joint angles throughout the operating range of a system. Changes in position due to suspension or foundation movement, for example, change the joint angles and result in non-constant velocity transmission. These changes in velocity produce torsional vibration in the drive train.

The maximum amount of acceptable torsional vibration depends on operating speed, support, and other units in the drive train. For example, a maximum angular acceleration of 400 radians/sec.^{2} is acceptable for passenger cars and trucks. Other equipment, such as agricultural equipment and stationary installations, may be able to tolerate higher levels of torsional excitation.

Torsional vibration produces oscillations on major supports within the system. These oscillations can excite motor mounts and rear axle supports, especially if they are near a resonance. The vibration is usually at operating frequency and torque-dependent; often very high amplitudes will occur within a short speed range under specific loading conditions.

The torsional vibration of supporting members can cause loading and unloading on the driving members. This tends to amplify gear noise and clatter that might not otherwise be noticeable. Torsional vibration usually occurs under full load, since joint angles are greatest and deflected supports transmit the most vibration.

15.2 Inertial Vibration

Inertial vibration is produced by the oscillating torque loads that occur when the drive shaft is being accelerated through non-uniform motion. This condition is present even when two universal joints are used in series at equal angles; the inertia of the intermediate shaft experiences changes in angular velocity, which results in oscillating torque loads. The level of these loads depends on the size of the joint angle and the inertia of the intermediate shaft.

Since inertial vibration is always present, it is this vibration that limits the maximum joint angle for a universal joint. Inertial vibration can be reduced in multi-shaft installations by carefully arranging joint angles and yoke phasing so that the acceleration of the shafts is opposed to each other.

Inertial vibration is most noticeable when the drive train is unloaded or very lightly loaded, since system components are free to reverse and clatter through their backlash when they are experiencing light loads. These vibrations are usually very disturbing, since system noise levels are lower in a no-load or light-load condition.

15.3 Secondary Couple Vibration

When a universal joint transmits torque at an angle, an oscillating bending is created by the change in direction of the torque. This bending moment is called a **secondary couple**, and is a function of the torque being transmitted and the joint angle. The secondary couple acts in the axial plane of the driver and driven yokes, and is transferred to connecting shaft members and supporting bearings.

Secondary couples are cyclic; their magnitude ranges from zero to maximum twice during each revolution. The resulting couple forces react on supporting bearings and other frame members. Vibration of the supports can cause a response in the drive train at one-half rotational frequency. The effect of the couple on the intermediate shaft is a 1*RPM vibration such as imbalance.

Vibrations from a secondary couple usually occur when the couple frequency coincides with a resonance within the system. Shaft support bearings are very susceptible to secondary couples; the supports are excited into resonance when the drive train is rotating at one-half the natural frequency of the support structure.

15.4 Other Vibrations

Bearing fault frequencies are usually not present in universal joints. Even though needle and roller bearings are used in universal joints, the bearings themselves rotate at such low frequencies that fault frequencies are not a problem.

However, it is very important that the bearings within a universal joint are completely lubricated. Lubrication is packed in the bearings when the joint is assembled, but the bearings must rotate to evenly distribute the lubricant. If a universal joint were perfectly aligned, the bearings would not rotate at all. This would cause the bearings to fail prematurely, with significant vibration.

16.0 FLEXIBLE COUPLINGS

Flexible couplings are used to join the shaft from the driving machine to the shaft of the driven machine. Besides their primary function, the transmission of power from the driver to the driven machine, flexible couplings have two advantages over older, rigid couplings:

1. They can accommodate some degree of shaft center line misalignment
2. They allow for end float (axial movement) of the coupled shafts

You should **not** assume that a coupling's ability to tolerate some shaft misalignment excuses you from properly aligning the driver and driven machines! Since perfect shaft center line alignment is impossible, a flexible coupling is designed to allow for slight misalignment under normal load (whereas bearings are not).

Several other factors cause shaft alignment to change (for example, unanticipated load, uneven heating, or foundation movement). A flexible coupling accommodates these alignment changes without damage to the coupling or the system.

But, **the coupled systems must be aligned as accurately as possible, even when using flexible couplings.** See the chapter on Shaft Alignment in Volume 1 for more information on aligning coupled shafts.

Unlike belts, chains, or U-joints, flexible couplings do not add unique vibration components to the system's vibration spectra. However, flexible couplings can lower the system's torsional resonance into the operating frequency range of the system. Excessive misalignment in flexible couplings can introduce torsional vibrations and can cause vibration components called **cranking vectors**. These are all discussed later in this section.

16.1 Types of Couplings

There are four basic types of flexible couplings:

1. **Mechanically flexible** - These couplings have rolling, sliding, or loose-fitting components that allow the shafts to flex. Mechanically flexible couplings usually require lubrication unless one of the moving parts is made of a material that does not require lubrication (such as nylon). Chain and gear couplings are two popular examples of mechanically flexible couplings.
2. **Elastomeric** - These couplings are made with a flexible material such as rubber or plastic. Shaft movement stretches or compresses the flexible material to stretch or compress.
3. **Metallic membrane** - These couplings are made with thin, flexible metal disks or diaphragms. Metallic membrane couplings allow the shafts to flex by bending the metal membranes.
4. **Miscellaneous** - This category consists mostly of couplings that use one or more of the previous design methods to obtain flexibility.

16.2 Vibration Considerations

Flexible couplings do not usually add vibration components that are unique to their design (as opposed to belts, chains, and U-joints, each of which can add their own unique vibration signatures). A vibration component can be traced to a coupling only under extreme conditions. In most cases, vibration components that are (incorrectly) associated with couplings result from excessive shaft center line misalignment, rather than from the couplings themselves.

16.2.1 Torsional Resonance

Torsional resonance becomes a concern in systems that use "soft" couplings (metallic membrane and elastomeric couplings that use elastic material).

All systems have a torsional resonance, just as they have other natural resonant frequencies. Resonant frequencies are a natural product of any structure. A system only *vibrates at one of these frequencies if a forcing function (or a harmonic of a forcing function) exists at or near a system resonance.*

Not all systems have torsional forcing functions. Some systems, however, do have torsional forcing functions. The cranking of an internal combustion engine, for example, introduces torsional vibrations into the drive shaft. Electric motors experience changes in operating frequency as load changes, and U-joint drive trains with improperly-phased joints can introduce torsional vibration (speed changes) into the driven shaft.

The presence of torsional forcing functions does not necessarily mean that the system is resonant at the forcing function frequency. In most cases, these speed changes have been accounted for in the design of the system. But if a soft coupling is used in a system with torsional forcing functions, the torsional flexibility of the coupling can lower the system's torsional resonance into the operating speed range.

16.2.2 Gear Couplings

Gear couplings deserve special mention, since an improperly-maintained gear coupling can lock up and lead to excessive vibration.

A typical gear coupling is shown in Figure 104.

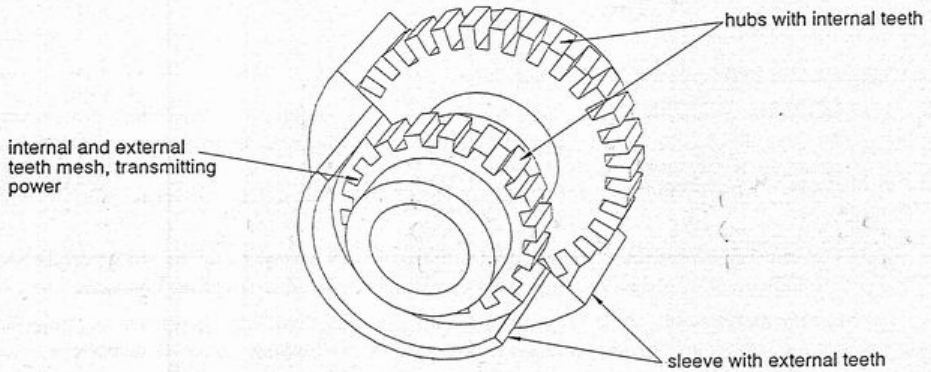


Figure 104 - *a gear coupling*

A hub with external gear teeth is mounted onto each shaft. These two hubs are covered with a sleeve containing internal gear teeth. The internal teeth of the sleeve mesh with the external teeth on the two hubs, allowing the transmission of power. Clearance between the teeth, and the shape of the teeth themselves (called the tooth profile), accommodate misalignment by allowing the hubs to move within the sleeve.

The name "gear" coupling may be an unfortunate name, since it seems to suggest that the teeth on the hubs and the sleeve are turning relative to each other, just as with normal gears. This is not the case; when the sleeve is attached to the hubs, the mesh is fixed - the hubs do not rotate relative to the sleeve. Therefore, gear mesh frequencies that are present in gear drives (see the Gears chapter in this Volume) are not present in gear couplings.

Gear couplings accommodate misalignment by allowing the hub teeth to pivot inside their mesh with the sleeve. Since the hub teeth are shorter than the sleeve teeth, the hub teeth slide back and forth in the mesh as end float occurs. Gear couplings require a small amount of misalignment, so that the movement of the hub teeth allows lubrication to work between the hub teeth and the sleeve teeth. If the two shafts were perfectly aligned, the hub teeth would remain in constant contact with the sleeve teeth. This would not allow lubricant to pass between the two surfaces, and would increase wear.

Under excessive conditions of misalignment and poor lubrication, the gear teeth wear and the clearances between the teeth increases. If this condition is not corrected, the excessive clearance allows so much lateral movement that the hub on the driver shaft locks up inside the sleeve. At this point the gear coupling ceases to be flexible, and the entire driver/driven shaft assembly behaves as one long, solid shaft that is bent in the middle.

This condition has two vibration symptoms:

1. **Asymmetric velocity** - The driven shaft changes velocity twice per revolution (see section 10.1, **Non-Uniform Motion**, for more information on asymmetric velocity). This results in high second-order (2*RPM) vibrations.

2. **Cranking vector** - An off-axis force that rotates around the shaft at 1*RPM. Imagine a hand crank used to start antique automobiles. A tangential force is applied continuously to the crank in order to turn it. This tangential force acts on the shaft in the same direction, but acts through the shaft centerline against the bearings. As the crank turns, the tangential force rotates around the shaft. Likewise, a cranking vector from a locked gear coupling is applied tangentially to the shaft center line, and rotates around the shaft at 1*RPM. Cranking vectors put high radial loads on the bearings on either side of the coupling, and lead to early bearing failure.

SUMMARY

Belt drives are the most common and the lowest cost way to transmit power from one shaft to another. Although they typically are not as durable as gear drives, they are much less expensive, run quieter, and absorb appreciable shock.

There are four major types of belts:

1. **Flat belts**
2. **Round belts**
3. **Timing belts**
4. **V-belts**

The V-belt is the basic style used for power transmission in most belt drives. V-belts are normally considered to be the best compromise for most belt drive systems because of their relatively low cost, high tractive force, reasonable operating speed range, and service life. They are the most popular belts for transmitting high loads between sheaves with short center distances.

Belt drives are one of the most common vibration forcing functions in industrial machinery. However, they are usually the last component to be suspected as the source of the vibration. Vibration components related to belts are usually related to

- **Sheave alignment**
- **Belt tension**
- **Primary belt frequency** - The number of times per minute that the belt makes one complete circuit. The predominant belt frequency is very seldom the primary belt frequency, but rather some multiple of the number of sheaves over which the belt passes and the number of belts in the drive times the primary belt frequency.
- **Belt strand frequency**

Chain drives are very much like flexible gears: the chain is similar to an endless gear rack, while the sprockets resemble drive, driven, and pinion gears. Chain drives are less flexible than belt drives, but chains can deliver more power than a belt drive.

There are three major types of chains:

1. **Roller**
2. **Block**
3. **Silent** (or **inverted tooth**)

Drive sprockets on roller chain drives should have 17 or more teeth for moderate speeds and 21 or more teeth for high speeds. Generally, roller chain sprockets should have a minimum of 12 teeth to avoid excessive wear and vibration. Silent chain sprockets should have a minimum of 17 teeth, while as many as 25 teeth should be used for high-speed applications.

As a general rule, if a chain has an even number of pitches, then the sprockets should have an odd number of teeth (and vice versa). This prevents a single tooth from contacting the same link each revolution, which would cause uneven wear and vibration.

There are certain sources of vibratory frequencies that can be prominent in chain drives:

- **Tooth-induced vibration**
- **Chordal action**
- **Inserting a new link in a wear-lengthened chain.** The new link has a shorter pitch than any of the other links, causing a shock each time the new link engages a sprocket.
- **Excess or insufficient chain tension** can lead to vibration. Too much tension puts too great a load on the shaft bearings, which can lead to bearing failure. Too little tension causes pulsations and whipping in the chain, which creates an irregular sprocket speed. These jumps in sprocket speed can lead to excessive torsional vibration.

A universal joint (also called a u-joint) is a device for transmitting torque and/or rotational motion from one shaft to another when the shaft axes are at fixed or varying angles of intersection. Common applications of universal joints are in automotive drive trains.

The universal joint designs that are in greatest use today are

1. **Cardan (or Hooke) joints**
2. **Ball and trunnion joints**
3. **Roller and trunnion joints**
4. **Sliding block and trunnion joints**

5. Rubber couplings

The most common design of universal joint is like that found in automotive drive trains. This design is referred to as either the *Cardan* or *Hooke* universal joint. This is a very old design; references to this type of joint can be found in architectural manuscripts dating from the thirteenth century.

When a Cardan joint operates at an angle, the angular velocity transferred from the driver to the driven shaft is not equal. If a Cardan joint must be operated at an angle, then the driven shaft will not rotate at a constant velocity. To achieve constant velocity across the shafts, it is necessary to use two Cardan joints operating at equal angles.

The non-uniform angular velocity characteristics of a universal joint produce varying speeds and torques, causing vibration excitation. Vibration in a universal joint can be broken into three categories:

1. **Torsional vibration**
2. **Inertial vibration**
3. **Secondary couple vibration**

Flexible couplings are used to join the shaft from the driving machine to the shaft of the driven machine. Besides their primary function, the transmission of power from the driver to the driven machine, flexible couplings have two advantages over older, rigid couplings

1. They can accommodate some degree of shaft center line misalignment
2. They allow for end float (axial movement) of the coupled shafts

Even though flexible couplings accommodate misalignment, **the coupled systems must still be aligned as accurately as possible.**

Unlike belts, chains, or U-joints, flexible couplings do not add unique vibration components to the system's vibration spectra. However, flexible couplings can lower the system's torsional resonance into the operating frequency range of the system. Excessive misalignment in flexible couplings can introduce torsional vibrations and can cause cranking vectors.

There are four basic types of flexible couplings:

1. **Mechanically flexible**
2. **Elastomeric**
3. **Metallic membrane**
4. **Miscellaneous**

Torsional resonance becomes a concern in systems that use "soft" couplings (metallic membrane and elastomeric couplings that use elastic material). If a soft coupling is used in a system with torsional forcing functions, the torsional flexibility of the coupling can lower the system's torsional resonance into the operating speed range.

Gear couplings require a small amount of misalignment, so that the movement of the hub teeth allows lubrication to work between the hub teeth and the sleeve teeth.

Under excessive conditions of misalignment and poor lubrication, the excessive clearance between teeth resulting from wear allows so much lateral movement that the hub on the driver shaft locks up inside the sleeve. The gear coupling ceases to be flexible, and the entire driver/driven shaft assembly behaves as one long, solid shaft that is bent in the middle.

BIBLIOGRAPHY

Belts and Chains, Fundamentals of Service-53, John Deere and Company.

Duggan, Terance V., Power transmission and vibration considerations in design, Copyright 1971, Butterworth and Co., Ltd., London.

Piotrowski, John, Shaft Alignment Handbook, Copyright 1986, Marcel Dekker, Inc., New York.

Mancuso, Jon R., Couplings and Joints, Copyright 1986, Marcel Dekker, Inc., New York.

Greenwood, Douglas C., Mechanical Power Transmission, Copyright 1962, McGraw-Hill Book Co., New York, NY.

American Chain Association, Chains for power transmission and material handling, Copyright 1982, Marcel Dekker, Inc., New York, NY.

Society of Automotive Engineers, Universal joint and driveshaft design manual, Advances in Engineering Series No. 7, Copyright 1979, SAE, Warrendale, PA.

Marks' Mechanical Engineers Handbook, Theodore Baumeister, Editor, Copyright 1958, McGraw-Hill Book Co., New York, NY.

Ulsoy, A.G., "Vibration Control in Rotating or Translating elastic systems," Journal of Dynamic Systems & Measurement Control, Vol 106, No. 1, 1984.

Rao, A.C., "Preventing Belt Vibration," Mechanical Design, Vol 49, No. 12, May 26, 1977.

Mote, C.C., "Vibration Coupling in Continuous Belt & Band Systems," Dept. of Mechanical Engineering, California University, Berkeley, Journal of Sound & Vibration, Vol 102, No. 1, Sept 8, 1985.

Claasz, A.F., "Compact Drives Possible with High Duty Flat Drives," Wireworld, Volume 30, No. 2, March 1988.

Tourret, J.; Birembaut, Y. & Deschamps, M., "Noise Generation of Synchronous Belts Using Acoustic Mapping," Noise Control Engineering Journal, Vol 30, No.1, Jan-Feb, 1988.

Abrate, S., "Noise and Vibration In Power Transmission Belts," SAE Technical Paper Series, Paper No. 860428.

"Round Woven Belts Enter a New Design Era," Design News, Vol 40, No. 11, June 4, 1984.

Smith, R.S. & Emerson, P.D., "Vibration Measurement of a Moving Tangential Drive Belt," ASME Paper, 81-DET-36.

Fawcett, J.N. & Burdess, J.S., "Experimental Investigation of the Vibration of Toothed Belts," ASME Paper 80-C2/DET-94.

Houser, D. & Oliver, L., "Vibration of V-Belt Drives Excited by Lateral and Torsional Inputs," The Ohio State University.

Rhodes, J.E., "Parametric Self-Excitation of a Belt into Transverse Vibration," Journal of Applied Mechanics, Transaction of the ASME, Vol 37, Series E, No. 4, Dec, 1970.

Yagishita, H., "Experimental Study on Variations of Rotational Speed and Flapping Vibrations in Belt Drives," ASME Paper No. ASME 76-DET-102.

16 ANALOG INSTRUMENTS

INTRODUCTION

The first dedicated electronic vibration measuring instruments for industry appeared on the market in the late 1930s. These instruments were analog: their output was proportional (ANALOGous) to the input. These analog meters measured the amount of vibration in mils or in inches per second and displayed this information on a panel meter.

Velocity transducers were used to measure vibration, since charge amplifier accelerometers were limited to laboratory use and eddy current displacement probes were unknown.

With the advent of World War II and the resulting impetus to electronic research and development, smaller and more capable vacuum tubes and circuit techniques were developed. This made possible battery-operated vibration meters with better signal-to-noise capability. Some of these units even used common mode rejection to further reduce the noise.

In the early 1950s, tunable filters, frequency meters, and strobe lights for phase reading were incorporated into the vibration meter to produce the analog vibration analyzer. Other than converting to transistors, which made battery operation practical, there has been little change in design over the past 40 years. A few years later, analog cross-correlation instruments, using analog multiplier circuits and a reference signal, were introduced. These units read phase on a meter and were used almost exclusively for field balancing. Although quite expensive, the analog vibration analyzer can be a powerful tool in the solution of machinery vibration problems.

1.0 BASICS OF ANALOG ANALYZERS

Analog analyzers read amount, frequency, and phase:

1.1 Amount

The vibration amount, usually called **amplitude**, is read on an analog panel meter. The magnitude may be measured as displacement in mils (Peak to Peak), velocity in inches per second (Peak), or acceleration in g's (Peak).

The vibration magnitude is a value calculated from the time domain average and may or may not be an accurate measure of the vibration energy, depending on the harmonic content and whether the instrument filter is being used. The amplitude accuracy may be as good as 2 or 3% and as bad as 25 or 30% of the full-scale reading from the panel meter.

Most of these instruments display the vibration amount on a linear meter scale and use a range switch with 3-to-1 or 5-to-1 scaling in order to cover a 60 or 70 dB dynamic range.

Some more recent units employ digital amount meters, which make it much more difficult to read the beat frequencies usually found in machinery analysis.

1.2 Frequency

Frequency is read on an analog or a digital meter in cycles per minute (CPM) or in Hz. Most units use dual scales so that the operator can read the frequency in both units.

The accuracy of the frequency readout varies between 3% and 5% of full-scale reading, and the instrument range is usually covered in several ranges using 10-to-1 scaling.

Most instruments read the number of zero crossings as the dominant vibration frequency. If there is a significant amount of harmonic content in the unfiltered vibration signal, the frequency meter appears to beat in a manner similar to that of the amount meter. In fact, the meter is moving between one frequency and another and gives the experienced operator an indication of two possible sources of vibration. Digital frequency meters may give more accurate readings but do not function well when the number of zero crossings is changing.

1.3 Phase

Phase is read either by a stroboscopic light fired by the vibration signal or on a phase meter using a reference (tach) signal.

A strobe light is most useful in vibration analysis when it is used in conjunction with a tunable filter that can also act as a variable frequency oscillator. The filter makes it possible to fire the strobe at discrete vibration peaks in the spectrum, which aids in machine diagnostics and phase measurement for field balancing.

When you use the filter as an oscillator, you can use the strobe light as a strobotach to determine the operating frequency of various components in the machine or to study their operating shape in slow motion.

The overall vibration magnitude is measured using the broadband or filter out setting on the analyzer. Since virtually all of these instruments are AC-coupled, they have a low frequency cutoff in the area of 1 to 3 Hz. And since there has been no standard set for the range of broadband vibration measurements, the high frequency rolloff of these units may be as low as 3KHz or as high as 10KHz.

As we mentioned earlier, the accuracy of the overall reading (as far as the actual vibration energy is concerned) depends on the harmonic content of the vibration signal.

Analog analyzers perform integration in the time domain, and the low frequency cutoff is very important when the signal is integrated. The lower the frequency response of the instrument, the more tribo (electrical noise introduced into a shielded cable when the cable is bent), electronic, and outside noise increase the indicated overall vibration level.

With a tunable filter you can "tune in" on one vibration component and read both the magnitude and the frequency of that component. Not only can you identify and study that component, but you can also read both the magnitude and frequency to the accuracy limit of the instrument.

Analog analyzers use a constant-percentage bandwidth filter, so the frequency resolution increases as the frequency to which the instrument is tuned decreases. For example, a 2% constant bandwidth filter would have a frequency resolution of .2 Hz when tuned to 10 Hz and a resolution of 20 Hz when tuned to 1,000 Hz. In most cases this is not a serious limitation, since it is preferable to have better frequency resolution at the lower frequencies when analyzing most machine vibration problems.

The filter response time essentially equals 1 over the bandwidth (the resolution). Therefore it takes much longer for the filter to respond at the lower frequencies than at the higher frequencies. In the case of the 2% constant bandwidth filter, it would take 5 seconds for the filter to reach the correct vibration magnitude at 10 Hz (since 1/.2 equals 5), and 1/20 of a second at 1,000 Hz. It is not uncommon for the inexperienced analyst to tune the filter much too fast at the lower frequencies and run the risk of missing an important low-frequency spectral component.

The tunable filter has a 180-degree shift from below to above the center frequency to which the filter is tuned. This is similar to that of the single degree of freedom system discussed in section 13 of the System Response chapter in Volume 1. The sharper the filter the more dramatic the phase shift, the more difficulty in tuning the filter, and the greater the phase shift with slight changes in the frequency of the vibration signal.

The better analyzers have a phase shift meter (often called a **tuning meter**) across the filter to help the operator precisely tune the instrument. The phase shift across the filter becomes extremely important when the analyzer is used for field balancing, since the filter phase shift becomes a phase error in the balance calculations.

2.0 STROBE LIGHTS

Almost all the analog vibration analyzers have an oscillate selection that utilizes positive feedback across the filter circuitry so that it functions as an oscillator rather than a filter. Use this selection to operate the unit as a strobotach. In this case the strobe can be used to "freeze" the shaft or rotor being balanced, which in turn tunes the filter so that there is no phase shift between balance runs.

The strobe light is a high-energy light source which is "fired" at a controlled rate and which emits a very short duration light pulse each time it is fired. The rate is controlled by the vibration signal or by the analyzer filter acting as an oscillator.

When the strobe is controlled by the vibration signal, you can use the light to

- identify components of the machine which are producing the vibration signal.
- study the relative phase relationships of machine components excited by the vibration.
- determine the major component when a beat is present.
- measure the relative phase during field balancing.

When the light is controlled by the oscillator, the strobe functions as a strobotach; use it to measure the frequency of various components in the machine or study their response in slow motion.

When the strobe is used as a tachometer, the short duration light pulse, coupled with the persistence of the eye, makes a moving object appear to stand still when the frequency of the light pulse is equal to the frequency of the target. When the firing frequency is slightly above or below the frequency of the target, the object appears to rotate slowly.

This is true not only when the frequencies are equal, but also when the frequency of the strobe light is any integer sub-multiple of the target frequency. Therefore it is not necessary for the strobe light to fire at the same frequency as the target in order to freeze it. Almost all vibration analyzers use this feature in order to have a bright light and still limit the power into the light source.

As the input frequency increases so that the power into the light is close to the maximum allowable, the firing of the light is "scaled" by 2, 4, 8, etc., so that the maximum power is never exceeded. This feature is barely noticeable when the light is used in machine analysis or in field balancing, but it can lead to serious error when the light is used as a tachometer.

The maximum frequency at which a strobe light fires can be as low as 40 Hz. Lights with lower power may fire at frequencies above 100 Hz but are difficult to use when the background light is high, for example, outdoors in daylight.

Since the operating frequencies of machine components run the gamut from very slow to very fast, the safest way to use a strobe light as a tachometer is as follows:

1. Use the lowest frequency range on the analyzer.
2. Starting at the highest frequency on that range, tune down in frequency until the first "frozen" image appears. Call this frequency *Fh*.
3. Tune on down, carefully, until the next "frozen" image appears. Call this frequency *Fl*.

The correct frequency of the target is

(Formula 16-1)

$$F = \frac{(Fh * Fl)}{(Fh - Fl)}$$

Almost all analog vibration analyzers have connector outputs of both the time domain signal and the strobe light firing pulses. Connect an oscilloscope to the analyzer in order to study the filtered or unfiltered time domain signal. Use the strobe pulses to trigger the oscilloscope for additional analysis or for remote phase indication.

In addition to these outputs, most analyzers provide analog DC outputs proportional to the magnitude of the vibration signal and to the frequency setting of the filter. These outputs can be connected to an X/Y plotter in order to obtain a hardcopy plot of the vibration spectra. Remember, the filter must be tuned slower at the low frequencies.

Analog analyzers are not very convenient for serious vibration analysis and have generally been replaced by the more capable and more convenient digital analyzers.

SUMMARY

Analog analyzers read amount, frequency, and phase.

The vibration amount, usually called **amplitude**, is read on an analog panel meter. The magnitude may be measured as displacement in mils (Peak to Peak), velocity in inches per second (Peak), or acceleration in g's (Peak).

Frequency is read on an analog or a digital meter in cycles per minute (CPM) or in Hz. Most units use dual scales so that the operator can read the frequency in both units.

Phase is read either by a stroboscopic light fired by the vibration signal or on a phase meter using a reference (tach) signal.

A strobe light is most useful in vibration analysis when it is used in conjunction with a tunable filter that also acts as a variable frequency oscillator. The filter makes it possible to fire the strobe at discrete vibration peaks in the spectrum, which aids in machine diagnostics and phase measurement for field balancing.

Analog analyzers perform integration in the time domain, and the low frequency cutoff is very important when the signal is integrated. The lower the frequency response of the instrument, the more tribo, electronic, and outside noise increase the indicated overall vibration level.

With a tunable filter you can "tune in" on one vibration component and read both the magnitude and the frequency of that component. Not only can you identify and study that component, but you can also read both the magnitude and frequency to the accuracy limit of the instrument.

Analog analyzers use a constant-percentage bandwidth filter, so the frequency resolution increases as the frequency to which the instrument is tuned decreases. For example, a 2% constant bandwidth filter would have a frequency resolution of .2 Hz when tuned to 10 Hz and a resolution of 20 Hz when tuned to 1,000 Hz. In most cases this is not a serious limitation, since it is preferable to have better frequency resolution at the lower frequencies when analyzing most machine vibration problems.

The filter response time essentially equals 1 over the bandwidth (the resolution). Therefore it takes much longer for the filter to respond at the lower frequencies than at the higher frequencies. In the case of the 2% constant bandwidth filter, it would take 5 seconds for the filter to reach the correct vibration magnitude at 10 Hz (since 1/.2 equals 5), and 1/20 of a second at 1,000 Hz. It is not uncommon for the inexperienced analyst to tune the filter much too fast at the lower frequencies and run the risk of missing an important low-frequency spectral component.

The tunable filter has a 180-degree shift from below to above the center frequency to which the filter is tuned. The sharper the filter the more dramatic the phase shift, the more difficulty in tuning the filter, and the greater the phase shift with slight changes in the frequency of the vibration signal. The better analyzers have a phase shift meter (often called a **tuning meter**) across the filter to help the operator precisely tune the instrument. The phase shift across the filter becomes extremely important when the analyzer is used for field balancing, since the filter phase shift becomes a phase error in the balance calculations.

Almost all the analog vibration analyzers have an oscillate selection that allow it to function as an oscillator, allowing you to operate the unit as a strobotach. The strobe light is a high-energy light source which is "fired" at a controlled rate and which emits a very short duration light pulse each time it is fired. The rate is controlled by the vibration signal or by the analyzer filter acting as an oscillator.

When the strobe is controlled by the vibration signal, you can use the light to

- identify components of the machine which are producing the vibration signal.
- study the relative phase relationships of machine components excited by the vibration.
- determine the major component when a beat is present.
- measure the relative phase during field balancing.

17 DIGITAL ANALYSIS

INTRODUCTION

This chapter introduces some of the functions of the digital analyzer that are important to the analyst.

1.0 DIGITAL ANALYZERS

The importance of vibration in the analysis of machinery faults, coupled with the advantages of digital analysis techniques, gave impetus to the development of highly sophisticated digital analyzers. This impetus was due in large part to the difficulty of using an analog analyzer to perform frequency analysis (separating the time signal into its individual frequency components).

1.1 Frequency Analysis with Analog Analyzers

If you wanted to perform a frequency analysis of the vibration present in a machine with an analog analyzer, you had to "tune" the analyzer across the desired frequency range. The analog filter was either hand tuned, or tuned steadily using a motor. The filter in these analyzers was usually a resonant circuit in the feedback loop around an amplifier or directly in the circuit path. The Q (sharpness) of these filter arrangements had to be kept low in order to make tuning practical.

The resonant filter design was a constant *percentage* bandwidth filter, rather than a constant bandwidth filter. For example, a 2% constant percentage bandwidth filter (the sharpest practical analog filter) has a resolution (bandwidth) of .2 Hz when tuned to 10 Hz, and a resolution of 20 Hz when tuned to 1,000 Hz.

To properly tune these filters, the frequency had to be varied slow enough to allow the circuit inertias to reach a steady-state value. The time it took to stabilize the circuit was nearly equal to one over the bandwidth. This means it took 5 seconds to obtain the correct amplitude of vibration at 10 Hz with a 2% constant percentage bandwidth filter.

It is not impossible to design a sharper filter; in fact, analog filters sharper than 2% are fairly simple to design. Sharper filters were not practical because they were too difficult to tune. Since the time required to stabilize the circuit equals one over the bandwidth, a sharper filter required a great deal of time to properly tune the filter. For example, a 0.1% constant-percentage bandwidth filter tuned to 10 Hz would require 100 seconds to obtain the correct amplitude. Small changes in the operating frequency of the system would occur during that time, making filter tuning impossible.

Producing a frequency domain display of the vibration over any reasonable frequency range required 2 to 3 minutes. Not only was this intolerable when you had to analyze many points, but you couldn't be sure that the low frequency components were really there at the same time as the high frequency components.

In addition, resolution was poor at the low frequency end, and even poorer at the high end. The argument was made that good resolution was not necessary at the higher frequencies, but this was before we knew about high-frequency vibration components like motor slot pass sidebands, gear mesh sidebands, and rolling element bearing fault sidebands. The lack of physical inertia and the advantages of real time frequency analysis were only two of the advantages of digital signal processing which led to the development of modern spectrum analyzers. Some of the others are

- stability
- linearity
- dynamic range
- accuracy
- real time
- constant bandwidth filtering
- no inertia
- transient data capture
- averaging
- mathematical manipulations

The first practical digital real-time analyzers appeared on the market in the late 1960s. The two most common techniques were to use a crude heterodyne frequency transformation called "time compression" or to use many individual analog filters in parallel. The heterodyne analyzer produced a linear frequency scale, whereas the parallel filter analyzer, usually employing 1/3 octave filters, produced a logarithmic frequency scale. The development of large-scale integration chips during the 1970s made it possible to produce practical real-time digital analyzers with digital conversion between the time and frequency domains. This digital conversion is known as the Fast Fourier Transform algorithm.

1.2 The Fourier Transform

Late in his life, Baron Jean Baptiste Fourier proposed that the complex waveform of the motion of a system in equilibrium could be represented as the sum of sinusoids (pure sine waves) and cosinusoids (pure cosine waves) of varying frequency and amplitude, separated by the repetition rate of their fundamental frequencies. Although many of his peers were vigorous in their opposition to his proposal, time has shown that Fourier was correct for a mass in equilibrium.

Vibration takes place in time. A vibration event is a discrete observation of what can be considered as an infinite stream of such events in history. This observation of the time history of the event is called a **time domain observation.** The Discrete Fourier transform is the mathematical means of determining the frequency composition of this time history of the vibration.

Moving from the time domain to the frequency domain is called the **Direct Fourier Transformation,** and moving back to the time domain is called the **Inverse Fourier Transformation.** Direct and Inverse Fourier transformations, although accurate, are very time-consuming. Therefore, a third type of Fourier transformation, called the **Discrete Fourier Transformation (or DFT),** was developed.

A DFT is a faster, more efficient Direct Fourier Transformation. That is, it does not make all of the normal calculations done in a direct transformation; it makes enough to give reliable accuracy. It was called "discrete" since it made a discrete number of calculations instead of the total necessary to make a direct Fourier transformation. With present-day computers, the Discrete Fourier Transformation can be accomplished rapidly and has come to be known as the **Fast Fourier Transformation (or FFT).**

1.3 Time and Frequency Domain Displays

The discrete time domain observation may be a single time history, an average of many single time histories, or a more complex computed time function such as the auto-correlation function. The auto-correlation function will be discussed later in this chapter.

In the simplest terms, the time domain display is a display of the time history of the amplitude variations of the vibration; that is, the amount of the vibration plotted against time. The frequency domain display is a display of the amplitude of the individual sinusoids which make up that time observation, plotted against frequency.

The vibration of a mechanical system is a time function, since the machine is vibrating in time. A vibration transducer is utilized to "transduce" the physical motion of the system into an electrical signal that is a faithful representation of the mechanical motion. The voltage output of the vibration transducer is called an analog signal since it is "analogous" to the actual motion of the vibrating mechanical system. In digital signal analysis the analog (time) signal is cut up into a number of individual values. This process is called **sampling,** and the rate at which the input signal is sampled is called the **sampling frequency** (or simply the "sample rate").

For example, suppose we have an input vibration signal at a frequency of 40 Hz. If we sample this signal 40 times each second as shown in Figure 105, each sample would have the same value, since the input signal is observed at the same point in each cycle. It would then be impossible to re-create the actual input signal.

Figure 105 - *sampling at the input frequency*

If we sampled the 40 Hz input signal twice as often (80 times each second), we would still not be able to rebuild the input, since the individual values would still occur at the same points on the input signal waveform. This is shown in Figure 106.

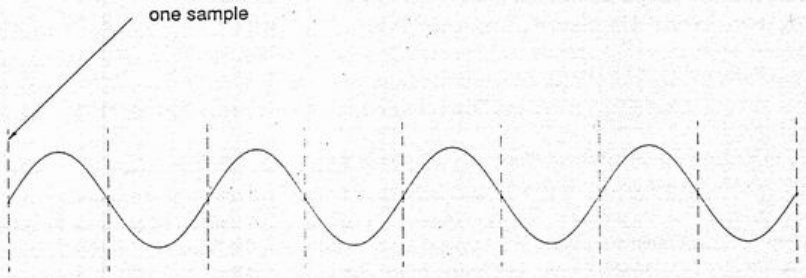


Figure 106 - *sampling at twice the input frequency*

Unless we sample the input signal at a rate higher than twice its frequency, it is not possible to confidently reconstruct it from the discrete values. This value of 2 * *input signal frequency* is sometimes called the **Nyquist frequency**.

In most modern digital analyzers, the sampling rate is usually chosen as 2.56 times the highest input signal frequency that is of interest to the analyst. For example, if the maximum frequency of interest (called the **baseband** or **MAX** frequency) was 400 Hz, the sampling rate would be 400 * 2.56 = 1,024 times each second.

1.4 Aliasing

One of the annoying side effects of digital sampling is **aliasing**. In Figure 107, an input frequency of 128 Hz is sampled at a sampling rate of 1,024 Hz. The input signal is sampled into 8 discrete values and can be reconstructed from these values. It may appear that Figure 107 shows 9 discrete values being obtained, but the ninth value is actually the beginning of the next cycle. Only 8 values are needed to reconstruct the wave.

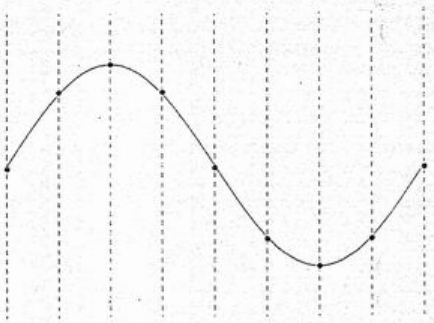


Figure 107 - *128 Hz signal sampled at 1,024 Hz*

Figure 108 illustrates an input signal of 1,152 Hz sampled at the same rate of 1,024 Hz. The 1,152 Hz signal is still sampled into the same 8 discrete values. However, the reconstructed signal is *not* 1,152 Hz; it is 128 Hz.

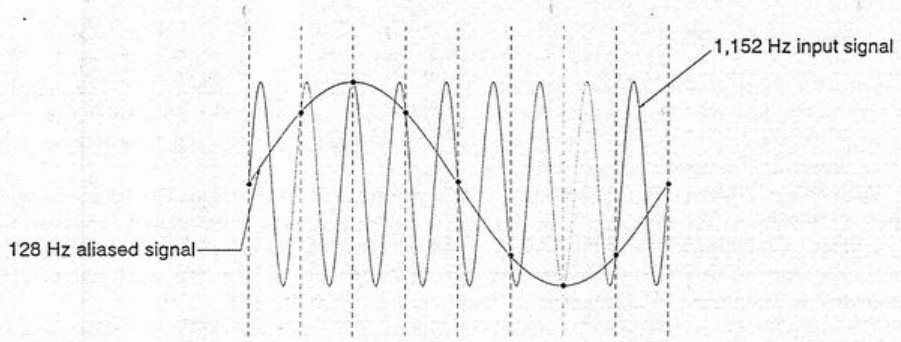


Figure 108 - *1,152 Hz signal sampled at 1,024 Hz*

The 128 Hz signal is not in the input signal, but is a "folded-in signal" that is a result of the relationship between the input signal frequency and the sampling rate (1,152 - 1,024 = 128). When vibration analysts observe aliased data in the frequency domain, they may spend time and effort trying to identify the source of a vibration that is not present in the machine.

When the input frequency is above the sampling frequency, you will detect an aliased frequency that is equal to the input frequency minus the sampling frequency. Likewise, you will detect aliased frequencies equal to the sampling frequency minus the input frequency when the sampling frequency is greater than the input frequency. In the example just cited, the input frequency of 1,152 minus the sampling frequency of 1,024 equals the 128 Hz aliased signal.

AN EFFECT OF ALIASING

Most of us have seen the effect of aliasing in western movies when spoked wagon wheels seem to reverse direction as the wagon slows down or speeds up. The picture on the screen is a reconstructed time signal from the "digital" data taken by the movie camera at a sampling frequency of 24 frames (discrete values) per second.

When the rotational frequency of the wheel equals the camera's frame rate (or equals an integer multiple of the frame rate), then the spokes appear to be in the same place from frame to frame. Since a spoked wagon wheel is a symmetrical object, it is impossible to tell one orientation of the wheel from another when the spokes all lie in the same place on the movie frame. When this occurs, the wagon wheel appears to be standing still while the wagon moves forward.

As the rotational frequency of the wheel changes, the position of the wheel's spokes changes from frame to frame. When the rotational frequency is below the frame rate, the wheel appears to rotate forward normally. However, if the rotational frequency of the wheel rises above the frame rate, the wheel appears to be moving backward. This apparent motion of the wheel is not real motion. The aliasing that results from the camera's sampling rate and the wheel's rotational frequency causes the appearance of backward motion.

To avoid aliasing, sample at a very high rate, or pass the input signal through a low-pass filter to remove all frequencies above the required baseband frequency. Most digital analyzers use sophisticated and expensive low-pass filters, called anti-aliasing filters, to remove all input frequencies above 40% of the sampling rate when sampling at a rate equal to 2.56 times the highest frequency of interest.

2.0 SIGNAL ANALYSIS

Once the input signal has been digitized, you can reconstruct and display the time signal or transform the data into the frequency domain and display it. For more detailed analysis, you can average the data in the frequency domain or in the time domain. The averaging process is discussed in the Linear Averaging section later in this chapter.

The frequency domain display is usually called a **spectra**, and the transformation process is called **spectral analysis**. The number of samples taken in the time domain determines the resolution, sometimes called **bandwidth**, in the frequency domain. The resolution is the smallest difference in frequency that can be detected between two closely spaced components. In most analyzers, the bandwidth is determined by the number of "lines" chosen by the operator.

One **line**, sometimes called a **cell** or **bin**, is one data point in the frequency domain. The resolution is equal to the baseband frequency divided by the number of lines. For example, choosing a 1,000 Hz baseband and 400 lines results in a maximum resolution of 1,000/400 = 2.5 Hz. As a result of the transformation, however, the energy is "smeared" over about 3 lines so that the actual resolution is somewhat less than this.

The finer the resolution (that is, the narrower the bandwidth), the more samples are required to make the necessary calculations. The number of samples required equals 2.56 times the number of lines. For a resolution equal to 400 lines, you must take 400 * 2.56 or 1,024 samples.

The time required to take this number of samples is called the **time window** and is equal to the number of samples required divided by the sample rate. A baseband frequency of 400 Hz sets the sampling frequency at 1,024 Hz, and a resolution of 400 lines requires 1,024 samples, so the sample time (the time window) is 1,024/1,024 = 1 second.

The price paid for resolution is time. Table 5 lists increasing number of lines for a constant baseband frequency showing the resolution (bandwidth), the sample rate, the number of samples required, and the time window.

NUMBER OF LINES	BASEBAND (Hz)	BANDWIDTH (Hz)	SAMPLE RATE	NUMBER OF SAMPLES	SAMPLE TIME (sec.)
100	1000	10.0	2560	256	0.1
200	1000	5.0	2560	512	0.2
400	1000	2.50	2560	1024	0.4
800	1000	1.25	2560	2048	0.8
1600	1000	0.63	2560	4096	1.6
3200	1000	0.31	2560	8192	3.2
6400	1000	0.16	2560	16384	6.4
12800	1000	0.08	2560	32768	12.8

Table 5 - *the effect of resolution on the time window*

Higher resolutions are obtained either by "zooming" in on a segment of the baseband frequency, or by displaying all of the lines and allowing the operator to expand the desired portion of the baseband.

THE PRICE IS TIME

As shown in Table 5, 12.8 seconds may not seem too long a time to obtain 12,800 lines of data. But bear in mind that it is 128 times longer than the time required to obtain a reading at 100 lines. Suppose you are taking 8 averages at 12,800 lines. This will require 102.4 seconds (nearly 2 minutes) to obtain your data. Two minutes can be too long when a system is vibrating so severely that it could be damaged. Some systems have safeguards against prolonged operation at high vibration, and may not even run for 2 minutes before cutting out.

The authors were called by a vibration analyst who complained that his data collection system was faulty: apparently, the analyst's equipment would not collect data at all. Upon inspection, the authors discovered that the analyst's system was set for 4,000 lines at a baseband frequency of 10 Hz. At these settings the resolution was 10/4,000, or .0025 Hz.

The time required to collect 4,000 lines of data at this resolution equals 1/.0025, or 400 seconds for each average. The analyst wanted to take 64 averages, which meant that the entire data collection for one run would take 64 * 400, or slightly more than 7 hours!

2.1 Line Spectra

The transformation from the time domain to the frequency domain results in two basic frequency domain spectra. The most common of these is called a "line" spectra, where the vibration energy is at definite frequencies that transform into discrete frequencies (lines) in the spectra.

It is natural to assume that since the vibration is at only one frequency there should be only a single "line" in the spectra. In fact, the result of the Fourier transformation fans out the energy into cells or lines on each side of the single frequency. Depending on the time window used, the data can cover from 3 to 5 lines.

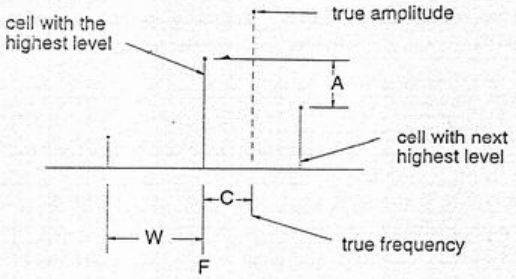
Most digital analyzers incorporate a "locate" routine similar to that shown in Figure 109 to calculate the actual amount and frequency of the signal. The lines on each side of the line with the highest amplitude are used to calculate the actual amplitude and frequency values.

An example of the line spectra is a fan vibrating due to an out-of-balance condition. In this case, it is logical to calibrate the amount of the vibration energy in engineering units, such as displacement in mils, velocity in inches per second, or acceleration in g's.

ACCURACY OF DISPLAYED VALUES

Almost all of the early analog analyzers determined the average value of the vibration signal from the transducer and calibrated it as a peak or peak-to-peak value. This was quite accurate as long as the vibration signal was a pure sine wave. As the harmonic content in the signal increased, the accuracy of the indicated amount of vibration present decreased.

Digital analyzers are inherently RMS devices, so that the amount of vibration is displayed in units that are proportional to the energy in the vibration signal in the time window. In cases where the harmonic content is high, there can be significant differences in the overall amplitude readings of the analog vs. the digital analyzer.



F = frequency of cell with highest level
A = dB difference between F and next highest cell
W = cell width

$$C = W * \frac{2 - 10^A}{20} / \frac{1 + 10^A}{20}$$

True frequency = $F \pm C$
plus if next highest level is above F
minus if next highest level is below F

Simple approximation: $C = W * \frac{0.5 - A}{12}$

Figure 109 - *locating a precise frequency*

2.2 Continuous Spectra

The spectrum that results from an impact is called a "continuous" spectra since energy is distributed across the entire baseband. There are an infinite number of frequencies across the band; the smallest energy at each frequency would still result in infinite power. Since infinite power is impossible, it is impossible to plot the energy for any given frequency in a continuous spectrum.

This makes it necessary to measure the energy in a band of frequencies rather than at one particular frequency. The unit of measure for the continuous spectrum is power spectral density, called **PSD**. The power spectral density is determined by squaring the RMS voltage in a frequency band and dividing by the center frequency of the band: $\frac{V^2}{Hz}$.

Almost all spectra are a combination of the line spectra and continuous spectra because of the discrete frequencies from machine components and the random background noise. It is good practice to examine the "skirts" (wide areas at the bottom of a frequency line) on the line components as well as the PSD of the continuous spectra. Wide skirts may be caused when

- two or more frequencies lie very close to each other in the spectra
- one of the frequencies is changing
- the amplitude of the frequency component is beating up or down
- the vibration data is mathematically manipulated

2.3 Time vs. Frequency Displays

One of the insidious byproducts of the older analog analyzers was the almost total neglect of the time domain while concentrating on the frequency domain. This is unfortunate, since the time domain is part of the digital analyzer, and you can obtain a great deal of information about a system by examining the time domain. Consider the following cases while remembering Fourier's basic assumption: the vibration waveform, no matter how complex, is generated by the sum of a series of sine and cosine waves at different frequencies.

2.3.1 Case 1 - First-Order Imbalance

A perfect high-speed spindle with a slight imbalance is generating only a single sine wave at a frequency equal to the operating speed of the motor. The time domain waveform is shown in Figure 110 and the frequency domain display in Figure 111.

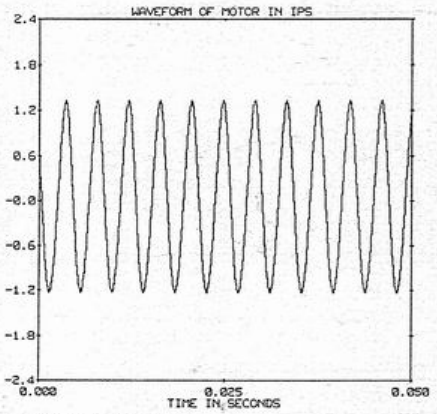


Figure 110 - *time domain waveform for Case 1*

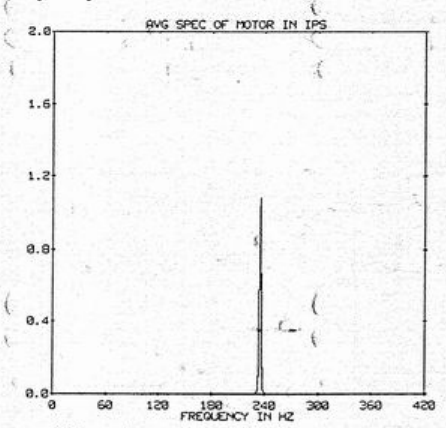


Figure 111 - *frequency spectra of Case 1*

The vibration signal is a pure sine wave, and all of the energy is periodic and at one frequency. The time and frequency domain displays are exactly what you would expect, and the frequency domain display is the preferred display.

The base of the frequency line in the frequency display is wider than just one single frequency; this is a result of the smearing mentioned earlier.

2.3.2 Case 2 - Restriction of Motion

The spindle described in Case 1 is mounted so that it can move easier to the right than to the left. The restriction to motion in one direction clips the waveform in that direction so that it is no longer a pure sine wave at a single frequency.

The result of this distortion, called **harmonic distortion**, adds a series of sine waves that are all exact multiples of the repetition rate of the signal, which in this case is the operating frequency of the spindle. Figures 112 and 113 show the time domain waveform and the frequency domain display.

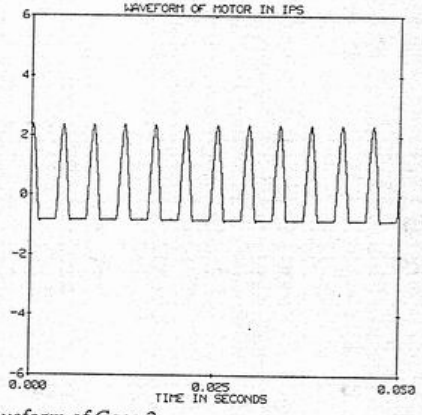


Figure 112 - *time domain waveform of Case 2*

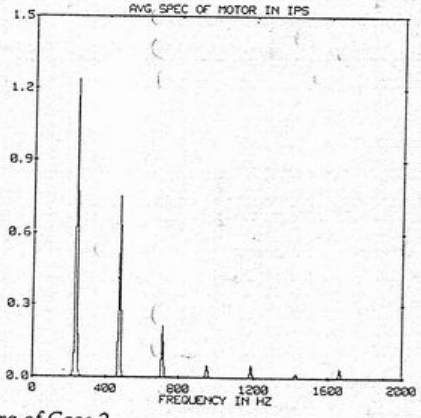


Figure 113 - *frequency spectra of Case 2*

To Fourier's credit, we see that the transformation from the time domain to the frequency domain reduced the complex time waveform to its individual sinusoids. In this case, however, what is immediately obvious in the time domain (the clipping of the waveform due to the restriction of motion) is not clear, even after detailed examination and analysis, in the frequency domain. Here the time domain display provides more useful information than the frequency display.

2.3.3 Case 3 - A Single Pulse

The harmonic content of a complex wave form becomes even more confusing when the repetition rate is reduced to zero. Consider a single pulse. The time domain display is shown in Figure 114.

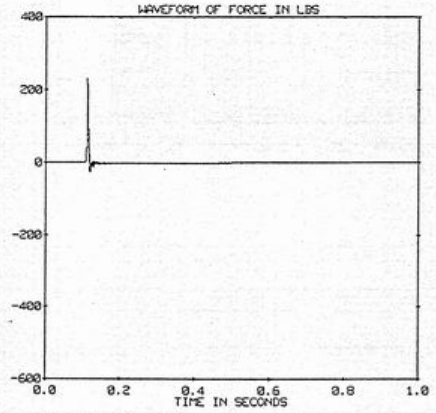


Figure 114 - *time domain waveform of Case 3*

Remember Fourier's basic premise: the complex waveform is the sum of harmonics of the repetition rate, which in this case is zero. Harmonics occur at all frequencies, and there is only one point at which they are all in phase to add together to produce the pulse shown in Figure 114.

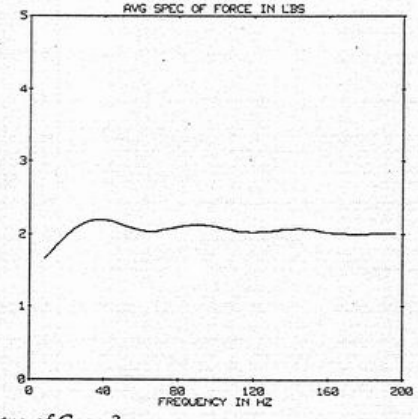


Figure 115 - *frequency spectra of Case 3*

The resulting frequency domain display, shown in Figure 115, does contain energy at all frequencies. Again, the time domain display provides more useful information.

2.3.4 Case 4 - Many Non-Periodic Pulses

Case 4 shows a vibration signal containing many non-periodic pulses. The complex waveform is shown in Figure 116, and the frequency domain display is shown in Figure 117.

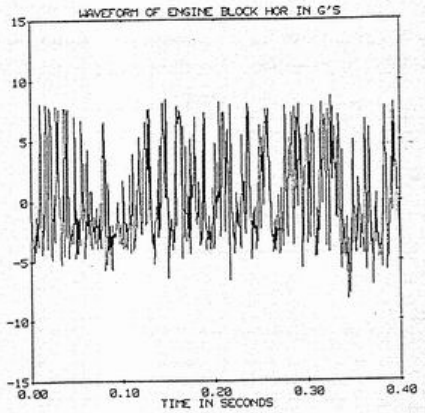


Figure 116 - *time domain waveform of Case 4*

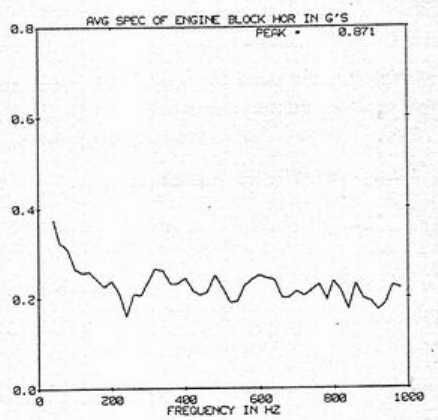


Figure 117 - *frequency spectra of Case 4*

The frequency spectra of a single pulse, many non-periodic pulses, or even random background noise are all continuous spectra (they contain energy at all frequencies). As shown in Figures 115 and 117, they all appear to represent essentially the same vibration problem.

Although the detailed analysis of these frequency domain spectra may never lead to the correct solution to the problem, a simple glance at the time domain display should be sufficient.

The time domain display is a powerful tool for the analyst in the determination of

- low frequency events
- random vs. periodic events
- simple or complex vibration
- impacting
- asymmetries
- distortion
- truncation, restriction to motion
- low frequency modulation
- beats
- electrical vs. mechanical signals
- slow changes in the signal, such as its amplitude

As we mentioned at the beginning of this chapter, time is generally equal to one over the bandwidth. In the digital analyzer, time is the length of the time window, and the bandwidth is the resolution chosen by the operator.

Since

$$T = \frac{1}{B}$$

where

T = time, the length of the time window
B = bandwidth (resolution)

it can be shown algebraically that

$$B = \frac{1}{T}$$

That is, the maximum resolution in Hz equals one over the sample time.

In our previous example of 400 Hz baseband and a time window of 1 second, the bandwidth equals 1 Hz. Given these values of B and T, it would only be possible to separate frequencies that are one cycle apart in the time window. If you needed to measure frequencies that were ½ Hz apart, you would need to increase the time window to 2 seconds.

2.4 Leakage

There is practically no possibility that the input signal contains sinusoids that all started and stopped exactly at the beginning and the end of the time window. There would be abrupt changes in signal level at the ends of the window that are not actually present in the input signal.

These sudden changes in signal level would transform into the frequency domain as energy in the first cell and higher frequency harmonics, which are usually called **leakage components**.

The usual solution to leakage is to multiply the input signal by a windowing function (this is done in the time domain). The most common window is called a **Hanning window**, sometimes called a **cosine window**. The Hanning window reduces the input signal to zero at each end of the window so that no leakage components appear in the spectra. The Hanning window also greatly reduces smearing in adjacent lines.

There are a variety of window functions provided in most digital analyzers; refer to advanced texts for a more detailed discussion. The following windows are used most often in machinery analysis:

1. **Uniform (or flat)** - The input signal is not multiplied by any function.

2. **Hanning** - A cosine function reduces the signal to zero at the beginning and the end of the window.

3. **Exponential** - The input signal is reduced to a level close to zero at the end of the window, using an exponential decay over the length of the window.

4. **Force-Exponential** - A combination of windows used for impact studies to minimize leakage from the impact and response.

3.0 INSTANTANEOUS SPECTRA

The transformation of a discrete time sample to the frequency domain is called an **instantaneous spectra**. Although it takes a finite time to collect the data (the length of the time window), all of the frequencies that appear in the spectra were there in the time window; this is why the digital analyzer is called a "real-time" analyzer.

A low-resolution, high-baseband instantaneous spectra allows you to quickly observe amplitude fluctuations and energy distributions of the vibration signal that may require more detailed analysis.

As an example, a 100 line, 1,000 Hz baseband instantaneous spectra requires only a tenth of a second of time data. That is, ten such spectra are presented each second. You can very quickly determine where to concentrate your data collection and analysis.

The analog signal from the vibration transducer is "flowing" into the analyzer, being digitized, flowing through the input buffer, and then disappearing as new data comes in. This process results in a stream of digitized time data flowing through the analyzer.

As pointed out in the previous discussions, the analog signal is digitized at the sample rate, and the number of discrete samples is determined by the resolution. The time it takes to obtain this number of samples is called the time window.

Once the time data has been digitized, it is transformed to the frequency domain. The time required to make the Fourier calculations is called the processing time. Most analyzers use parallel processing, where a new time sample is taken while the analyzer is processing the previous time sample.

The real-time rate is defined as the baseband frequency that results in the time window equaling the processing time. At all baseband frequencies below this value, the analog signal is digitized in contiguous blocks since the data can be processed in less time than the time window.

Your abilities are much more important than the real-time rate of the analyzer. Very sophisticated work can be accomplished with analyzers having low real-time rates. As a general rule, the real-time rate is not important when analyzing steady state signals, whereas a high real-time rate is desirable when analyzing signals that are changing rapidly, such as in a machine coastdown.

4.0 TRIGGERING

In the untriggered condition, the analyzer is processing data as fast as it can. If the baseband frequency has been chosen below the real-time rate, the data is sampled and processed in a continuous unbroken stream. If the baseband frequency is above the real-time rate, there are gaps in the digitized data equal to the difference between the processing time and the window time.

In either case, the analyzer is sampling the signal as fast as possible. The start of each time window is determined by the analyzer real-time rate, the selected base band frequency, and the chosen resolution.

Since the data is flowing through the analyzer, it is possible to "trigger" the start of the time window. The trigger event can be internal to the analyzer (such as a selected level of the input signal), or some external event such as a tach pulse from the machine, a valve closure, or a switch closure.

In most cases, it is an advantage to take data that has a common starting point, such as the trigger event. The trigger can be delayed with respect to the time window so that the trigger occurs before or during the time window. Trigger delay makes it possible to examine data that occurs before or after the trigger event.

5.0 AVERAGING

One of the most powerful advantages of the digital analyzer is the averaging process.

5.1 Linear Averaging

The most common averaging is linear averaging, also called summation averaging, ensemble averaging, or normal averaging. In this case data is taken in the time domain, transformed to the frequency domain and averaged in the normal fashion (added together and divided by the number of samples).

In some analyzers the data can be averaged "up" (plus) and then "down" (minus). *Averaging up* is simply the normal method of adding the values together and dividing by the number of samples. *Averaging down* involves subtracting the last value and dividing by the number of samples minus one. The process is repeated, subtracting the last value and dividing by the number of remaining values.

If data is averaged up while the machine is running and then averaged down when the machine is shut off, it is possible to average out the background. This can be a very useful technique when several machines of the same type and speed are mounted on a common platform.

Both triggered and untriggered sampling can be used in the averaging process. Linear averaging reduces the signals, noise, and the background to their average value.

5.2 Exponential Averaging

Exponential averaging is quite similar to linear averaging, except that the most recent data is weighted more heavily than the older data. Once again the signals, the background, and the noise are all reduced to their average values, but in this case the most recent data has the greatest effect on the values.

This technique can be advantageous when averaging data over long periods of time, such as during the startup of a machine, where the data at operating temperature is more important than earlier data.

5.3 Peak Hold

Although peak hold is not an averaging process, it is always included in a discussion of the averaging capabilities of the digital analyzer. In peak hold, the analyzer holds the highest value that occurs in each line during the averaging time. The data is transformed to the frequency domain and compared with the values already stored in each cell. If the new value is larger than the stored value, the stored value is replaced with the new value.

Peak hold is used when the analyst is interested in the highest frequency domain values encountered over a period of time. This is especially useful during coastdowns or startups.

You can also compare the peak hold values with the linear averaged values to detect the presence of a beat in the data. The difference between the average value and the peak hold value is equal to one-half the amplitude of the beat.

In peak hold, the noise and the other background signals are all held at their peak values.

5.4 Synchronous Averaging

All of the averaging techniques just discussed can be triggered or untriggered. Triggered averaging is called **synchronous averaging**. In this case, the time window start is controlled by the trigger event so that data which is synchronous with the trigger frequency occurs at the same place in each time window.

This technique ensures that the data of interest is in the time window and makes it possible to display the phase relationship between the trigger event and the vibration signals harmonically related to the trigger event.

With triggered averaging it is possible to display the frequency domain spectra in orders of the tach pulse, rather than in Hz or in CPM. This is usually called **orders tracking**. In this case the signals phase-locked to the tach pulse do not change location on the display, even though their frequencies may change.

5.5 Synchronous Time Averaging

In all of the cases just discussed, the sample rate is set by the analyzer at a frequency equal to 2.56 times the chosen base band frequency, and the number of samples required is set by the chosen resolution (the number of lines times 2.56).

The averaging is done in the frequency domain, and both the noise and the signals average to their average values. It is not uncommon to have signals whose frequency changes slightly from one time sample to the next, which results in an additional smearing of the skirts. It is important to determine whether wide skirts are the result of such frequency jitter or whether there is more than one frequency very close together.

In synchronous time averaging, the triggered averaging is done in the time domain rather than the frequency domain, so that all non-synchronous components average to zero. The number of samples required is still set by the chosen resolution, but the sample rate is set equal to 2.56 times the trigger frequency times the number of orders you wish to see.

For example, a tach pulse from an 1,800 RPM motor would be at 30 Hz. To look at 10 orders of motor vibration, use a sample rate of 30 x 2.56 x 10 = 768 Hz. In this case, all of the components not phase-locked to the tach pulse would average to zero. In addition, since the sample rate is tied to the tach pulse, small changes in the frequency of a component would not result in a smearing of the skirts.

Synchronous time averaging is a very powerful tool used to separate low-level signals which are phase-locked to the tach pulse from much higher amplitude nonsynchronous components, since the nonsynchronous data sums to zero.

Using synchronous time averaging from reference pulses taken from various components in a machine, you can determine the vibration energies that are tied to each of the machine components. In addition, you can read the phase between the reference pulse and orders of vibration phase-locked to the tach pulse. Remember that all of the components that are not phase-locked to the tach pulse will sum to zero. This means that significant data, such as that from a defective rolling element bearing, will also sum to zero.

Synchronous time averaging must be used in conjunction with the other capabilities of the analyzer to perform a complete analysis.

6.0 CROSS-CHANNEL CAPABILITIES

More powerful analysis techniques are possible with multi-channel analyzers. A single channel instrument is sufficient for 70% of general machinery analysis problems, but you need a multi-channel analyzer to solve the other 30% and to do advanced machinery analysis or modal analysis.

We are not going to provide you with a rigorous mathematical basis for any of the cross-channel functions, nor do we discuss all of the possible functions and their usage. However, we need to define certain terms in order to introduce some of the most common functions:

- **Auto-Correlation Function** - The extent to which a signal correlates with a version of itself displaced in time. This was used in older analyzers to detect periodic components in the signal that are buried in noise. With modern analyzers, the power spectrum is a better way to detect these components.
- **Cross-Correlation Function** - The extent to which one signal correlates to another signal displaced in time. Use the cross-correlation function to detect the presence of a signal buried in noise as well as to determine the correlation of one signal to another.
- **Cross-Spectrum Function** - The forward Fourier transformation of the cross-correlation function. Its primary use is in the determination of the transfer function.

The following cross-channel functions are the most useful to the industrial vibration analyst. These functions all require averaging, and confidence in the readings increases with the number of averages. Depending on the quality of the signals, 8 to 32 averages should be taken for each reading.

6.1 Coherence

In the simplest terms, coherence is a measure of the linear relationship between two signals. Since this measurement is based on the multiplication of individual frequency input values by the same frequency output values, the coherence at the first average is always 1, or 100%. As the number of averages increases, the coherence function approaches zero because the contribution of the non-linear components increases.

Thus, in frequency ranges where the coherence function approaches 1 (or 100%), the impedance of the mechanical system can be assumed to be linear. This is extremely important when the transfer function contributes a great deal to the system's "character" (for example, mechanical impedance and damping).

The actual coherence value is usually somewhat dependent on the number of averages, so that a value of 60% at 32 averages might be 95% at 4 averages. The transfer function is valid for coherence values as shown in Table 6.

NUMBER OF AVERAGES	COHERENCE
4	.90 - .99
8	.80 - up
16	.70 - up
32	.60 - up

Table 6 - *coherence as a function of the number of averages*

If two signal components A and B are coherent, then you know at least one of the following is true:

- The mechanical system between A and B is linear.
- A is the cause of B.
- B is the cause of A.
- Both A and B are caused by a yet unknown source.

One common error is to assume that if two signals are coherent, the smaller amplitude signal is caused by the larger amplitude signal. This assumption is based on the hope that there are no resonances in the system and that there is not a source C which is the cause of both A and B.

With the coherence function, you can:

- Determine the frequency ranges over which the transfer function is valid
- Detect the presence of noise and system non-linearities
- Verify the vibration transmission path between measuring locations

Examples of system non-linearities include excessive clearances, looseness, cracks, non-linear spring constants, and non-linear damping. Figure 118 is a coherence plot taken on a machine spindle that had a crack in the housing.

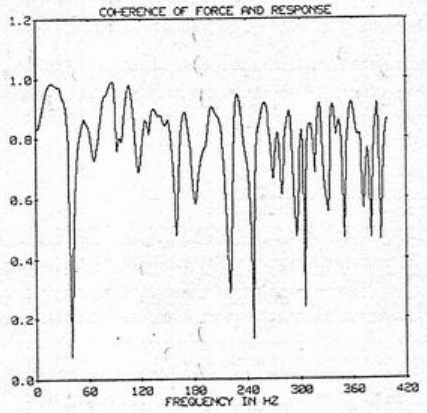


Figure 118 - *coherence plot of cracked machine spindle*

Figure 119 is a coherence plot taken at the same location on a new replacement spindle.

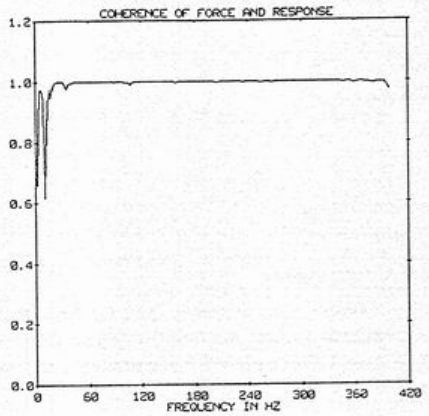


Figure 119 - *coherence plot of a new machine spindle*

The crack in the first spindle made the spindle housing become a non-linear mechanical system (which it should not be). The coherence function indicated that something was not normal in the system. Further inspection located the crack, and the spindle was replaced.

The coherence function can also be used to check the transmission path between an inaccessible measuring point and an accessible one. For example, suppose you need to measure vibration on a machine with a very hot bearing. Mounting a transducer on the hot bearing may be impossible, or at least may damage the transducer.

By briefly mounting a transducer on the hot bearing and checking other points on the machine where coherence is high, you can find other measurement points that are cooler and more accessible. The high coherence at these points means that there is a good transmission path from this cooler point to the hot point on the bearing. Vibration measured here (at the cooler point) is a faithful representation of the vibration at the hot point.

Finally, the coherence function can be used to optimize the number of measurement points. For example, if one measurement point on a machine has good coherence to three other points, then there may be no need to measure vibration at those other points. Bear in mind that coherence establishes the validity of the transmission path; it does not define the relative amplitudes between the two points. To define the relative amplitudes between the two points, you use the transfer function.

6.2 Transfer Function

In the frequency ranges where the coherence approaches 1 (or 100%) the **transfer function** is a frequency domain description of how the mechanical system operates on one signal to produce the other. The transfer function is the ratio of the cross-spectrum density to the input spectrum density and makes it possible to predict the system response to a known input.

If there is good coherence between points A and B, then the transfer function is the ratio of the two signals. To measure vibration at point B, you first measure vibration at point A. Then, divide the reading at A by the transfer function. The result is the vibration at B.

As discussed in earlier chapters, there are six transfer functions:

1. Inertance - acceleration/force
2. Mobility - velocity/force
3. Compliance - displacement/force

and their reciprocals:

4. Apparent mass - force/acceleration
5. Impedance - force/velocity
6. Apparent stiffness - force/displacement

The first three display resonant frequencies as peaks, and the last three display antiresonances (frequencies where the system is stiff) as peaks. Knowing any one of the six allows us to calculate any of the others. For example, Figure 120 is a plot of the inertance measured by impacting a turbine generator.

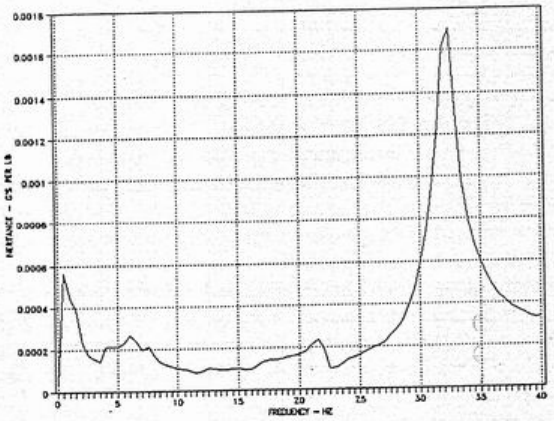


Figure 120 - *inertance plot of turbine generator*

Figure 121 is a plot of the apparent stiffness, calculated from the inertance.

Since the turbine generator ran at 1,800 CPM (30 Hz), the plot in Figure 120 shows that the inertance was very high at 30 Hz. This enabled the analyst to determine the cause of recurrent high vibration and balance problems with the unit. Figure 121 supports the analyst's conclusions by showing that the stiffness of the turbine generator is very low at 30 Hz.

There are two ratios of output to input: one is composed of the components which are in phase, called **real** or **coincident**, and the other is composed of the 90 degree components, called **imaginary** or **quadrature**.

The transfer function can be displayed on a PC monitor, or plotted as an amplitude:

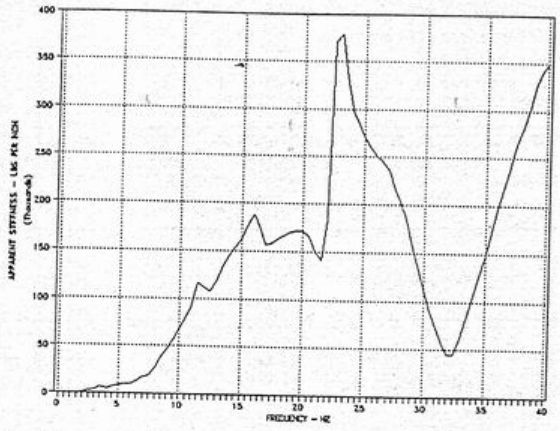


Figure 121 - *apparent stiffness calculated from inertance*

- the ratio of output to input vs. frequency
- the phase lag between output and input vs. the frequency

Either of these amplitude plots result in a polar presentation of the data. If the real and imaginary components are plotted against each other, the result is a series of circles, one for each resonance.

Transfer function data can also be presented in rectangular format as:

- the magnitude of the transfer function versus frequency
- the real components versus frequency
- the imaginary components versus frequency

For the six transfer functions, resonances are displayed as peaks, valleys, or zeros on the rectangular display, depending on which function is plotted:

FUNCTION	MAGNITUDE	REAL	IMAGINARY
Inertance	Peak	Zero	Peak
Mobility	Peak	Peak	Zero
Compliance	Peak	Zero	Peak
Apparent mass	Valley	Zero	Valley
Impedance	Valley	Valley	Zero
Apparent stiffness	Valley	Zero	Valley

Table 7 - *how resonances are displayed in rectangular transfer function plots*

6.2.1 Response to a Known Force

One of the main uses of the transfer function is to identify the response of a system to a known input. Figure 122 is a plot of the imaginary portion of the inertance shown in Figure 120. The

resonance appears as an inverted peak, which indicates that the direction of motion was away from the accelerometer used to measure the response.

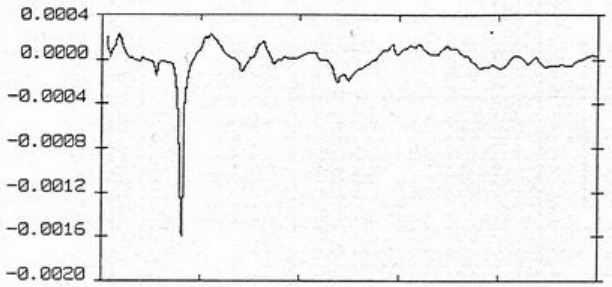


Figure 122 - *imaginary portion of Figure 120*

In this case the transducer was mounted at the 9 o'clock position on the outboard bearing of the turbine, and the generator was impacted at the 9 o'clock position on its outboard bearing using an instrumented force hammer.

The distance between the two locations was 18.5 feet and, contrary to what some would consider normal, the unit did not pivot as a stiff system at a point between the bearings. Rather, the outboard end of the turbine moved in the same direction as the outboard end of the generator. A few more impacts at several other locations on the turbine generator were all that was required to identify the mode shape of the system at the resonant frequency of 32.5 Hz.

Figure 123 is a sketch of the system showing the other impact locations, and Figure 124 is a plot of the mode shape at 32.5 Hz, showing that the unit was flexing near the middle. Mode shape will be discussed more fully in the chapter on modal analysis.

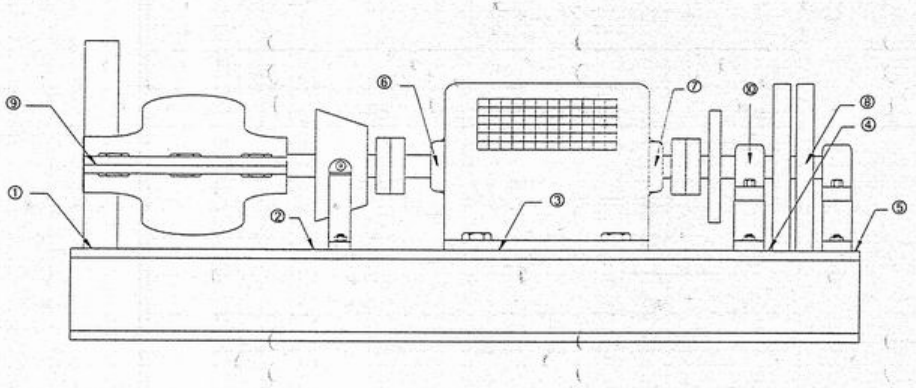


Figure 123 - *impact locations on turbine generator*

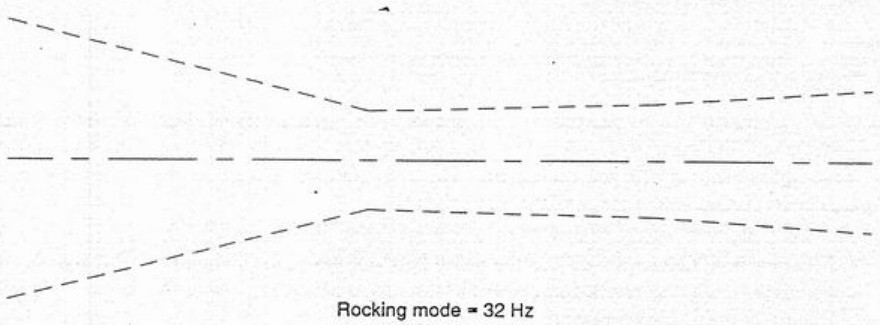


Figure 124 - *rocking mode of turbine generator*

6.2.2 Energy Transfer through the System

Where the coherence is high, the transfer function is an indication of the energy transfer through the system. For example, the effectiveness of vibration isolators can be evaluated by measuring the transfer function across the isolator. Be careful when using this method to evaluate the isolators if there is more than one path of energy transfer: the other transmission path could be interpreted as a shorted isolator.

In order to determine whether or not there is a good impedance between the machine and the foundation, measure the transfer function across the mounting feet of machinery.

6.2.3 Determining Amount and Location of Imbalance

Since the transfer function has magnitude and phase, you can use it to determine the amount and location of imbalance in a rotor. The system phase lags, discussed in earlier chapters, can be determined directly from the transfer function.

If the amplitude of vibration is mils peak-to-peak and the true phase of the displacement is measured, you can impact the system in order to obtain the inertance. The amount of the imbalance is given by the equation:

$$\text{Amount} = \text{Compliance} * \text{Amplitude}$$

or

$$U = .008 * \text{mils peak-peak} / \text{Inertance (g's/lb)}$$
$$= \text{imbalance in oz-in}$$

and the location of the imbalance is given by

$$P = K - B - A$$

where

P = Angle of heavy spot from the reference mark (in direction of rotation)
K = System phase lag
B = True phase of vibration
A = Angle between reference signal and vibration transducer

6.3 Cross Channel Phase

It is also possible to measure the phase at any frequency between the signals input to a multi-channel digital analyzer. This is a true measure of the time delay (phase) between any two of the signals; there is no added phase shift due to the instrument. The measured phase is somewhat dependent on the harmonic content, that is, the shape of the input signals.

Some useful applications of cross-channel phase are listed below:

1. Measure both the amplitude and the phase between the horizontal and vertical signals on a machine bearing to check for a possible resonance condition. Note that no tach signal is required for this measurement.
2. Measure the horizontal-to-horizontal, the vertical-to-vertical, and the axial phase across a coupling to check for shaft alignment without using a reference signal.
3. Measure the horizontal-to-horizontal and vertical-to-vertical phase relationships on a machine to check for unbalance, bent shaft, resonance, or alignment. Again, no reference signal is required.
4. Use one transducer at a fixed location on the machine as the reference, and the other to take amplitude and phase measurements at various locations on the machine. In this case, the measurements can be taken not only without a tach signal but on sealed equipment where the components cannot be seen. In addition, the phase can be measured for any frequency, not just that of a tach reference.
5. Use one transducer as the fixed transducer and the other as the roving transducer to take operating deflection shape data. See the chapter on modal analysis for more information.

SUMMARY

With analog analyzers, producing a frequency domain display of the vibration over any reasonable frequency range required 2 to 3 minutes. This was not only intolerable when many points were to be analyzed, but you couldn't be sure that the low frequency components were there at the same time as the high frequency components. In addition, the resolution was poor at the low frequency end, and even poorer at the high end.

The lack of physical inertia and the advantages of real-time frequency analysis were some of the advantages of digital signal processing that led to the development of modern spectrum analyzers.

18 MEASUREMENT AND COLLECTION TECHNIQUES

INTRODUCTION

This chapter presents an overview of three machinery maintenance methods, and discusses considerations for taking vibration data as part of a maintenance program.

1.0 MACHINERY MAINTENANCE

Machinery maintenance is performed in one of three ways:

1. Catastrophic (Hysterical) - Fix it when it breaks.
2. Preventive (Historical) - Overhaul on a calendar basis.
3. Predictive (Histogram) - Predict and repair only when necessary.

1.1 Catastrophic (Hysterical) Maintenance

Catastrophic maintenance has been around since the first toothache, and is the normal procedure in many industrial facilities. There are those who still believe that the most efficient use of production machinery is to run it until it fails and then fix it.

Although this may not have been too far from the truth years ago in low-speed machinery, it is a foolish method of maintenance today. The initial cost and the repair cost of modern, high speed

equipment, coupled with the continued effort to improve product quality, makes it necessary to perform required maintenance before complete failure of the machine occurs.

1.2 Preventive (Historical) Maintenance

Since critical components such as anti-friction bearings have a finite (yet predictable) life, it seems reasonable to shut down and repair and replace on a periodic basis. In fact, it is quite common today to find industry performing maintenance on a calendar basis whether the machine needs it or not.

The major disadvantages of preventive maintenance are as follows:

1. Maintenance is performed more than is necessary.
2. Parts and labor costs are higher than necessary.
3. Unexpected catastrophic failure may still occur before the next scheduled maintenance, affecting throughput and raising repair costs higher than normal.
4. Maintenance effort is not concentrated on the machines that need it the most.

1.3 Predictive (Histogram) Maintenance

A program that continually monitors the health of machinery and provides the ability to predict not only when a machine will fail, but also *why* it will fail, solves all of these major disadvantages.

With a program that provides this information, the shutdown of equipment can be scheduled to have the least impact on the production process, and the necessary spare parts and trades can be on hand when the required maintenance is done.

Since machines are constantly monitored, there is a growing body of information on the characteristics of each machine, and the primary maintenance effort is concentrated where it is needed the most.

Corrective action is taken when a defect is detected rather than when it has propagated to failure, so repairs are simpler to make and less expensive. In addition, the spare parts inventory can not only be reduced, but be tailored to only those parts that are actually needed.

There is abundant evidence that predictive programs do work. They not only pay for themselves in reduced maintenance and spare parts cost, but there is usually a significant increase in machine up-time. An additional benefit that is sometimes overlooked is the increase in product quality that comes from machines that are kept in good running condition.

A good predictive maintenance program starts with an evaluation of all of the equipment in the plant to determine its importance to process and throughput. At the very minimum, machines should be classified as critical or non-critical to the process.

A critical machine is one where catastrophic failure would:

- injure or kill someone
- stop the plant throughput
- stop people from performing their job
- be expensive to repair
- have a long lead time on spare parts

Non-critical machines are those whose failure might be an annoyance but would not injure personnel or adversely affect the process.

Once the critical machines have been identified and cataloged, they are analyzed to determine the data necessary to predict both their failure and the cause of that failure. That data may include parameters such as

- temperature
- pressure
- flow
- horsepower
- voltage
- current
- torque
- flow, basis weight, phase, and torsional vibration
- vibration

Of all these measurable parameters, vibration time and frequency domains contain the most information about machine health. That is not to say that other parameters should not be measured, but it does say that, above all other parameters, vibration data should be measured.

Nor are we suggesting that it is possible to extract all of the information that exists in the vibration data. But as instrumentation and experience increase, you can extract more and more of that information, and use it to evaluate machine condition.

In almost all cases, a well-executed vibration data collection and analysis program provides not only the best, but the least expensive means to monitor the health of production equipment and prevent catastrophic failure.

Once the critical machines have been identified, take initial data to determine the present condition of these machines. Schedule maintenance so that all of these machines are operating at the present level of acceptability. As the program develops, establish a better baseline for each machine.

2.0 VIBRATION DATA COLLECTION

The advantages of a consistent program of vibration data collection and analysis as the basis for a predictive maintenance procedure have been well known for many years. With the development of computer-compatible data collectors, the technique is now not only readily available, but is easy to implement in the plant.

Once you decide to use vibration survey and analysis for machinery health monitoring, you need to answer some basic questions. The answers to these questions determine the value and the success of your program. It is important that you understand these points before making the final selection of the procedures, the instrumentation, and the transducers to be used in the program.

Ask the following important questions:

- Will data be collected and stored as broadband data or as spectral data?
- What frequency range should be covered by the program?
- What units will be used for machine health baseline consideration?
- How much vibration is too much vibration?
- What transducers will be used to collect these survey units?
- How will the transducers be mounted?

- How many measuring locations are required?
- How often will data be collected?
- Will machines be loaded or unloaded during data collection?
- Who will collect and analyze the data?

Let's examine each of these issues in turn.

2.1 Collect Broadband or Spectral Data?

Although broadband, sometimes called **filter out**, data is much better than no data, and much easier to collect and store, changes in machine health will not be detected as early nor as accurately as with spectral and time domain data.

In the case of catastrophic failure, it is almost impossible to pinpoint the failure from broadband information. The analysis of stored spectral data may well indicate the cause of the failure, and allow the program to be updated to be on the lookout for indications of this problem developing again.

Spectral data taken and stored on the computer makes it possible to establish more accurate trend data, to more precisely predict incipient failure, and to preserve the ability to do post-collection data analysis.

Good data management software makes it possible not only to calculate trends for several frequency bands within the spectrum, but also to set separate alert and alarm limits for each of these bands. In addition, the collector can be flagged to store the time domain information when the alert levels are exceeded in any one of the bands.

2.2 What Frequency Range Should Be Covered?

In a typical plant, there could be low-frequency data from cooling tower fans, as well as high-frequency data from gear boxes. A successful program may require more than one data collection route, using different instrumentation and procedures for different machines. However, if only broadband data is taken, the frequency range is determined by the transducer and the instrumentation used to collect the data.

If spectral data is taken on the equipment, and seismic velocity transducers are used to collect the data, the frequency range is limited by the transducer, from about 10 to 2,000 Hz. An accelerometer with internal electronics could extend that range from below 1 to as high as 20,000 Hz.

When a program includes data points where frequencies above 1000 Hz are to be taken, an accelerometer should always be used, and the mounting of the transducer becomes more critical as the maximum frequency increases. Transducer mounting is discussed later in this chapter, and is covered in detail in the chapter on Transducers in Volume 1.

Since the accelerometer output is proportional to acceleration, the presence of high-level, high-frequency vibration signals will probably swamp the accelerometer so that low-frequency vibration cannot be measured when high frequencies are present.

A gear box is a good example of this problem, where there may be low-frequency pitch line runout in the presence of high-level gear mesh. The high-frequency gear mesh will probably make it impossible to use an accelerometer to measure the vibration caused by the pitch line runout, so it may be necessary to use both a seismic velocity transducer and an accelerometer, in conjunction with different spectral limits, to determine the condition of the box.

A hybrid velocity transducer (an accelerometer with "built in" integration so that the output is proportional to velocity) will not solve this problem since the integration is after the fact, that is, after

the accelerometer element. Small mechanical filters, available from several manufacturers, can be mounted between the accelerometer and the machine in order to filter out the high frequencies when low-frequency signals are to be measured.

The program should include the capability to divide the spectra into several frequency bands for trend analysis. As a minimum, spectral energy should be trended for the sub-synchronous, first order, second order, third through blade or vane pass orders, bearing fault frequency area, and a high frequency band. These bands should be established for each measurement location and the limits refined as the knowledge base improves.

A good rule of thumb for determining the frequency range is as follows:

- if the system contains journal bearings, the upper limit of the frequency range should be 12 to 15 times shaft RPM.

1. if the system contains roller bearings, the upper limit of the frequency range should be 50 times shaft RPM.

2.3 What Units Are Used for Machine Health Baseline?

For many years peak, or peak-to-peak displacement, was the criteria of choice for machine health; however, this makes it necessary to know the frequency of the vibration before consideration can be given to its severity. This point alone makes displacement a poor choice for broadband survey and general machine condition monitoring.

Since vibratory energy is approximately proportional to velocity, it is more logical to use velocity for most machinery monitoring, at least in the frequency range from 5 to 1,000 Hz. Since vibration severity levels in velocity are independent of frequency, this parameter is ideally suited for broadband monitoring.

For low-frequency disturbances, such as those found in cooling towers, it may be necessary to use displacement rather than velocity as the survey parameter. On the other hand, for high-frequency problems, such as those in gear boxes, acceleration may be a more logical choice.

For machines with anti-friction bearings, some data should be taken with an accelerometer at each bearing location. If possible, axial data in the bearing load zone usually gives the best bearing fault trend data.

2.4 How Much Vibration is Too Much Vibration?

The two basic reasons for setting limits on the amount of vibration present in a system are the mechanical health of the machine, and the quality of the product produced by that machine.

One of the greatest advantages, though many times it is not recognized, of a good vibration survey program coupled with maintenance follow-up is improvement in the quality of the product. The effect of vibration on product quality can be determined as the vibration survey program develops, and the acceptable limits revised as this information becomes available.

If no other information is available at the beginning of the program, reasonable velocity limits in inches/sec. for machine health (not product quality) would be as shown in Table 8.

For anti-friction bearing frequencies, a good limit should be much lower than .10 inches/sec., something around .01 to .02, until a more realistic limit can be established based on the trend.

One of the most common mistakes made in setting acceptable limits of vibration is to set the same level for all components (all frequencies) in the spectrum. It should be obvious that a machine could

FREQUENCY RANGE	GOOD	ALERT	ALARM
1 - 10 Hz	.06	.12	.36
10 - 80 Hz	.10	.30	.60
80 - 150 Hz	.08	.20	.45
150 - 1,000 Hz	.02	.15	.30

Table 8 - *categorizing levels of vibration*

tolerate a much higher vibration level at its basic rotational frequency than at rolling element bearing frequencies.

A good vibration survey program lets you set different limits for different machines, as well as different limits for different regions of the frequency domain spectra for the same machine.

2.5 What Type of Transducer Should Be Used?

Displacement transducers, such as the eddy current probe or the LVDT (linear variable differential transformer) transform mechanical motion into an analog signal that is proportional to the displacement (the space occupied by the vibrating system). Seismic velocity transducers produce an analog signal proportional to the velocity of the motion, while an accelerometer produces an output signal representing the change in velocity (acceleration) of the motion.

Each transducer has advantages and disadvantages, some of which are discussed here. For a more complete discussion of transducers, see the Transducers chapter in Volume 1.

2.5.1 Displacement Transducers

The non-contact eddy current probe has become the most popular displacement transducer today, and has found its major application in measuring the relative motions between the bearing journal and the bearing housing in machines with sleeve bearings. Mounting two transducers at 90 degrees to each other in a sleeve bearing makes it possible to see and analyze the shaft orbit.

Because of the need for an outside power source, and the surface condition of the target material, it is difficult to estimate the signal-to-noise ratio of such transducers, but a reasonable range is from 20 to 60 dB.

Some of the advantages of non-contact displacement transducers are

- small size
- light weight
- relatively inexpensive
- simple to calibrate and check
- measures static displacement

Some of the disadvantages are

- measures relative motion so that a stable datum plane is required
- requires an outside power supply
- sensitive to variations in the target material
- sensitive to magnetic spots in the target material

- limited frequency range
- difficult to mount
- requires double differentiation to cross all parameters

2.5.2 Seismic Velocity Transducers

A seismic velocity transducer, normally a coil in a magnetic field, transforms mechanical motion into an analog signal proportional to the velocity of the vibration of the system.

The mounting of the coil or the magnet in these transducers is a typical k/m system with a resonance between 3 and 12 Hz, which requires damping close to critical. The damping is normally magnetic or viscous, either of which changes with temperature, so that the output of the transducer also changes with temperature, especially in the low-frequency range.

In addition to the change in damping with temperature, both the magnetic field strength and the coil resistance change with changes in temperature. Some of these units utilize the input resistance of the instrument as part of the damping. Don't use seismic velocity transducers in situations where there may be significant changes in temperature, and make sure you operate them into the correct impedance.

The signal-to-noise ratio of the velocity transducer is limited only by the integrity of the lead shielding, the electronics, and the presence of electromagnetic noise; however, a normal range is 60 to 100 dB.

Some of the advantages of the seismic velocity transducer are

- measures velocity, the parameter which is proportional to vibration severity
- does not require outside power
- excellent signal-to-noise ratio
- sensitive
- does not require stable datum plane
- rugged
- easy to use
- requires only one differentiation or one integration to cross all parameters

Some of the disadvantages are

- large size
- heavy
- relatively expensive
- limited temperature range
- limited frequency range
- sensitive to electromagnetic fields
- output varies with mounting position

In addition to the seismic velocity transducer, hybrid units are available which are accelerometers with internal integration. Although this arrangement increases the frequency range, it also decreases the signal-to-noise ratio and is susceptible to the problem of the high-level, high-frequency vibration swamping the accelerometer element.

2.5.3 Accelerometers

Accelerometers transform mechanical motion into an analog signal proportional to the vibration acceleration in the system. Almost all accelerometers are basic high-impedance piezoelectric devices and are available with or without internal electronics.

Those without the internal electronics require an external charge or voltage amplifier with very high input impedance so that both the lead length and the lead configuration between the accelerometer and the charge amp is critical. Both types are also sensitive to base strain and connector strain if proper care is not taken when mounting to machinery.

The great advantage of units with internal electronics is the low output impedance, which eliminates sensitivity to lead configuration and length and usually results in a significant increase in signal-to-noise ratio. Typical signal-to-noise ratios of these units in a normal industrial environment would be

- charge amplifier - 20 to 50 dB
- internal electronics - 40 to 60 dB

Although accelerometers operate over a very wide frequency range, it is important to remember that high frequency acceleration can swamp the piezoelectric element so that low frequencies can not be measured.

Some of the advantages of the accelerometer are

- small size
- light weight
- relatively inexpensive
- wide frequency range
- research is ongoing to improve

Some of the disadvantages are

- requires an outside power supply
- high-impedance units sensitive to lead length and configuration
- somewhat delicate
- medium signal-to-noise ratio
- high frequency acceleration can swamp the unit
- requires a double integration to cross all parameters
- sensitive to electrostatic interference

Remember that accelerometers have a "settling time" after they are attached to the machine being monitored. The impact when an accelerometer is mounted (especially if you are using a magnetic base) creates a high voltage signal that temporarily swamps the circuitry. You must wait for the high voltage to pass through the system before you attempt to read vibration.

Some accelerometers have a settling time of as little as 3 seconds, while others may take up to 20 seconds. Check the literature on your accelerometer for the appropriate settling time, or calculate it yourself by "plunking" the accelerometer down and watching the time domain signal.

Once the vibration units have been selected, the obvious conclusion would be to choose a transducer that transforms the vibration directly into these units; however, this is not always wise or possible.

If the transducer of choice were an accelerometer, and the units of choice were velocity, it would be necessary to convert the signal proportional to acceleration to one proportional to velocity by

integration; and if the units of choice were displacement, a second integration would be required to convert the acceleration signal to one proportional to displacement.

This process is illustrated both graphically and mathematically in Figure 125. The acceleration signal at the top, $a = A * \sin(\omega t)$, is integrated to produce the velocity signal

$$v = \frac{-A}{\omega} * \cos(\omega t) + C1$$

where

a = instantaneous acceleration (in/sec^2)
A = maximum acceleration (in/sec^2)
ω = circular frequency (radians/sec)
t = time (seconds)
v = instantaneous velocity (in/sec)
$C1$ = the constant of integration

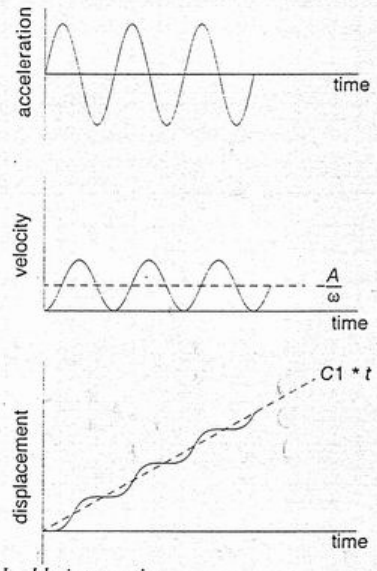


Figure 125 - *integration and double integration*

Solving the velocity equation for $C1$ when $t = 0$ and $v = 0$ results in $C1 = A_\omega$, where $\omega = 2 * \pi * f$. Thus the integrated signal proportional to velocity from an accelerometer is offset by a level equal to A_w.

A second integration converts the signal to displacement

$$d = \frac{-A}{\omega^2} * \sin(\omega t) + C1 * t + C2$$

Solving this equation for $C2$ when $t = 0$ and $d = 0$ results in $C2 = 0$. It is then apparent that the double integrated signal proportional to displacement from an accelerometer is increasingly offset by a level equal to $C1 * t$, that is

$$A * \frac{t}{\omega}$$

The offset of the first integration, and/or the offset that increases with time of the second integration, must be removed from the result of the integration in order to read the peak velocity or the peak-to-peak displacement of the dynamic vibration. This is done by simple capacitor coupling or by setting the "knee" of the integrator at some reasonable low frequency. In the industrial environment, the lowest frequency for charge amplifier systems should be about 10 Hz, and for accelerometers with internal electronics, about 1 or 2 Hz.

Accelerometers, and in particular, charge systems, are sensitive to capacity-coupled electrical noise and triboelectric noise (noise associated with flexing of the signal cable). Since this noise is most prevalent when low-frequency vibration is being measured, don't use charge amplifiers below 10 Hz for displacement measurement, and use accelerometers with internal electronics below 1 or 2 Hz.

It is also clear that the double integration results in a division by ω^2, so that very low-frequency noise in the acceleration signal results in very high and erroneous displacement data. If at all possible, avoid the double integration required to convert acceleration to displacement at frequencies below 10 Hz.

Differentiation (converting a displacement signal to one proportional to velocity or acceleration, as shown in Figure 126), results in no errors due to low-frequency noise, and has no offsets due to the integration constants. In this case, however, the high-frequency electronic and process noise is exaggerated so that, given the choice, most of the time it is preferable to integrate rather than differentiate.

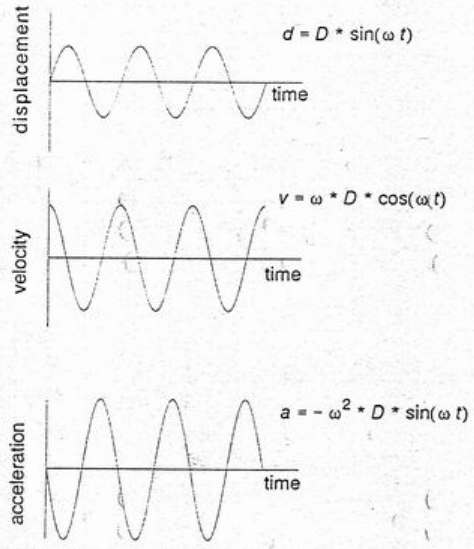


Figure 126 - *differentiation and double differentiation*

Your choices for transducer selection are, in order of preference:

1. Use a transducer whose output is proportional to the desired vibration units.
2. Integrate once, with the integration cutoff at 1 to 5 Hz.
3. Differentiate once, with the high frequency cutoff at 500 to 1,000 Hz.
4. Integrate twice, with the integration cutoff at 2 to 10 Hz.

The ideal transducer for choice 1 would be a displacement transducer for measuring low frequencies, and an accelerometer for measuring high frequencies. If the data collection route includes anti-friction bearings, take at least some of the bearing data using an accelerometer without integration.

2.6 How Will the Transducers Be Mounted?

The success of the entire program may well depend on how the transducers are mounted when the data is taken. Many vibration survey programs use operators who hand hold the transducer during data collection. This is probably the easiest and fastest means of gathering the data, but you must ensure that variables such as pressure, angle, location, and probe tip remain as constant as possible.

The resonant frequency of the stinger or probe is probably between 600 and 1,500 Hz, so any vibration signal that coincides with the probe resonance is amplified. If components in this range appear in the spectra, use a different transducer mounting and investigate the area of concern.

Ideally, transducers should be permanently mounted at each location and hard-wired to a central computer for continuous data sampling. However, the number of locations and the cost of the transducers and installation make this impossible for most applications.

Some of the most common provisions for transducer mounting in a single transducer survey program are as follows:

1. Hand hold the probe. The probe should be as short as possible, and the tip should not be so sharp that it deforms the surface being probed. Take care to ensure consistent data, and note that frequencies above 800 Hz may be amplified by the probe resonance.
2. Use a spot face and transducer mounting stud at each measuring location so that the pickup can be securely attached to the machine. The spot face must have a surface finish better than 10 to 20 micro-inches, be perfectly clean, and the stud must be square with the face.

   The transducer must be torqued to specification so it does not influence the data by base strain, and the transducer cable should be secured so that the connector is not strained during data collection. With care, the frequency range should be as good as the transducer itself.
3. Use machined blocks with a surface finish better than 10 micro-inches which are welded or glued to the machine at each measurement location. These blocks may have studs for transducer mounting as in number 2, or they may be used for magnetic mounting.

   If studs are used and the blocks are properly attached to the machine, the frequency range should be nearly as good as stud mounting. As a general rule, the thinner the adhesive used to attach the block, the higher the frequency response.

   If the blocks are used for magnetic mounting, and rare earth ground magnets are used, the frequency range can be as high as 8,000 Hz. If the more normal Alnico 5 magnets are used, the frequency range is only about 1,000 Hz.

4. Use magnets to hold the transducer to the machine. Make sure that the transducer does not rock, or that dirt is not under the magnet. The maximum frequency capability is about 600 Hz.

Section 1.3.2 of the Transducers chapter (Volume 1) illustrates the frequency response of various methods of transducer mounting. For more detailed analysis once a machine fault has been identified, it is very convenient to attach the transducer directly to the machine with a quick-setting adhesive. The transducer is securely mounted to the machine, and the high frequency capability of the transducer is not greatly reduced.

2.7 How Many Measuring Locations Are Required?

Equipment covered by the survey program should be divided into at least two classes: critical and non-critical. Once the machines have been classified, the information necessary to maintain the health of the critical machines should be carefully considered and catalogued. This may include such parameters as voltage, current, temperature, pressure, flow, lube oil analysis, horsepower, and torque. However, one of the measured parameters should be vibration.

The purpose of a vibration survey program is to learn more about the equipment, predict necessary repairs, anticipate imminent failure, and improve product quality. In order to collect the data necessary to accomplish this, the selection of measurement locations is very important.

At the beginning of the program, taking horizontal, vertical, and axial data at each bearing is the bare minimum. For example, to detect rolling element bearing fault frequencies, it is best, if possible, to take axial data through a hard metal path to the load zone. If axial data is not taken at any of the locations, at least 20 to 25 percent of the machine problems will be missed.

As the program develops and more is learned about the machine as well as the data necessary to maintain it, the number of data points and locations can be modified to improve the efficiency of the program.

2.8 How Often Will Data Be Collected?

Once again, at the start of any survey program, taking data more often than necessary is preferable to taking data less often than necessary. As the program continues, the frequency of data collection can be tailored to the maintenance requirements of the equipment.

2.9 Will Machines be Loaded or Unloaded During Collection?

Whether data is taken when the machines are running loaded or unloaded depends on the machine and the load. The machine conditions should be as constant as possible, not only while the data is being taken, but also from one data collection period to the next.

An air handler, for example, would probably be loaded as close to the average load as possible when the data is taken. On the other hand, data should be taken on metal cutting machines or hammer mills in the unloaded condition.

A good program takes into account the variable from the load or process and schedules data collection so that the machine condition is the same each time the data is taken.

2.10 Who Will Collect and Analyze the Data?

Although the collection of the vibration data is critical to the success of the program, the skill required to take data is not as great as the skill required to analyze it. It is best to utilize personnel for collection and analysis from the same department, and these people should be thoroughly trained for the job, work well as a team, and have the enthusiastic support of management.

The person charged with data collection should understand the value of the data and how that data is influenced by the transducer mounting and data recording. If he or she thinks that the job is routine and unimportant, the program will fail.

The person responsible for data analysis must be trained in vibration analysis and must direct the collection of the data. There are computer programs that can be integrated with the database that quickly perform most of the routine analysis of the data and list those machines that require the analyst's attention.

Store vibration data so that data history and trends can be plotted for each critical machine. Analysis and maintenance records for each machine should be stored with the vibration data, so that the accuracy of analysis, and the results of the maintenance performed, can be correlated with the vibration data.

This data is then used to fine-tune the data collection and analysis program. For example, you might want to update

- the number of data points
- the time between data collection periods
- the transducers used and their mounting
- the alert and alarm levels for bands in the spectra
- the base band frequency at each point
- the frequency resolution (number of lines)

If careful consideration is given to these basic questions when a vibration data collection program is put into practice, and if the personnel responsible for the collection and analysis of the data are not only well-trained, but also convinced that the program is valuable and will work, then the program *will* work.

SUMMARY

Machinery maintenance is performed in one of three ways:

1. Catastrophic (Hysterical) - fix it when it breaks.
2. Preventive (Historical) - overhaul on a calendar basis.
3. Predictive (Histogram) - predict and repair only when necessary.

Catastrophic maintenance is the normal procedure in many industrial facilities, but it is a foolish method of maintenance today. The initial cost and the repair cost of modern, high-speed equipment, coupled with the continued effort to improve product quality, makes it necessary to perform required maintenance before complete failure of the machine occurs.

Preventive maintenance seems reasonable, since critical components such as anti-friction bearings have a finite (yet predictable) life. It is quite common today to find industry performing maintenance on a calendar basis whether the machine needs it or not. The major disadvantages of preventive maintenance are that maintenance is performed more than is necessary, parts and labor costs are higher

than necessary, and unexpected catastrophic failure may still occur before the next scheduled maintenance.

A program that continually monitors the health of machinery and provides the ability to predict not only when a machine will fail, but also *why* it will fail, solves all of these major disadvantages.

With a program that provides this information, the shutdown of equipment can be scheduled to have the least impact on the production process, and the necessary spare parts and trades are on hand when the required maintenance is done. Since the machines are being constantly monitored, there is a growing body of information on the characteristics of each machine, and the primary maintenance effort is concentrated where it is needed the most.

A good predictive maintenance program starts with an evaluation of all of the equipment in the plant to determine its importance to process and throughput. Once the critical machines have been identified and cataloged, they are analyzed to determine the data necessary to predict both their failure and the cause of that failure.

Once the critical machines have been identified, initial data should be taken to determine the present condition of these machines. Maintenance should be scheduled so that all of these machines are operating at the present level of acceptability. As the program develops, a better baseline will be established for each machine.

19 EXPERIMENTAL MODAL ANALYSIS

INTRODUCTION

Modal Analysis is the process of characterizing the dynamic properties of an elastic structure in terms of its modes of vibration. The goal of modal analysis is to develop a dynamic model of a structure or machine that can be used for simulation, troubleshooting, diagnostics, and/or design.

Modal Analysis is divided into two separate categories:

1. Experimental modal analysis
2. Numerical (or analytical) modal analysis

Experimental modal analysis (EMA) uses experimentally-determined data taken from a test structure. Analytical methods are used to obtain an estimate of the structure's modal parameters from the measured data.

Numerical modal analysis uses a mathematical model which approximates the stiffness, distribution of mass, and the effect of surrounding boundary conditions on the dynamic characteristics of the structure. The Finite Element Method is currently the most popular numerical technique for modal analysis.

This chapter is dedicated to experimental modal analysis. The following chapter covers the finite element technique.

1.0 EXPERIMENTAL MODAL ANALYSIS

Experimental modal analysis (EMA) differs from the conventional signal analysis approach to vibration analysis, in that it provides a complete systems analysis approach to vibration analysis.

Conventional vibration signal analysis consists of experimentally obtaining a system's dynamic response to an unknown excitation during normal operating conditions. The data is analyzed in both the time and frequency domains, and a probable cause for vibration is identified and corrected based on the analysis results.

However, if one of the forcing functions occurs at (or near) a natural frequency of the system, then the dynamic characteristics of the structure must be modified, in addition to correcting the source of excitation. The affect of structural modifications on the dynamic characteristics of a structure cannot be evaluated by conventional signal analysis. This evaluation must be done by either experimental modal analysis or numerical modal analysis.

Experimental modal analysis allows you to quantify the dynamic characteristics of a structural system with experimental data. The major advantage of this method is that, unlike analytical or numerical techniques, you do not have to know the distribution of mass and stiffness. This is important if engineering drawings are not available, and you cannot obtain dimensions, thicknesses and material properties by field measurements. Also, since the modal model is developed upon experimental data alone, boundary condition effects do not influence the results to the degree they do in numerical analysis.

Plant personnel often attempt to solve resonance excitation problems by arbitrarily increasing the stiffness of a structure or machine component. In some cases, the indiscriminate addition of a brace could adversely affect the dynamic characteristics of a structure instead of correct the problem. Modal analysis provides you with a tool to completely understand the problem, thus enabling you to proceed with an educated approach to vibration troubleshooting. The case histories included at the end of this chapter include an example of a vibration problem that would have been compounded by adding structural stiffening.

1.1 Disadvantages of Experimental Modal Analysis

Experimental modal analysis has several major disadvantages. The most obvious is that the structure must exist before you can perform an EMA. Therefore, this method is not acceptable as an engineering design tool. However, experimental modal analysis is a valuable engineering tool for product evaluation and mechanical troubleshooting.

Another disadvantage is that the modal model consists of fewer degrees of freedom than what is typically included in a numerical analysis. This makes it difficult, during structural modification evaluation, to quantify the structural stiffness that needs to be added between two data points of a complex structure.

1.2 A Comparison with Finite Element Analysis

Numerical modal techniques such as finite element analysis can overcome some of the disadvantages of experimental modal analysis. However, there are disadvantages to finite element analysis that are not present in the experimental approach.

Experimental modal analysis can verify and improve the accuracy of finite element models by providing an independent check of the finite element analysis accuracy. Experimental measurements are taken on the actual structure to provide a physical check of a mathematical model's accuracy. If the finite element analysis model predicts the same modes that are measured, you can assume that the model is accurate, and that it can be used for further analysis and simulation.

This chapter presents the basic theory behind experimental modal analysis, testing procedures, and case histories.

2.0 EXPERIMENTAL MODAL ANALYSIS THEORY

Realizable structures (structures that can be built) contain multiple degrees of freedom. However, you must understand the mathematics governing a single degree of freedom vibrating system before the theory governing multiple degree of freedom vibrating systems can be developed.

2.1 A Single Degree of Freedom System

Figure 127 illustrates a single degree of freedom vibrating system consisting of a vibrating mass, a spring, and a damper.

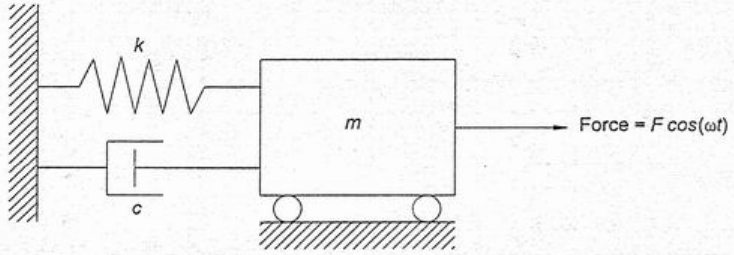


Figure 127 - *a single degree of freedom vibrating system*

The equation of motion for forced harmonic excitation of this system is:

$$m\ddot{x} + c\dot{x} + kx = F\cos(\omega t) \qquad \text{(Formula 19-1)}$$

where

m = mass
$\ddot{x}$ = acceleration
c = damping
$\dot{x}$ = velocity
k = stiffness
x = amplitude
$F\cos(\omega t)$ = magnitude of the applied dynamic force

Figure 128 shows that $F\cos(\omega t)$, the magnitude of the applied force, varies harmonically with respect to time t. The maximum value of the applied force is F.

The frequency at which force F is applied is

$$\omega = \frac{2\pi}{T}\ (radians/sec.)$$

or

$$f = \frac{1}{T}\ (Hz)$$

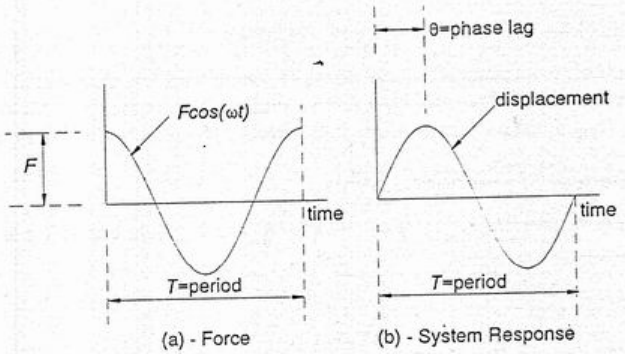


Figure 128 - *magnitude of force varies with time*

where

T = the period of the applied force

Dividing both sides of Formula 19-1 by mass m, multiplying both the denominator and numerator on the right hand side of the equation by the stiffness k, and defining the undamped natural frequency as $\omega_n = \sqrt{k/m}$ yields:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \omega_n^2(F/k)e^{j\omega t}$$

where

$\ddot{x}$ = acceleration
ζ = damping ratio c/c_c (c_c = critical damping)
ω_n = undamped natural frequency $\sqrt{k/m}$ (radians/sec)
$\dot{x}$ = velocity
x = amplitude

The Greek character ζ is pronounced "zeta".
The solution to this differential equation is

$$x = \left[\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}\right]\frac{F\cos(\omega t - \theta)}{k} \qquad \text{(Formula 19-2)}$$

The total displacement is a complex quantity that lags behind the applied force by the angle θ, which equals

$$\theta = \tan^{-1}\frac{2\zeta\omega/\omega_n}{1-\left(\omega/\omega_n\right)^2}$$

The total displacement can be broken down into the following real and imaginary parts:

$$Real(x) = \left[\frac{1-\left(\omega/\omega_n\right)^2}{\left(1-\left(\omega/\omega_n\right)^2\right)^2 + (2\zeta\omega/\omega_n)^2}\right]\frac{F\cos(\omega t)}{k} \qquad \text{(Formula 19-3)}$$

$$Imaginary(x) = \left[\frac{2\zeta\omega/\omega_n}{\left(1-\left(\omega/\omega_o\right)^2\right)^2 + (2\zeta\omega/\omega_o)^2}\right]\frac{F\sin(\omega t)}{k} \qquad \text{(Formula 19-4)}$$

The real component of the displacement is in phase with the applied force, and is said to be **coincident** to the applied force. The imaginary component has a phase lag of 90 degrees behind the excitation, and is said to be in **quadrature** with the applied force.

Figure 129 shows the relationship between the complex total displacement, the real and imaginary components, and the applied harmonic force. Total displacement equals the vectorial addition of the real and imaginary components, and lags behind the force vector by the angle θ.

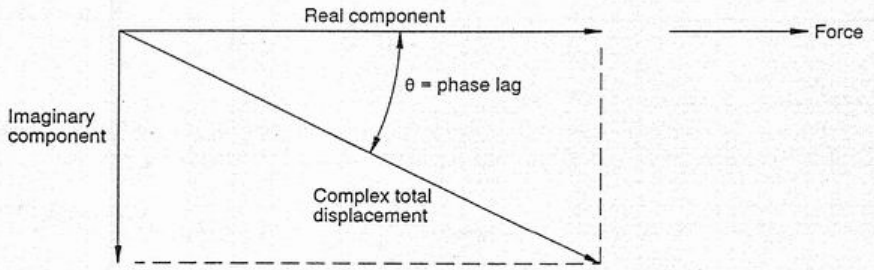


Figure 129 - *real and imaginary components of total displacement*

The total displacement can be written in the form

$$x = \left[\frac{1}{\sqrt{\left(1-\left(\omega/\omega_n\right)^2\right)^2 + \left(2\zeta\omega/\omega_n\right)^2}}\right]\frac{F}{k}\sin(\omega t - 0) \qquad \text{(Formula 19-5)}$$

The quantity in brackets in Formula 19-5 is the absolute value of the complex frequency response, and is generally referred to as the **magnification factor**.

2.1.1 The Magnification Factor

The magnification factor (also called the quality factor) is a dimensionless ratio between the amplitude of the dynamic displacement and the static displacement (F/k). The magnification factor increases as the frequency of the applied force approaches the natural frequency of the structure. This is illustrated in Figure 130.

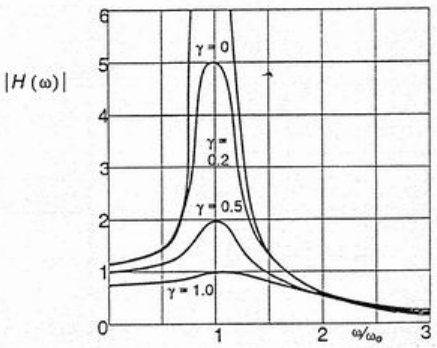


Figure 130 - *the magnification factor*

For cases of light damping (approximately 10% of critical or less), the magnification factor reduces to

$$|H(\omega)| - Q = \frac{1}{2\zeta} \qquad \text{(Formula 19-6)}$$

Most structures are typically lightly damped.

If experimental data is available, you can obtain the magnification factor by reading the frequencies at the half-power points (-3 dB points), and entering these values into the following formula:

$$\frac{1}{2\zeta} = \frac{\left(\omega_2/\omega_1\right)^2 + 1}{\left(\omega_2/\omega_1\right)^2 - 1} \qquad \text{(Formula 19-7)}$$

The damping is the reciprocal of the quality factor:

$$\frac{1}{2\zeta} = \frac{\left(\omega_2/\omega_1\right)^2 - 1}{\left(\omega_2/\omega_1\right)^2 + 1}$$

It is difficult to define the frequency associated with the -3dB power point by visual inspection.

2.1.2 Imaginary Frequency Response Curve

In addition to finding the damping by obtaining the magnification factor, you can also use the imaginary frequency response curve to estimate the damping in a single degree of freedom system. Figure 131 contains a typical imaginary frequency response curve. Note that there is a positive and negative peak value for the frequency response curve on either side of the natural frequency. You can easily obtain the frequency values associated with these peaks using the FFT analyzer's cursor.

You can calculate the amplification factor from the imaginary frequency response curve:

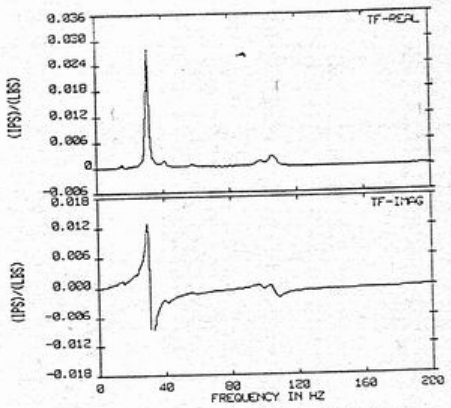


Figure 131 - *a typical imaginary frequency response function*

(Formula 19-8)

$$Q = \frac{\left(\omega_h/\omega_l\right)^2 + 1}{\left(\omega_h/\omega_l\right)^2 - 1}$$

where

ω_h = frequency of peak above resonance
ω_l = frequency of peak below resonance

2.2 A Multiple Degree of Freedom System

Figure 132 illustrates a two degree of freedom system. This system consists of two lumped vibrating masses, springs and dampers between the masses, and springs and masses between each mass and ground.

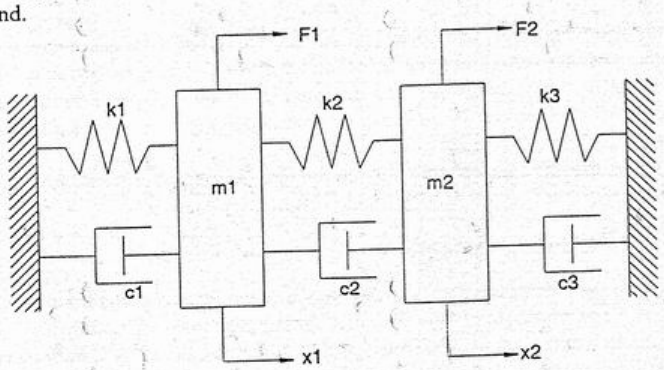


Figure 132 - *a multiple degree of freedom system*

The term "lumped" mass assumes that all the mass is concentrated at one point. The equations of motion for this multiple degree of freedom system are:

$$m_1\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 - c_2\dot{x}_2 + (k_1 + k_2)x_1 - k_2x_2 = F_1 \qquad \text{(Formula 19-9)}$$

$$m_2\ddot{x}_2 - c_2\dot{x}_1 + (c_2 + c_3)\dot{x}_2 - k_2x_1 + (k_2 + k_3)x_2 = F_2 \qquad \text{(Formula 19-10)}$$

Both equations can be expressed in the following matrix form:

$$[m]\{\ddot{x}\} + [c]\{\dot{x}\} + [k]\{x\} = \{F\} \qquad \text{(Formula 19-11)}$$

where

$[m]$ = mass matrix
$[c]$ = damping matrix
$[k]$ = stiffness matrix
$\{F\}$ = force vector
$\{\ddot{x}\}$ = acceleration vector
$\{\dot{x}\}$ = velocity vector
$\{x\}$ = displacement vector

The mass, damping and stiffness matrices for this 2 degree of freedom system are

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$[k] = \begin{bmatrix} k_1 + k_2 & -k_1 \\ -k_2 & k_2 + k_3 \end{bmatrix}$$

$$[c] = \begin{bmatrix} c_1 + c_2 & -c_2 \\ -c_2 & c_2 + c_3 \end{bmatrix}$$

These matrices are symmetric. However, the stiffness and damping matrices are non-diagonal (off-diagonal terms are non-zero) since the equations of motion consist of two coupled differential equations. The mass matrix is symmetric and diagonal since we assumed a lumped-mass system. Had we made a distributed or consistent (uniform) mass assumption, we would have obtained a non-diagonal mass matrix.

Modal analysis is the process of deriving the system response by transforming the coupled equations of motion into a set of independent linear differential equations.

2.2.1 Converting to Diagonal Matrices

The mass, stiffness, and damping matrices can be converted into diagonal matrices under certain conditions (referred to as **orthogonality**). The orthogonal properties of a structure's mode shapes (eigenvectors) provide a coordinate transformation that decouples the system both inertially (mass) and elastically (stiffness) simultaneously, providing diagonal matrices. The x coordinates can be transformed to the modal coordinate system by the following formula:

$$\{x\} = [\Phi]\{\eta\} \qquad \text{(Formula 19-12)}$$

where

$[\Phi]$ = matrix of mode shapes (eigenvectors)
$\{\eta\}$ = modal coordinates

By definition of the orthogonality conditions of the eigenvectors, the following transformations provide diagonalized mass [M], stiffness [K] and damping [C] matrices:

$$[\Phi]^T [m][\Phi] = [M]$$

$$[\Phi]^T [k] [\Phi] = [K]$$

$$[\Phi]^T [c][\Phi] = [C]$$

Backsubstitution of these diagonalized matrices into Formula 19-11, transformed into the modal coordinate system, yields:

$$[M]\{\ddot{\eta}\} + [C]\{\dot{\eta}\} + [K]\{\eta\} = [\Phi]^T\{F\} \qquad \text{(Formula 19-13)}$$

Formula 19-13 represents a set of linear differential equations for the damped single degree of freedom system, the *i*th equation being:

$$M_i\ddot{\eta}_i + C_i\dot{\eta}_i + K_i\eta_i = \{\varphi_i\}^T\{F\} = F_i$$

Furthermore, any particular eigenvalue (natural frequency) and eigenvector (mode shape) will satisfy the following formula:

$$K_i = \omega_i^2 M_i$$

Substituting the above equality into Formula 19-13 and dividing through by mass yields:

$$\ddot{\eta}_i + 2\zeta_i\omega_i\dot{\eta}_i + \omega_i^2\eta_i = \frac{F_i}{M_i} \qquad \text{(Formula 19-14)}$$

This transformation is significant, since it has uncoupled the equations of motion into a set of linear independent differential equations that can be described by the modal parameters (modal frequency, modal damping and scaled mode shapes) of the structure. This transformation forms the basis for all modal analysis techniques.

2.2.2 Curve-Fitting Experimental Data

The analytical expressions used in commercially-available experimental modal analysis software to curve fit experimental data are based on a Laplace transformation of the equations of motion in the time domain to the frequency domain. The subspace iteration technique used in finite element analysis

to obtain the modal parameters of a finite element analysis model are also based on the transformed equations of motion.

An important assumption must be made when using the Laplace transformation of the equations of motion from the time domain to the frequency domain: that damping is viscous and proportional to stiffness and/or mass such that

$$[c] = \alpha[m]$$
$$= \beta[k]$$
$$= \alpha[m] + \beta[k]$$

It is usually difficult to determine which type of damping is present in a particular structure or machine component. The damping characteristics could be a combination of all types of damping. However, in most realizable structures, damping is small and you can make the simplifying assumption of proportional damping without significantly impacting the accuracy of the solution.

3.0 FREQUENCY RESPONSE FUNCTIONS

Experimental modal analysis techniques consist of obtaining modal parameter estimates by curve fitting experimentally-determined frequency response functions to an analytical expression. Therefore, it is imperative that you understand what a frequency response function consists of and how it is obtained.

3.1 Basic Frequency Response Functions

There are six basic frequency response functions. The first three are the measured ratio of the structure's response in acceleration, velocity, and displacement to a known excitation. These frequency response functions are

- Inertance = Acceleration / Force
- Mobility = Velocity / Force
- Compliance = Displacement / Force

The next three frequency response functions are simply the reciprocals of the first three. They represent the amount of force required to provide a response at a given frequency. These functions are usually referred to as:

- Dynamic Mass = Force / Acceleration
- Dynamic Impedance = Force / Velocity
- Dynamic Stiffness = Force / Displacement

The frequency response functions are the calculated ratio of output/input (or input/output). They provide a measured value of the magnitude of displacement, velocity, or acceleration per pound of force over the entire frequency range of concern.

A structure's dynamic response is magnified at frequencies near the structure's natural frequency, so that the displacement, velocity, or acceleration per pound of force increases rapidly. The structure's stiffness "appears" to decrease as the structure's response per pound of force increases. Thus, this frequency response function is sometimes referred to as **apparent stiffness**. Likewise, the structure's mass appears to decrease, so the Dynamic Mass frequency response function is sometimes referred to as **apparent mass**.

According to the derivation contained in Section 2.1, the response for a single degree of freedom damped vibrating system can be expressed as

$$x = Q\left(\frac{F}{k}\right) \qquad \text{(Formula 19-15)}$$

where

$$Q = f\ (\omega, \omega_n, c)$$

Formula 19-15 shows that the static stiffness ($k = F/x$) is a constant at all frequencies.

4.0 EXPERIMENTAL MODAL ANALYSIS PROCEDURES

The basic steps in a complete experimental modal analysis are:

1. Develop a test procedure
2. Perform experimental test
3. Identify modal parameters
4. Verify results

4.1 Develop a Test Procedure

There are three important components of a test procedure:

1. Define all the mode shapes in the frequency range of concern.
2. Choose an excitation method.
3. Identify data point locations.

The most common methods used to excite a structure's natural frequencies are an instrumented force hammer or a tuneable electrodynamic sinusoidal shaker. Prior to multi-channel FFT analysis, the traditional method of excitation was the sinusoidal shaker. Multi-channel FFT analyzers allowed the measurement of response to broadband excitation (impact). Excitation methods are discussed in more detail later in this chapter.

You must define data point locations that will insure that all of the structure's mode shapes will be properly identified. If desired, perform a preliminary finite element analysis to obtain information relative to the number of natural frequency modes that can be expected and the approximate mode shape of each. This analysis identifies critical degrees of freedom that should be included in the modal model. It also provides insight into the number of different reference locations that might be required to experimentally define each mode shape.

The following guidelines on choosing data point locations will help you perform a successful experimental modal test:

1. The total number of data points must be sufficient to accurately define the mode shape. Using too few data points could lead to visual aliasing of the mode shape. For example, the second and fourth mode shapes of a simple beam are identical if you use only one data point at the center of the beam. In fact, these mode shapes would not even be detected since the displacement at the center of the beam is zero for both of these modes.
2. Locate extra data points at or near areas of concentrated mass.

3. If the structure's primary motion is in one direction, but distribution of stiffness and/or mass induces motion perpendicular to this direction, then define data points in both directions.

4.1.1 Data Point Locations

The curve-fitting process uses information obtained from the data points used in field testing to estimate the system's modal parameters. Essentially, the accuracy of the modal parameter estimation increases as more data points are used in the test. Also, you must use a sufficient number of data points to prevent either visual aliasing or totally missing a mode shape.

For example, consider the cantilevered beam in Figure 133. If you chose only one data point at the end of the beam, then the first mode shape would be the only possible mode shape that the EMA software could properly synthesize and display.

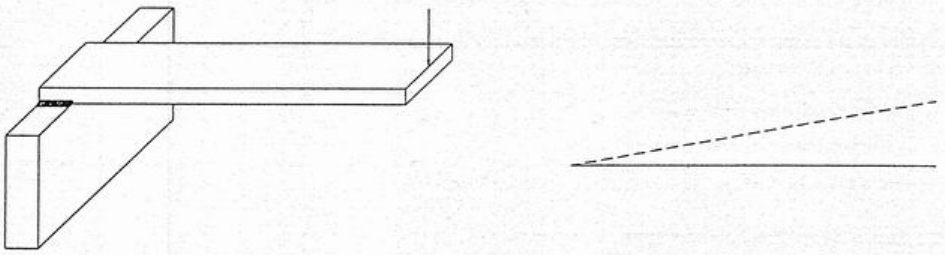


Figure 133 - *mode shape obtained from one data point*

However, if you define five equally-spaced data point locations along the length of the beam as shown in Figure 134, your software could synthesize and display several mode shapes.

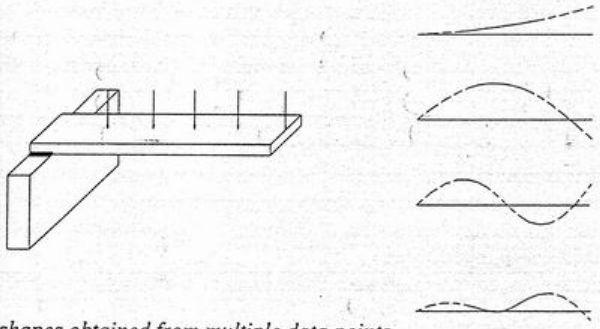


Figure 134 - *mode shapes obtained from multiple data points*

4.1.2 Visual Aliasing

Consider the third mode shape shown in Figure 134. Assume that you only took data at two locations: point #1 at the end of the beam, and point #2 at the beam's midpoint. If you only took data

at these two points, point #1 would show a displacement at third mode, but point #2 would not. This could lead to an erroneous mode shape display as illustrated in Figure 135.

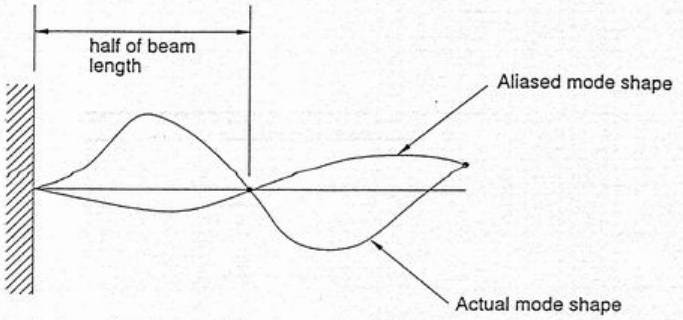


Figure 135 - *erroneous mode shape display resulting from aliasing*

The accuracy of the modal parameter estimation increases as more data points are selected. However, the time required to perform the field testing procedure also increases as the number of data points increases. You must obtain enough data to accurately estimate the modal parameters, yet you must minimize the amount of redundant data so that the test procedure does not become inefficient and too time-consuming. For structures and machines that have already been placed in service, production requirements generally limit the amount of time available to perform a test.

Clearly mark and identify all selected data point locations prior to performing the test. You must maintain accurate records, including the location of data points, the location of reference points, and the direction of the response transducer at each data location.

4.2 Field Test

You must excite the structure to obtain the required modal data. The most common excitation methods used to excite a structure's natural frequencies are sinusoidal excitation using a tuneable electrodynamic shaker, and broadband excitation (impact) using an instrumented force hammer.

The impact method is the most popular method for the experimental modal analysis of small and medium structures; it is sometimes referred to as the **transfer function method**. An impact (a broadband excitation) excites all of the structure's modes simultaneously. Both force input and dynamic structural response data can be obtained using a multi-channel FFT analyzer. The FFT transforms the data and mathematically manipulates the data to produce a frequency response function.

In all experimental modal analysis procedures, a series of frequency response functions are measured between a fixed reference point and other data points of interest on the structure. Since you must obtain the force and response data simultaneously in order to construct the frequency response function vs. frequency curve, a multi-channel FFT analyzer is required.

4.2.1 Broadband Excitation

The input force from impact is distributed over a broad band of frequencies; therefore it is also referred to as a broadband excitation method. Although the total energy imparted to the structure by the force hammer may be substantial, the input force at any frequency could be small. Figure 136

contains both the time domain plot of a typical impact and a frequency domain plot of the force resulting from the impact. The total impact force was approximately 1,650 lbs. However, the average force over the entire spectrum was only 8.7 lbs.

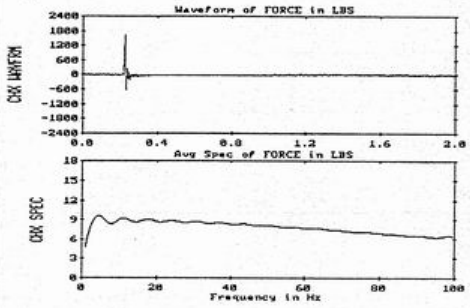


Figure 136 - *impact and response plots*

You must use the proper hammer tip to ensure that force energy will be provided over the entire frequency range of concern. Figure 137 illustrates the effect of various tip hardnesses on the frequency band that will be excited. Generally, the harder the tip, the higher the frequency range that will be excited and the lower the average force will be over that frequency range.

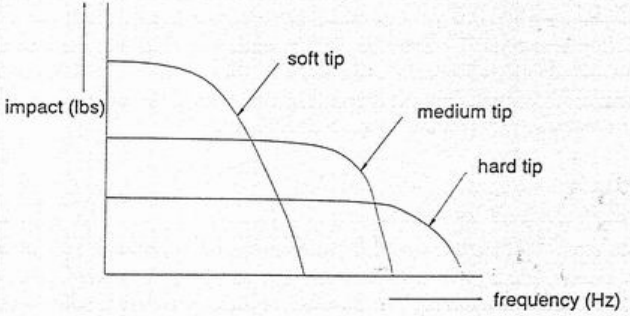


Figure 137 - *effect of tip hardness on excited frequency band*

The impact method has several major advantages:

- It is a simple test.
- It is relatively easy to implement.
- It excites the structure over a wide range of frequencies.
- The transducer types attached to the structure can be chosen so that the mass loading is insignificant.

The major disadvantages of the impact method include:

- Complex structures with closely coupled modes do not lend themselves to this approach

- The input energy is spread across the entire frequency range, so that the energy at any particular frequency may not be sufficient to excite the natural frequency. This is particularly true for large structures with low frequency modes.

Force-Exponential Window

To reduce errors due to noise and leakage, define a force-exponential window when obtaining modal data using an impact hammer. The force window is set up for the channel measuring the impact and the exponential window is set up for the channel measuring the response.

Figure 138 shows a time domain plot of an impact and the resultant structural response, sometimes referred to as the "ringdown". A force window is superimposed on the time domain of the impact, and an exponential window is superimposed on the time domain of the ringdown response.

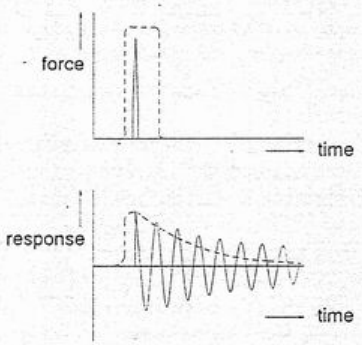


Figure 138 - *a force-exponential window*

Design the force window to enclose the entire impact, and to ensure that all points outside the window read zero. Design the exponential window so that it is zero before the ringdown, increases to 100% immediately before the ringdown, and exponentially decays to nearly zero at the end of the window.

Coherence

Coherence ia a measure of the linear relationship between input and output. It is a good indicator of the presence of noise in the input or output signals as well as non-linearities in the machine structure.

The presence of noise in the data, or of non-linearities such as cracks and non-linear damping at bolted joints, can adversely affect the accuracy of the frequency response function. Since the experimental modal model uses the measured frequency response functions to estimate the modal parameters, noise in the data also affects the accuracy of the experimental modal analysis modelling techniques. Therefore, you must check the coherence function prior to accepting the frequency response function data as being valid.

Figure 139(a) contains a coherence plot measured during an impact test of a machine component that had a cracked casting. Figure 139(b) shows the coherence after the casting had been replaced. Obviously, the first set of data would not have provided good frequency response function data for the experimental modal model.

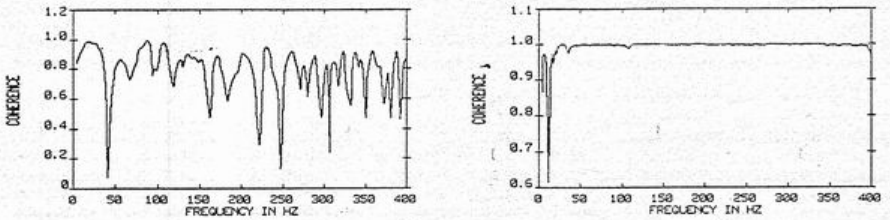


Figure 139 - *coherence before and after repairs*

The coherence function is an average function; the coherence of the frequency response function obtained for the first impact will be unity over the entire frequency range. Therefore, you must obtain enough averages at each data point to ensure that the coherence value is valid.

Maxwell's Reciprocity

One of the basic assumptions of experimental modal analysis is Maxwell's **reciprocity**: that the structure exhibits both elastic and inertial symmetry. In other words, a linear relationship exists between the applied force and resultant response, such that motion at one data point caused by a force at a second data point is equal to motion at the second point caused by the force at the first point.

This is a significant assumption, since it minimizes the amount of data that must be obtained during the test. You can move the force hammer between data points while keeping the response transducer stationary, or vice versa depending upon which is more convenient.

Maxwell's reciprocity assumes that the structure behaves in a linear-elastic fashion. Non-linearities adversely affect the accuracy of the experimental modal model.

4.2.2 Sinusoidal Excitation

An electrodynamic excitor inputs energy to the structure at one location, and the response transducer is used to rove to all of the other data points.

The main advantage of this method is that all of the energy is concentrated at a single frequency. The main disadvantages include the mass loading imposed on the structure by the excitor, and the relative complexity of the test setup.

4.3 Modal Parameter Identification

The modal parameters derived from experimental data are:

- Modal frequency
- Modal damping
- Mode shape

Commercially-available experimental modal analysis software packages use analytical curve-fitting techniques to extract these parameters from experimentally-measured frequency response functions. The frequency response functions measured during the field test are transferred from the FFT analyzer to the experimental modal analysis software for processing. An analytical expression for the frequency response function is fit to the experimental data, thus identifying the modal parameters.

The curve-fitting algorithms are based on the solution to a set of linear differential equations. Decoupling of the differential equations is accomplished in a manner similar to that presented in Section 2.2.

4.3.1 Curve Fitting

Most experimental modal analysis software packages require that you define two parameters:

1. The number of modes or the frequency range to consider
2. The curve fitting method to use

Curve fitting algorithms usually fall into one of two groups: single degree of freedom methods and multiple degree of freedom methods. Single degree of freedom curve fitting methods include the Coincident Peak Method, Quadrature Peak Method, Complex Peak Method, and the Polynomial Method. A Single degree of freedom curve fitting method is recommended only for structures with well-defined and lightly coupled modes of natural frequency.

The measured frequency response function is a sum of the resonance curves for each mode of vibration in the structure. An adjacent mode can affect the magnitude of response at the frequency of consideration (this is referred to as **modal coupling**). The amount of modal coupling is dependent upon the amount of damping and the frequency separation between modes.

Figure 140 illustrates both light modal coupling and heavy modal coupling. A single degree of freedom curve fitting method would suffice for the lightly coupled example. A multiple degree of freedom curve fitting method would be necessary to accurately calculate the modal parameters for the heavily coupled example.

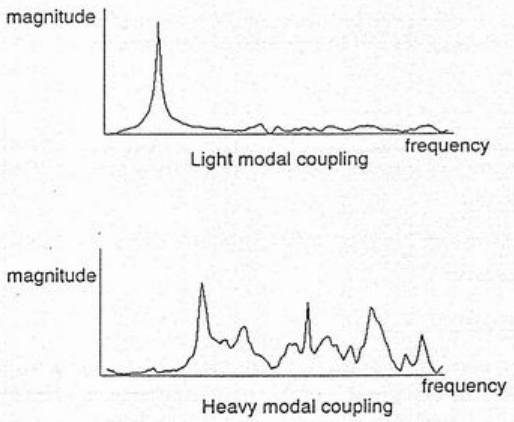


Figure 140 - *light and heavy modal coupling*

Most experimental modal analysis software packages contain curve fitting methods for both single mode (lightly coupled) and multiple mode (heavy damping, closely spaced) curve fitting. A single degree of freedom curve fitting method does not require that only one mode be present in the data. Rather, it requires that the modes be spaced such that the effect of coupling is insignificant.

Rational Fraction Least Squares Method

The Rational Fraction Least Squares (RFLS) method is a popular polynomial method that can be used for both single mode (single degree of freedom) and multiple mode (Multiple degree of freedom) curve fitting. This method consists of fitting a polynomial function, in rational fraction form, to the measured frequency response function data. This curve fitting model takes the form:

$$H(j\omega) = \frac{(r2_k\sigma_k + r1_k\omega_k + jr2_k\omega)}{(\sigma_k^2 + \omega_k^2 - \omega^2 + 2j\sigma_k\omega)} + A_0 + A_1(j\omega) + A_2(-\omega^2) \qquad \text{(Formula 19-16)}$$

where

A_0, A_1, A_2 = coefficients
σ_k = modal damping of the k^{th} mode
ω_k = modal frequency of the k^{th} mode
$r_k = r1_k + j(r2_k)$ = complex residue of the k^{th} mode

Formula 19-16 is derived from a Laplace transformation of the equations of motion (time domain) into the frequency domain. Since the peak frequency (ω) and the frequency response functions are available from the test data, estimates of the modal damping, modal frequency, and complex residue can be obtained by curve fitting the experimental data to this expression using a least squares method.

The RFLS Method fits extra polynomial terms (A_0, A_1, A_2) to the data to compensate for the effects of modes which are located outside of the frequency band of consideration. This eliminates the possibility of modal parameter error due to off-frequency modes. Many other curve fitting methods are available with experimental modal analysis software packages. Refer to the technical literature for more information on the theory and background of these methods.

4.4 Model Verification

You should verify the modal model prior to using it for any analysis such as structural dynamic modification analysis or forced response analysis. There are several verification procedures:

1. Mode shape display
2. Comparison of measured and synthesized frequency response functions
3. Finite element analysis

4.4.1 Mode Shape Display

Displaying the mode shape determined by the curve-fitting routine is the first method you should use to check the validity of the modal model. Almost all experimental modal analysis software packages can graphically animate the mode shape. A mode shape display is very useful for finding errors in measurement or curve fitting results. Seemingly arbitrary phase angles in the animated display can indicate that you have incorrectly identified transducer response directions.

4.4.2 Comparison of Frequency Response Functions

Another method of checking the validity of the modal model is to compare a synthesized frequency response function (developed using the calculated modal parameters) with the actual frequency

response function obtained from the test data. Discrepancies between the synthesized and actual frequency response function can be caused by:

- Non-linearities in the structure affecting the accuracy of the program's analytical expression. Remember that the coordinate system transformations used to uncouple the differential equations assumed that the system was linear and elastic. Also, your test procedure was developed assuming structural reciprocity, which is not strictly valid for a nonlinear structure.
- An incorrect curve-fitting algorithm resulting in an inaccurate assessment of the modal parameters. Closely-spaced mode shapes require multiple degree of freedom curve fitting methods. Also, solution accuracy can be affected if you fail to properly consider residual terms outside of the frequency range of concern.
- Incorrectly defining the structure's geometry. Defining the structure's geometry within the experimental modal analysis program is similar to node and element definitions in finite element analysis (see the Finite Element Analysis chapter for more information). The experimental modal analysis software cannot detect if you incorrectly defined the structure's geometry. If you define the structure's geometry incorrectly, then the modal parameters calculated by the curve-fitting procedure could contain significant error.
- Errors in the identification of transducer direction affecting the accuracy of the modal parameter estimation and the synthesis of the frequency response function. You must define the direction of the response transducers. Many of these errors are easily spotted by visual inspection of the animated mode shape. However, the error could be subtle enough to escape detection during the mode shape animation.
- Bad experimental data. This is the most obvious reason for a discrepancy between the synthesized and measured frequency response function. This could be due to noise, leakage, or the collection of data at poorly-defined locations.

4.4.3 Finite Element Analysis

The finite element method is a numerical technique used to estimate a structure's modal parameters using a mathematical model instead of experimental data. Finite element models generally contain more degrees of freedom than experimental models, making it possible to describe a structure in greater detail.

Finite element analysis can be used to verify the results of an experimental modal analysis. Conversely, experimental modal analysis can be used to verify a finite element analysis. Since these methods represent two totally different approaches to modal parameter estimation, it is reasonable to assume that the modal model developed by both techniques is valid if the results of the experimental model is confirmed by the finite element analysis.

Additional information regarding finite element analysis as it pertains to modal analysis is included in the Finite Element Analysis chapter.

5.0 STRUCTURAL MODIFICATION METHODS

Most experimental modal analysis software packages include a subroutine that estimates the changes in the dynamic properties of a structure due to modifications in the structure's mass, stiffness, and/or damping properties. This procedure is referred to as **structural dynamics modification.**

Because the number of degrees of freedom typically used to define the experimental modal model are limited due to time and accessibility factors, it is sometimes difficult to estimate the magnitude of stiffness that a modification adds to the modal model at a particular degree of freedom. This is especially true for complex stiffened shell structures. For these cases, you should consider the calculated response to structural modification obtained from a structural dynamics modification routine approximate at best.

The finite element method is recommended for evaluating the effect of structural modifications on the dynamic characteristics of complex structures and machines.

6.0 CASE HISTORY #1

Experimental modal analysis can be a valuable engineering tool. It can be used to identify not only the magnitude of the natural frequencies of a structure or machine, but it can be used to estimate other modal parameters such as damping and mode shape. Once a dynamic modal model has been developed based on experimental data, you can use structural dynamics modification techniques within experimental modal analysis software packages to evaluate the effect that proposed modifications might have on the dynamic characteristics of a structure.

However, these techniques are usually limited to defining a change in stiffness between two data points, or a change in mass at a single data point. Quantifying the stiffness for all degrees of freedom at that data point could be difficult if the structure is complex.

This case history is an example of a problem that required both experimental modal analysis and finite element analysis techniques to:

- accurately define the magnitude of natural frequencies and associated mode shapes
- identify the source of excitation
- evaluate design modifications to minimize the structure's dynamic response to the excitation

6.1 Statement of Problem

A large chain-driven rotary breaker used to grind slag at an aluminum producing facility had been experiencing premature failure of roller elements due to excessive vibration. Preliminary analysis of the vibration signal identified that the pattern worn into the roller elements corresponded to vibrations at a frequency near the mesh frequency of the chain drive.

The authors suspected a resonant condition in the structure that supported the rotary breaker. A total dynamic systems analysis approach was required, including both experimental and numerical modal techniques, since vibration signal analysis was not sufficient, by itself, to quantify the problem and evaluate the probable success of potential modifications.

6.2 Experimental Modal Analysis

The authors used the impact method to experimentally determine the modal parameters of the structural system consisting of the rotary breaker, storage bins, dust covers, and supporting framework. Due to the size of the instrumented impact hammer (a 50,000 pound force hammer) required to excite the 78,000 pound (empty) rotary breaker and the massive structural steel framework, it was convenient to use the impact point as the stationary reference point and move the response transducer to all data points of interest.

Figure 141 contains a typical frequency response curve obtained during the test. The first three natural frequency modes were identified as 5.0 Hz, 13.5 Hz, and 24.0 Hz. The 13.5 Hz natural frequency corresponded to 1*RPM sprocket tooth modulation of the sprocket-chain mesh frequency. The mesh frequency was approximately 14.8 Hz and the sprocket shaft frequency was 1.2 Hz, resulting in a principal dynamic force at 14.8 Hz with sidebands at 13.6 Hz and 16.0 Hz.

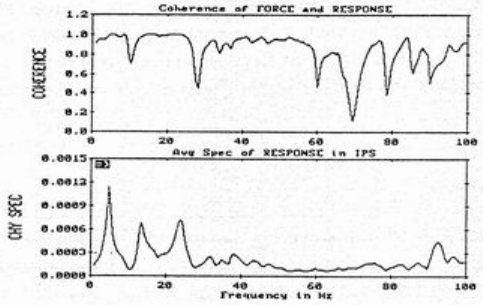


Figure 141 - *frequency response curve for Case 1*

The experimental data was input to a computerized experimental modal analysis software program, which analyzed, curve-fit, and displayed the mode shapes. The first mode shape at 5.0 Hz consisted of a simple in-phase flexural pattern of the structural columns. The second mode shape at 13.5 Hz is illustrated in Figure 142 and consisted of a torsion mode where one column line was 180 degrees out of phase with the other.

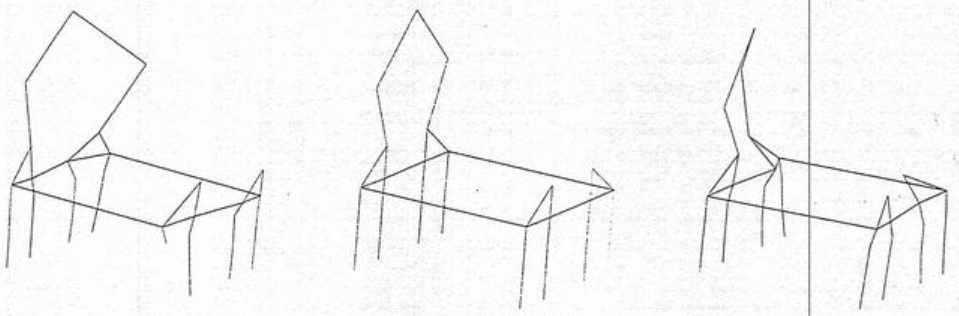


Figure 142 - *second mode shape for Case 1*

Although this second mode shape was not the principal mode of the structure, it was the major contributing factor to the excessive vibrations since the magnitude of the natural frequency associated with this mode was near the mesh frequency of the chain drive.

The modification objective was to increase the magnitude of the natural frequency of the structure's second mode, and to decrease the rotational speed of the crusher so that the excitation did not occur

within the dynamic magnification range associated with the natural frequency. Also, the modification should not increase the magnitude of the first mode so that it would be sensitive to the excitation source.

The mode shapes generated by the experimental modal analysis indicated that the second mode contained a considerable amount of torsional warping of the rather deep, flexible beams supporting the rotary breaker. Due to the large degree of elastic coupling associated with these beams, it was difficult to accurately define the stiffness between data points in the experimental modal model, so the structural modification techniques in the experimental modal analysis software were not able to provide an accurate assessment of the effect of any modification on the dynamic characteristics of the complete structural system. Therefore, the finite element method was used to evaluate potential structural modifications.

6.3 Finite Element Model

The finite element method is described in more detail in the Finite Element Analysis chapter. Refer to this chapter for additional information on implementing this method for modal analysis.

Figure 143 illustrates the finite element analysis model developed for this analysis. The rotary breaker is included since it provides both stiffness and mass to the structural system. Other components such as dust covers and bins, which provided mass but not stiffness, were not modelled. The mass of these elements were lumped into the finite element analysis model at appropriate locations.

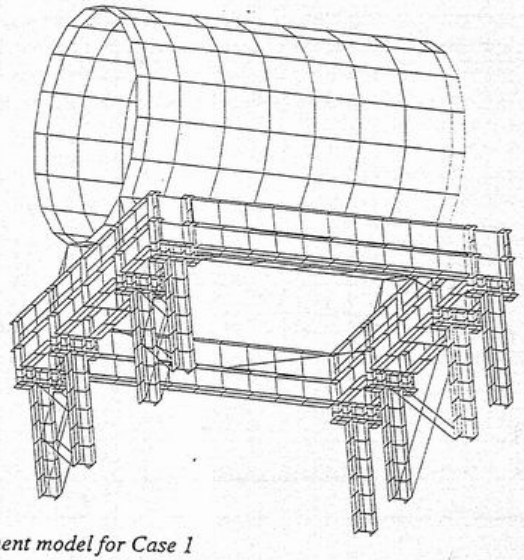


Figure 143 - *finite element model for Case 1*

The beams, columns and stiffeners were modelled with three-dimensional thin shell elements, instead of beam type elements, in order to accurately describe the torsional properties of the structural system.

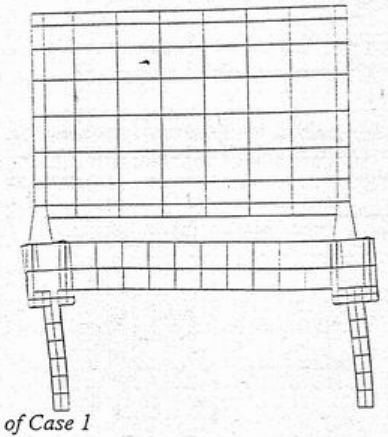
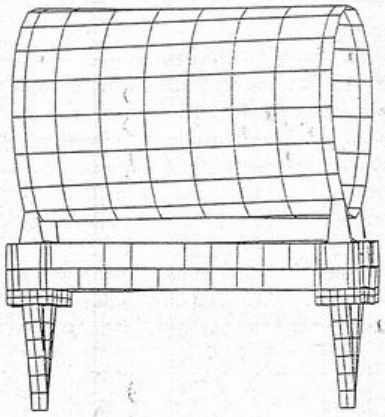


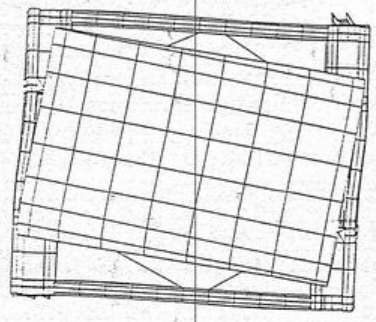
Figure 144 - *first mode shape of Case 1*

The first natural frequency mode predicted by the finite element analysis was at 4.91 Hz. The mode shape is shown in Figure 144. Both the magnitude of the natural frequency and the mode shape were consistent with the animations developed by the experimental modal analysis.

The finite element analysis procedure estimated the magnitude of the second mode natural frequency as 13.5 Hz. Figure 145 illustrates both the side and top views of this mode shape. Once again, both the magnitude of the natural frequency and the mode shape were consistent with the results of the experimental modal analysis.



front view of second mode shape - 13.5 Hz



top view of second mode shape - 13.5 Hz

Figure 145 - *side and top views of second mode shape of Case 1*

6.4 Structural Modifications

The first modification investigated consisted of adding cross bracing to the columns, to increase both the lateral and torsional stiffness of the frame. This modification would impact both of the first two modes of natural frequency.

The modification was evaluated using both finite element analysis and the structural dynamics modification subroutine of the experimental modal analysis software. The predicted results for the natural frequency of the modified structure are summarized in Table 9.

MODE	FEM PREDICTION	EMA PREDICTION
1	7.57 Hz	7.30 Hz
2	14.40 Hz	16.90 Hz

Table 9 - *comparing FEM and EMA predictions*

Both the experimental modal analysis and finite element analysis predicted similar results for the magnitude of the modified structure's first natural frequency. These predictions are close since the mode shape associated with this natural frequency is rather simple, and requires only a relatively small number of degrees of freedom to properly describe it. Therefore, for this mode shape, the experimental model provides a good basis for structural dynamic modification evaluation. The additional refinement from the added degrees of freedom of the finite element analysis model do not provide any greater degree of accuracy.

However, there is a significant difference between the finite element analysis and experimental modal analysis predictions for the magnitude of the natural frequency of the second mode shape. This mode shape consisted of not only torsional racking of the frame, but also of twisting of the main support beams. The additional refinement of the finite element analysis model was necessary to calculate the effect of the modification on the natural frequency.

The significance of this case history is that, if the cross bracing were just arbitrarily added to the structure without performing the complete modal analysis, the new natural frequency of the second mode would have been approximately 14.40 Hz. This new natural frequency would have been very close to the sprocket-chain meshing frequency. Since the magnitude of the dynamic force associated with the meshing frequency is greater than the modulated sideband force, the forcing function exciting this natural frequency would have been greater than that which excited the second mode of the unmodified structure. Therefore, the levels of vibration probably would have increased rather than decreased.

The finite element analysis model was then used to evaluate additional modifications which increased the natural frequency beyond the frequency of the dynamic force. Together, with a slight reduction in the rotational speed of the drum, this modification provided a successful solution to the vibration problem.

7.0 CASE HISTORY #2

This case history involves the analysis of a large axial fan. The operating speed of the fan was approximately 1,780 RPM (29.7 Hz). The outside diameter of the fan impeller was 66 inches. The fan was powered by a 300 HP motor. Both the fan and motor were located inside of, and were

supported by, a circular section of ductwork. The ductwork was supported by a frame constructed of welded structural steel.

7.1 Statement of Problem

The vibration levels at the motor bearings of an air recirculating fan at a power generation plant had been excessive since being placed into service. The high vibrations occurred at 1*RPM. Imbalance had first been assumed to be the source of the dynamic forcing function causing the vibration. However, field balancing provided only a modest improvement in the vibration level.

Preliminary impact data indicated the probability that a system natural frequency existed near the operating speed of the fan. An experimental modal analysis was performed to identify the modal parameters, and to assist in the design of modifications intended to change the natural frequency of the fan and its supporting structure.

7.2 Experimental Modal Analysis

The authors used the impact method to experimentally determine the natural frequency, damping, and mode shape of all resonance modes near the operating speed (or at multiples of operating speed) of the equipment.

Figure 146 contains coherence and frequency response curves obtained during the test. Test data indicated that two modes of vibration existed in the frequency range of concern. Structural resonance

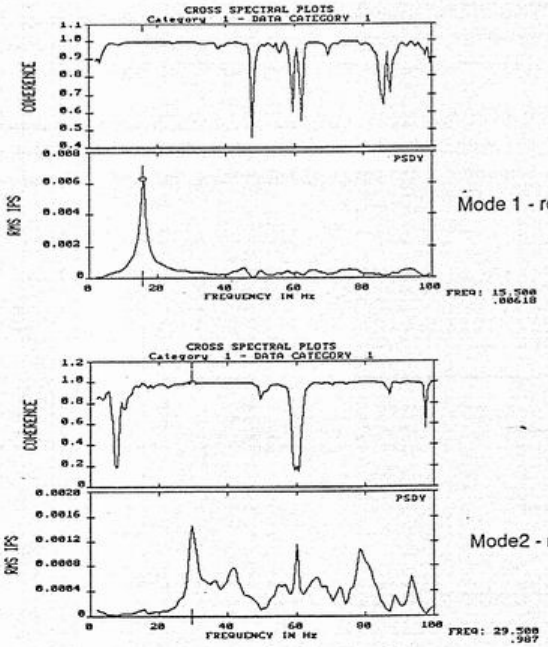


Figure 146 - *coherence and frequency response plots for Case 2*

was identified at around 15.5 Hz and 29.5 Hz; coherence was nearly 100% at both of these frequencies. This indicated that the measured response was due to the impact and not background noise.

Then, the authors used experimental modal analysis software to analyze, curve fit, and animate the experimental data. The modal parameter estimates for natural frequency and damping that the software provided are shown in Table 10.

MODE	NATURAL FREQUENCY	DAMPING
1	15.7 Hz	4.40 %
2	29.8 Hz	3.20 %

Table 10 - *natural frequency and damping estimates for Case 2*

The magnitude of damping predicted by the analysis is consistent with damping values typically seen in systems with this type of construction. This consistency provided one check of the validity of the analysis.

Since the fan operated at around 29.7 Hz, and a natural frequency occurred at around 29.8 Hz, even an extremely small imbalanced condition would cause excessive vibration levels due to the dynamic magnification of the structural response. This explained why field balancing was unable to effectively reduce the levels of vibration.

The objective of any modification would be to change the natural frequency of the second mode, without significantly affecting the magnitude of the first mode natural frequency. If the modification were not properly defined, the first mode could increase dramatically and effect the dynamic response of the structure. There are many examples in industry where solving one vibration problem creates another.

Information on the first mode shape to be modified is essential if any modification is to be properly assessed. Figure 147 shows the animated mode shape of the first mode (15.72 Hz). It consists of a simple horizontal rocking motion of the structure supporting the ductwork and motor. This mode shape shows a great deal of motion in the base.

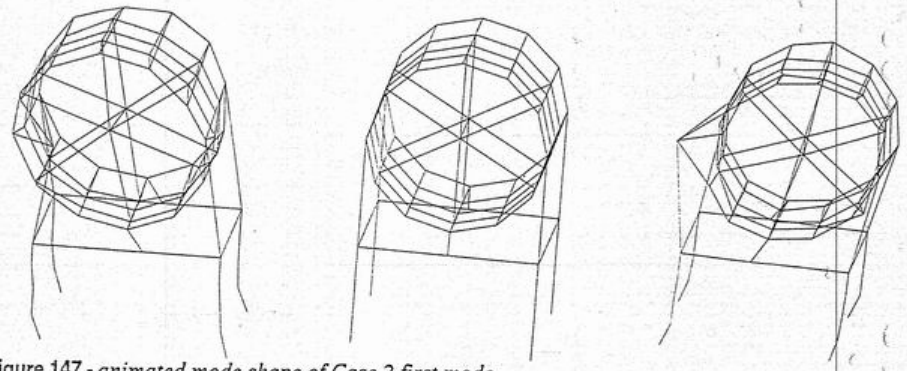


Figure 147 - *animated mode shape of Case 2 first mode*

Figure 148 shows an end view of the animated shape for the second natural frequency mode. The distortion of the ductwork is predominant in this mode.

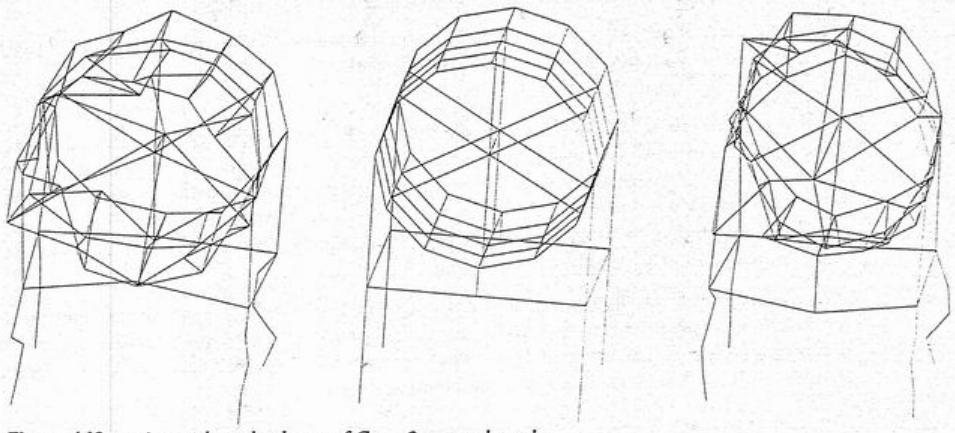


Figure 148 - *animated mode shape of Case 2 second mode*

In order to change the magnitude of the second natural frequency mode without significantly affecting the first mode, the structural modification concentrated on increasing the ductwork stiffness instead of that of the supporting frame. Both axial and circumferential stiffeners were added to the circular ductwork. The thickness of the motor mounts was also increased.

8.0 OPERATING DEFLECTION SHAPE ANALYSIS

Operating deflection shape analysis is a multi-channel testing procedure used to define the system structural deformation pattern resulting from a dynamic forcing function at a specific frequency. The same commercially-available experimental modal analysis software can be used to curve-fit and animate the operating deflection shape.

An operating deflection shape analysis, by itself, does not provide enough information to determine a structure's modal parameters. The operating deflection shape analysis produces a display of the relative structural deformation of a system resulting from an actual forcing function. If the frequency of the forcing function coincides with a natural frequency of the supporting structure or of a machine component, then the operating deflection shape provides a good estimate of that particular mode shape. If the forcing function frequency does not coincide with any natural frequency, then the operating deflection shape is merely a display of the normal structural response of the system and not a mode shape.

The major advantages of operating deflection shape analysis are as follows:

- The test is performed while the equipment is operating. This is a major advantage in industrial facilities that operate around the clock, with only limited downtime for scheduled maintenance.
- The test is much quicker to perform than an experimental modal analysis test.

- The operating deflection shape can be used to identify areas of structural weakness or looseness, which could be the cause of excessive vibration (even in cases where a natural frequency is not involved).

The major disadvantages of operating deflection shape analysis are as follows:

- The test does not necessarily ensure that a natural frequency is being excited. Additional testing techniques are required to verify that a natural frequency exists at (or near) the equipment's operating frequency.
- Once you have identified the existence of a natural frequency using other means, operating deflection shape analysis provides a good estimate of the mode shape. However, the operating deflection shape analysis may not provide information regarding the existence of other mode shapes which may be in close proximity to the forcing function. This can lead to the development of structural modifications that may favorably affect one natural frequency mode but adversely affect another.
- The operating deflection shape data is not scaled properly for either structural dynamics modification analysis or forced response simulation analysis. You cannot use the operating deflection shape model to investigate what effect adding (or subtracting) mass and/or stiffness will have on natural frequency and mode shape.

8.1 Test Procedure

The test procedure for operating deflection shape analysis is similar to multi-channel testing techniques used for experimental modal analysis. The main difference is that the equipment provides the excitation for the response to be measured, instead of a force hammer or shaker. The test is performed while the equipment is operating, as opposed to the experimental modal test which is performed when the machine is shut off.

Figure 149 shows the typical operating deflection shape setup. Use two transducers to obtain relative phase between data points:

1. a reference transducer that remains at a single location throughout the test.
2. a second, roving transducer to take data at all other data points.

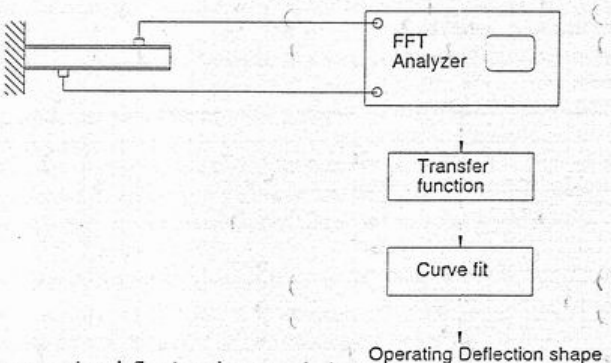


Figure 149 - *setup for operating deflection shape analysis*

Use an FFT analyzer to calculate a "pseudo" frequency response function, which is actually a measure of transmissibility. Pseudo frequency response function units are g's/g's (acceleration), ips/ips (velocity), or mil/mil (displacement).

Figure 150 contains the pseudo frequency response function for a fan operating at 1,800 RPM. This frequency response function contains many "junk" peaks, which are a result of dividing the noise floor at the response transducer by the noise floor at the reference transducer.

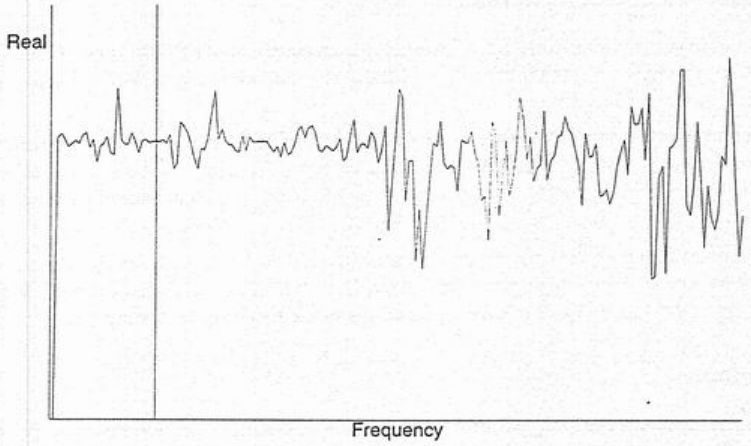


Figure 150 - *pseudo FRF for an 1,800 RPM fan*

This preponderance of junk peaks is the reason you should restrict curve fitting to the forcing function frequency when you want to obtain an animated operating deflection shape.

Place the reference transducer near a location of peak response. Do not place the reference transducer near a modal inflection (antiresonance) point, nor at a point where the response is low. If you place the reference transducer at a point of low response, the resulting pseudo frequency response function will be very large since you are dividing by a number close to zero. The magnitude of this pseudo frequency response function could exceed the dynamic range of your analyzer!

8.2 Curve Fitting

Since the pseudo frequency response function contains so many junk peaks associated with the noise floor, restrict curve fitting to single degree of freedom techniques. You should restrict all curve fitting to the frequency at which the force is applied. For example, if you need to find the system response to imbalance, restrict curve fitting to 1*RPM. Likewise, restrict curve fitting to 2*RPM to obtain response from misalignment. Never use polynomial curve fitting over a band of frequencies.

In most cases, you can obtain the real part of the operating deflection shape with a coincident curve fitting technique. This provides the normalized operating deflection shape, since relative motion between data points is either in phase (0 degrees) or 180 degrees out of phase. You can use a complex curve fitting technique to provide the actual phase angles of the response relative to the reference transducer. Animating the complex operating deflection shape sometimes appears as if the structure is waving or swimming like a fish.

In some cases, you can obtain the imaginary part of the operating deflection shape using a quadrature curve fitting technique. This technique provides structural response information when the load is 90 degrees out of phase with the reference transducer. For example, you can obtain an operating deflection shape for the response of a structure to the vertical component of a rotating imbalance, even though the reference transducer is placed horizontally.

8.3 Single-Channel Operating Deflection Shape

You can develop an operating deflection shape using a single-channel FFT analyzer. Obtain phase from a tach pulse instead of a reference transducer. Single-channel operating deflection shape analysis has several disadvantages, including:

- Using a tach pulse as a reference bases the operating deflection shape on measured peak responses rather than a normalized frequency response function. If the system response is not stationary with respect to time, an operating deflection shape analysis will provide erratic results.
- You cannot interpret the operating deflection shape data for any frequency other than the rotational speed from which you obtain the tach pulse. Therefore, you cannot investigate the system's response to other forcing functions without additional testing.

8.4 Case History

A large overhung centrifugal fan had been experiencing excessive vibration levels (1.08 in./sec peak to peak). The fan was removed from service for a short period of time and impacted with an instrumented force hammer. Data was collected using a multi-channel FFT analyzer. Figure 151 shows the frequency spectrum of the response, and the coherence between the force input and the measured response.

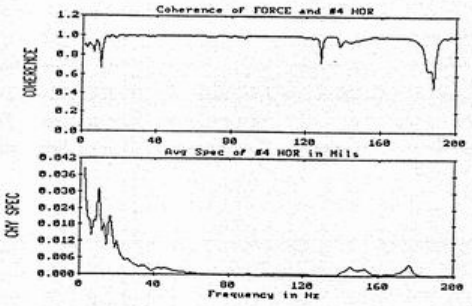


Figure 151 - *coherence and frequency response plots for ODS Case 1*

Several spectral peaks were measured. Background noise from adjacent equipment dominated the system below 12 Hz, where coherence was less than 66%. However, a spectral peak existed around 16 Hz, which is close to the fan's operating speed. Coherence of .995 indicated that this peak was quite probably associated with one of the system's natural frequencies.

It was not possible to remove the fan and adjacent equipment from service for the time required to perform a complete experimental modal analysis. Therefore, the authors decided to obtain the mode shape by performing an operating deflection shape analysis on the fan, the support pedestal, and the foundation during operation. This was possible since the frequency of the dynamic force was close to the natural frequency.

Figure 152 shows the result of the operating deflection shape analysis. The best description of the mode shape is that the foundation and pedestal were rocking as a rigid body on top of the soil.

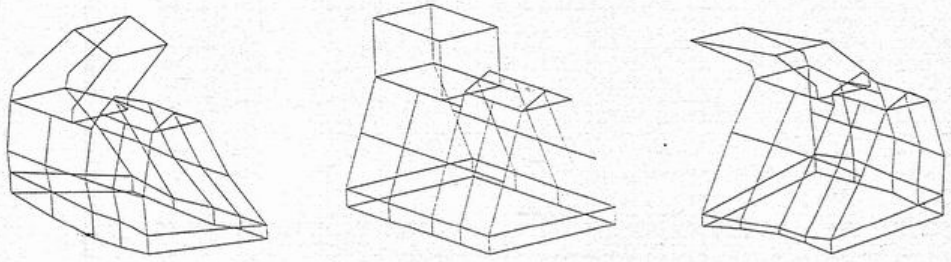


Figure 152 - *result of ODS analysis*

The operating deflection shape indicated that the soil/foundation interface needed to be modified. The mode shape also indicated that the motor was 180 degrees out of phase with the fan, pedestal, and foundation.

The authors used an experimental modal analysis during a plant shutdown to verify the mode shape predicted by the operating deflection shape. However, the experimental modal analysis identified another mode shape at 10.7 Hz, consisting of the rigid rocking of all components in phase with one another.

This case history illustrates both a major advantage and disadvantage of operating deflection shape analysis:

- If the forcing function occurs at (or near) the structure's natural frequency, you can obtain the mode shape while the system is running.
- The operating deflection shape analysis, however, does not guarantee that other mode shapes exist in the frequency range of concern.

If the foundation had been stiffened in an attempt to move the natural frequency away from the forcing function produced by the fan, other natural frequencies that had not been excited before might have been increased until they coincided with the forcing function frequency. This would have resulted in little change in system response; the vibration levels could have increased.

20 FINITE ELEMENT ANALYSIS

by Robert J. Sayer, P.E.

The finite element method (FEM) is a numerical analysis technique that can be used to provide approximate solutions to a broad range of engineering problems. The technique can be employed to solve complex problems in stress analysis, vibration analysis, fluid mechanics, and heat transfer analysis of both linear and non-linear systems subjected to steady state or transient loading conditions. The finite element method is normally used when geometric and/or material complexities prevent you from obtaining a "closed form" analytical solution.

In the field of vibration engineering, finite element analysis can be separated into the following two basic categories:

1. The analysis and design of systems that have not yet been built or installed.
2. The analysis of systems that are in operation.

For the former case, expensive prototype testing or finite element analysis are the only techniques available to evaluate the dynamic characteristics of complex structural systems. Even in cases where a prototype is required by customer or agency specification, finite element analysis can evaluate the design prior to constructing the prototype. This reduces the number of prototypes required to justify the design.

You can perform a vibration analysis of systems that are in operation by either experimental modal analysis or finite element (numerical modal) analysis techniques. In some cases, it is desirable to analyze the problem using both experimental and numerical techniques.

Many references are available which address the theoretical development of the finite element method. It is beyond the scope of this chapter to present an in-depth development of the theory. However, you should possess a general understanding of finite element theory before considering

using the method to solve a vibration problem. Therefore, a brief description of basic finite element theory is included in this chapter. Refer to the references [1], [2], [3] for further information.

Many references pertaining to applied finite element analysis are also available. These are usually available in the form of software manuals, journals of applied analysis, conference proceedings and technical magazines. As in the case of theoretical finite element analysis, it is beyond the scope of this chapter to present in-depth instruction in applied finite element analysis. However, a brief description of the procedures required to prepare a finite element analysis are presented to provide a general understanding of the practical application of the technique. Examples and case histories are included to provide a greater insight into the method.

1.0 HISTORY

The concepts and procedures currently employed in modern finite element analysis were first used in 1943 by Courant [4] to approximate the St. Venant torsion problem with an assemblage of triangular elements. Courant used a linear warping function for each element and formulated the problem using the principle of minimum potential energy.

Clough [5] was the first person to introduce the term "finite element" in a paper published in 1960. During the following decade, many of the principles included in modern finite element analysis computer packages were developed. For instance, static and kinematic matrix reduction techniques were developed by Guyan [6] in 1965 and an efficient frontal solution technique was developed by Irons [7] in 1970. Also during this period, NASA funded the development of a general-purpose finite element computer program (NASTRAN), capable of analyzing the complex structures and loading conditions that are common in the space industry.

By the early 1970's, several large general-purpose finite element computer programs were commercially available. Also, comprehensive textbooks on the subject began to be published during this time period.

The capabilities of commercially available finite element analysis software continued to expand through the remainder of the 1970's to the present time. Finite element codes continue to incorporate the latest developments in higher order isoparametric elements, numerical integration techniques, error norm approximation, enhanced graphics capabilities, and automatic adaptive mesh generation. Today many universities offer courses in both theoretical and applied finite element analysis.

2.0 PRINCIPLES OF FINITE ELEMENT ANALYSIS

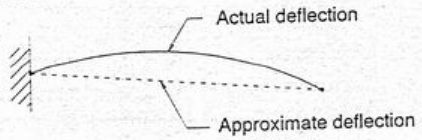
Finite element analysis focuses on **continuum problems**: calculating the behavior and response of a continuous surface or object. In a continuum problem, a field variable such as pressure, temperature, displacement, or velocity, contains an infinite number of possible values since it is a function of each and every point in the continuum. Since solving a continuum problem with an infinite number of points is impossible, a method must be used to simplify the calculation.

The finite element method simplifies the task of solving the continuum problem by dividing the continuum into a finite set of subdivisions called **elements**. The elements are interconnected at **nodal points** into an assembly called a **mesh**.

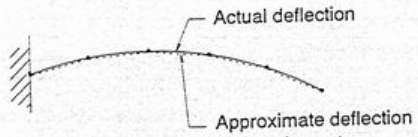
The field variable at any point within the element is defined in terms of its value at the node points. The value of the field variable at the nodal points is approximated using a set of assumed interpolation functions.

Since the finite element solution is based on a set of approximate functions, you can increase the accuracy of the method by refinement of this assumed function. For instance, if a quadratic displacement of a beam is to be modelled by a single element, then use a higher order quadratic interpolation function for the formulation of the beam element. A simple linear displacement function would not provide a good approximation to the actual problem.

Another method of increasing the accuracy of the solution is to increase the number of elements in the finite element mesh. You can closely approximate the actual displacement of the beam using a linear displacement interpolation function, provided you use multiple elements to discretize the beam. This is illustrated in Figure 153.



Approximate deflection using one element



Approximate deflection using five elements

Figure 153 - *approximating beam deflection*

As stated above, the most important factors which affect the accuracy of the solution are the finite element mesh chosen to discretize the continuum and the interpolation functions used to approximate the field variable. The interpolation functions for elements residing in the libraries of many popular finite element software packages are fixed and cannot be changed. Therefore, when using a finite element software package, the analyst can only increase the accuracy of the solution by refinement of the finite element mesh.

Any finite element analysis requires the following solution steps:

- Idealize the continuum as an assembly of elements.
- Establish the equilibrium requirements, in terms of field variables, for each element.
- Establish a set of simultaneous equations for the unknown field variables based on the element interconnection defined by the mesh.
- Solve the simultaneous equations, and calculate the element response using the element equilibrium requirements.
- Interpret the results, revise the model if necessary, and perform the analysis again.

If you are using finite element analysis, you must remember that discretization of a continuum involves approximation; an exact solution is *never* attained. A great deal of experience is required to determine the degree of mesh refinement necessary to provide accurate results and minimize computer resource requirements.

3.0 STATIC ANALYSIS

The basic principles for the solution of engineering problems using the finite element technique will first be demonstrated for the static analysis problem, then extended to the dynamic analysis problem. The static analysis problem can be stated, in matrix form, as

$$[K]\{u\} = \{F\} \qquad \text{(Formula 20-1)}$$

where

$[K]$ = the global stiffness matrix
$\{u\}$ = the vector of unknown displacements
$\{F\}$ = the applied force vector

The matrix equation in Formula 20-1 represents a set of simultaneous equations. The unknown displacement vector is obtained by inverting the stiffness matrix and multiplying both sides of the equation by this inverse, such that

$$\{u\} = [K]^{-1}\{F\} \qquad \text{(Formula 20-2)}$$

Knowledge of matrix algebra is a prerequisite of understanding finite element theory. Refer to a text on linear algebra for information pertaining to matrix inversion and other matrix manipulations.

The global stiffness matrix in the static analysis is assembled from the stiffness matrices for each individual element. As an example, consider a simple two-dimensional truss element with a linear displacement function. The stiffness matrix for the element is

$$[K] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \text{(Formula 20-3)}$$

Consider a finite element model consisting of two uniaxial truss elements of equal length, cross section area and material properties, coupled in series as illustrated in Figure 154.

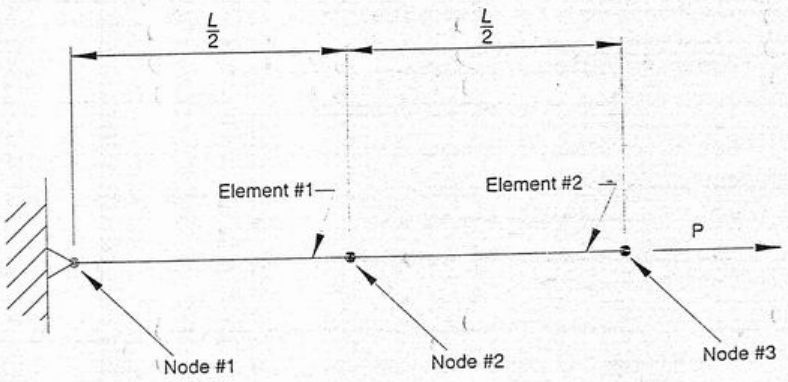


Figure 154 - *two uniaxial truss elements coupled in series*

The individual stiffness matrices would be

$$[K]_1\{u\} = \frac{2AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} \qquad \text{(Formula 20-4)}$$

$$[K]_2\{u\} = \frac{2AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} x_2 \\ x_3 \end{Bmatrix} \qquad \text{(Formula 20-5)}$$

Assembling the local stiffness matrices into a global stiffness matrix and substituting into the static analysis equation yields

$$\frac{2AE}{L}\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}$$

By the statement of the problem, the truss is constrained against translation at node #1, such that

$$x_1 = 0$$

Also, a loading condition has been specified such that

$$F_1 = R = \textit{Reaction at Boundary}$$

$$F_2 = 0$$

$$F_3 = P = \textit{Applied Load}$$

Substituting the boundary and loading conditions into the equilibrium equation yields the following:

$$\frac{2AE}{L}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ P \end{bmatrix}$$

Inverting the stiffness matrix and multiplying both sides of the equation by that inverse yields

$$\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \frac{L}{2AE}\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ P \end{bmatrix} = \frac{PL}{AE}\begin{bmatrix} .5 \\ 1 \end{bmatrix}$$

from which the displacement at node #2 and node #3 are determined as:

$$x_2 = \frac{PL}{2AE}$$

$$x_3 = \frac{PL}{AE}$$

This analysis indicates that the unconstrained end of a linear elastic truss member stretches twice as much as the center of the member when a uniaxial force is applied to the end. This is consistent with the closed form analytical solution.

4.0 DYNAMIC ANALYSIS

The dynamic equation of motion can be written in matrix form as:

$$[M]\{\ddot{x}(t)\} + [C]\{\dot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \qquad \text{(Formula 20-6)}$$

where

$[M]$ = the mass matrix
$[C]$ = the damping matrix
$[K]$ = the stiffness matrix
$\{x(t)\}$ = the nodal displacement vector
$\{\dot{x}(t)\}$ = the nodal velocity vector
$\{\ddot{x}(t)\}$ = the nodal acceleration vector
$\{F(t)\}$ = the applied load vector

The dynamic equilibrium equation contains two additional matrices *[M]* and *[C]*, for mass and damping, that were not included in the static equilibrium equation. All vectors for displacement, velocity, acceleration and applied load are a function of time.

Mathematically, the equation of motion represents a set of second-order linear differential equations. You can obtain a solution by several different methods, including:

- Modal superposition
- Frequency response
- Direct integration for transient response

The modal superposition method is normally used to solve the dynamic equation of motion for linear elastic systems. This method is the most popular, since the majority of real world problems are approximately linear elastic. Therefore, this chapter focuses on the modal superposition method.

4.1 The Modal Superposition Method

The modal superposition method is a two-step procedure. First, obtain the mode shapes of a structure by solving the free vibration problem. Then calculate the response to the dynamic forcing function for each mode separately and sum them for the total response. The mode shapes are obtained from the equation of motion for undamped free vibration, which can be written in matrix form as:

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = 0 \qquad \text{(Formula 20-7)}$$

where

$[M]$ = the mass matrix
$\{\ddot{x}(t)\}$ = the nodal acceleration vector
$[K]$ = the stiffness matrix
$\{x(t)\}$ = the nodal displacement vector

Since $\{\ddot{x}(t)\} = -\omega^2\{x(t)\}$ for free vibration, the equation can be rewritten in the following form:

$$[K]\{x(t)\} - \omega^2[M]\{\ddot{x}(t)\} = 0 \qquad \text{(Formula 20-8)}$$

where

$[K]$ = the stiffness matrix
$\{x(t)\}$ = the nodal displacement vector
ω^2 = the natural frequency
$[M]$ = the mass matrix
$\{\ddot{x}(t)\}$ = the nodal acceleration vector

For the non-trivial solution $\{x(t)\} \neq 0$, the magnitude of the resonant frequencies can be obtained by the generalized eigenvalue solution to:

$$[K] - \omega^2[M] = 0$$

The stiffness matrix for the finite element mesh is developed in the same manner as described for static analysis. There are several different methods for approximating the mass matrix. Two of the most common approaches used in finite element analysis are the lumped mass and consistent mass methods.

The lumped mass method, as its name suggests, consists of evenly dividing the total mass of the element and lumping that mass at each node. Using this technique for the uniaxial truss element, the mass matrix would be:

$$[M] = \begin{bmatrix} \frac{m}{2} & 0 \\ 0 & \frac{m}{2} \end{bmatrix} = \frac{\rho AL}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The consistent mass method is a more precise approach of estimating the mass matrix. The consistent mass matrix is derived from the displacement functions of that element. For the uniaxial truss element, this method yields the following mass matrix:

$$[M] = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \text{(Formula 20-9)}$$

Unlike the lumped mass matrix, the consistent mass matrix is non-diagonal. A **non-diagonal** matrix contains off-diagonal terms other than zero. It is a better representation for the distribution of mass and provides a more accurate estimate for the natural frequency of the structure.

Regardless of which method is used to develop the mass matrix for the individual elements of the finite element mesh, the global mass matrix for the entire structure is assembled in the same manner as the stiffness matrix.

RESPONSE TO DYNAMIC LOADING

Once the eigenvalues and eigenvectors for each mode shape are obtained, the response to dynamic loading can be calculated using the modal superposition procedure which is based on the following transformation:

$$\{x(t)\} = [\Phi]\{y(t)\}$$

where

$[\Phi]$ = matrix of eigenvectors
$\{y(t)\}$ = vector of modal displacements

For the case of linear transient dynamic loading, substituting the above expression for the displacement vector into the dynamic equilibrium equation results in a set of uncoupled equations of the form:

$$\ddot{y}_i(t) + 2\omega_i (c/c_c)\dot{y}_i(t) + \omega_i^2 y_i(t) = f_i(t) = \{\Phi_i\}^T\{F(t)\}$$

The ratio of (c/c_c) in the above expression is the effective damping ratio, measured as a percentage of critical damping. A numerical time-integration scheme is used to solve the set of equations to obtain the dynamic response. The harmonic response of structures can be obtained in a manner similar to the linear transient dynamic problem.

5.0 MESH REFINEMENT

The degree of accuracy of the finite element solution is affected by the discretization of the finite element mesh that you choose. The effect of mesh refinement can best be demonstrated by an example. Consider the steel cantilever slab illustrated in Figure 155. The slab is 60 inches long by 10 inches deep. Let us assume that the slab is very wide so that the response can be modeled as a two-dimensional plane strain problem.

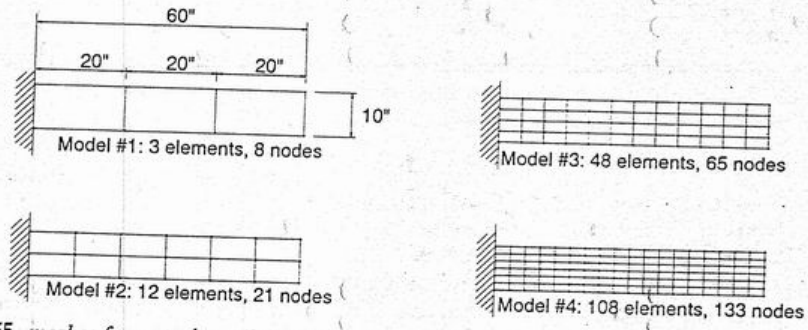


Figure 155 - *meshes for a steel cantilever slab*

Four different models were developed to predict the first three bending modes of natural frequency. The models are illustrated in Figure 155. All elements are based on a simple linear displacement function.

The first model uses 8 nodes and 3 elements to describe the beam. Since all degrees of freedom are rigidly constrained against translation at the boundary which represents the support point, the model contains 12 total degrees of freedom.

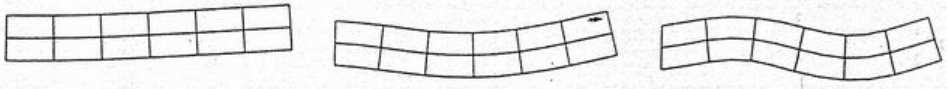
The second model employed 21 nodes and 12 elements to define the mesh. The model contains 36 total active degrees of freedom. The finite element mesh for this model is considerably more refined than the first model.

The finite element mesh of the final two models are refined even more. The third model contains 65 nodal points and 48 elements. The last model contains 133 nodes and 108 elements. The results of the analysis are summarized in Table 11.

MODEL	# ELEMENTS	MODE #1 (Hz)	MODE #2 (Hz)	MODE #3 (Hz)
1	3	140.9	880.8	2499
2	12	104.3	611.1	1656
3	48	92.2	525.8	1354
4	108	88.7	508.6	1295

Table 11 - *calculated mode shapes using models from Figure 155*

The three mode shapes determined from the second model are illustrated in Figure 156.



First mode - 104.3 Hz Second mode - 611.1 Hz Third mode - 1,656 Hz

Figure 156 - *plotted mode shapes from Table 11*

You can make the following conclusions from the results of the finite element analyses:

1. The solutions monotonically converge toward the closed form solution as the number of elements used to model the beam are increased.
2. The predicted magnitude of natural frequency decreases as the number of elements increase. This is due to the fact that the stiffness of the finite element model decreases as the number of elements increase.
3. Because of the increasing complexity of the higher order mode shapes, more elements are required for the same degree of solution accuracy as the order of resonance increases.

Generally, the accuracy of the solution increases as the finite element mesh is refined. However, the computer resource requirement also increases. You must decide what degree of accuracy is economically justified.

The density of a finite element mesh does not have to be uniform throughout the model. You can increase the mesh density in areas of concentrated mass or stiffness. Reducing the mesh density in other areas reduces the computational resources required to perform the analysis.

Unlike static stress analysis, local geometric discontinuities such as fillets and small holes, which do not effect the overall global stiffness, can be neglected without affecting the accuracy of a modal analysis. However, these items must be included in the finite element model if stresses due to dynamic loading are desired from the analysis.

6.0 HIGHER ORDER DISPLACEMENT FUNCTIONS

Another method for increasing the accuracy of the solution is to use elements based on higher order displacement functions. For instance, if the deflected shape of a structure to be analyzed is quadratic, then a finite element model comprised of elements with quadratic displacement functions will provide accurate solutions using fewer elements than a model comprised of elements with linear displacement functions.

This can be demonstrated by the cantilevered slab problem presented in Section 5.0. This time the models will be developed with higher order elements. Figure 157 contains two models of the cantilever slab which will be used for this example. The elements used in these models are based on a quadratic displacement function. Additional nodes are required to define each element because of this higher order displacement function. One node is located at each corner of the element and one midside node is included between each corner node.

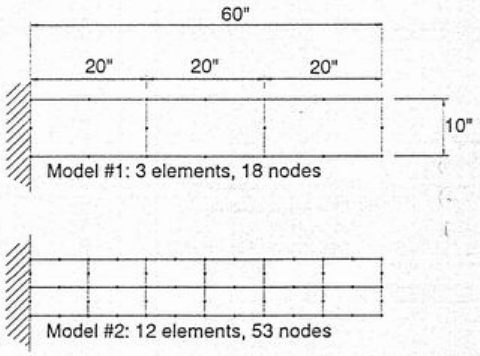


Figure 157 - *higher order displacement functions*

Each element contains a total of eight nodes. This results in 16 degrees of freedom for each two-dimensional plane strain element. Because of the additional degrees of freedom, this element requires more computer time and resources than the comparable linear displacement element. The first model contains 3 elements and a total of 18 nodes. The second model of the beam contains 12

elements and 53 nodes. The solution for the first three modes of natural frequency as predicted by each model are summarized in Table 12.

MODEL	ELEMENTS	FREQUENCY #1 (Hz)	FREQUENCY #2 (Hz)	FREQUENCY #3 (Hz)
1	3	89.2	513	1380
2	12	87.8	497	1261

Table 12 - *mode shapes calculated using a quadratic displacement function*

If you use a higher-order displacement function, you can obtain a very good estimation of the natural frequency with relatively few elements (especially for lower-energy mode shapes).

The results of the models using higher order quadratic displacement functions are compared to the results obtained in Section 5.0 for the models containing linear displacement functions in Table 13.

ELEMENTS	FUNCTION	FREQUENCY #1 (Hz)	FREQUENCY #2 (Hz)	FREQUENCY #3 (Hz)
3	Linear	140.9	881	2499
3	Quadratic	89.2	513	1380
12	Linear	104.3	611	1656
12	Quadratic	87.8	497	1261
48	Linear	92.2	526	1354

Table 13 - *linear vs. quadratic displacement functions*

The model containing 3 quadratic elements provided a better estimation for the first two modes of natural frequency than the model containing 48 linear elements. The model containing 12 quadratic elements provided a better estimation for all three modes of natural frequency. However, the computer time required to perform the analysis of the model containing 12 quadratic elements was comparable to the time required to obtain the solution for the linear model containing 48 elements. Generally, you can obtain an accurate estimate of the magnitude of resonant frequency with fewer elements if a higher order formulation is used. However, the computer resource requirement will not necessarily be reduced.

Some finite element analysis codes provide a compromise between the simple linear elements and the higher order varieties. This is done by introducing an extra displacement shape to the linear element formulation. Refer to the literature [8] for information regarding the theory of extra displacement shape functions. These elements containing an extra shape function are defined by the same number of nodes as the linear elements. Solution accuracy is usually increased significantly with only a modest increase in computational time.

The cantilever slab problem was solved again using several models constructed from linear displacement elements with extra shape functions. A comparison of these results with the solutions obtained from the previous analyses is included in Table 14.

The comparisons in Table 14 show that, although the extra shape function does not provide as accurate of a solution for the slab bending problem as does the quadratic element, it does provide a significant increase in accuracy over the simple linear displacement formulation, especially for models

ELEMENTS	FUNCTION	FREQUENCY #1 (Hz)	FREQUENCY #2 (Hz)	FREQUENCY #3 (Hz)
3	Linear	140.9	880	2499
3	Extra shape	88.7	626	2150
3	Quadratic	89.2	513	1380
12	Linear	104.3	611	1656
12	Extra shape	88.3	550	1452
12	Quadratic	87.8	497	1261
48	Linear	92.2	526	1354
48	Extra shape	87.9	504	1302

Table 14 - *solution of the cantilever slab problem using several models*

containing only a few elements. The advantage of the higher order elements diminished as the number of elements in the finite element mesh increased.

The type of element to be used in any finite element model is your decision. All of the analyses used here for the cantilever slab considered only the bending modes of natural frequency. Since bending of a beam is approximated by a quadratic function more closely than a linear function, the higher order elements provided a significant increase in accuracy. However, if the stretching modes of natural frequency were critical to the analysis, the use of higher order elements would not provide the same advantage. As an example, the first stretching mode of resonance was calculated for the simple three element model using elements with linear, extra shape and quadratic displacement functions. The results are included in Table 15.

SHAPE FUNCTION	STRETCHING FREQUENCY	RELATIVE CPU TIME
Linear	848	1.00
Extra Shape	846	1.08
Quadratic	838	1.27

Table 15 - *calculating first stretching mode using the three displacement functions*

The finite element model consisting of quadratic elements provided only a modest increase in accuracy over the finite element model developed from a simple linear displacement function, but required an additional 27% of computer time.

This example illustrates the importance of both a basic understanding of the theory of the finite element method and experience in applying the theory. Both theoretical and practical experience is necessary to provide economical solutions to complex vibration problems using the finite element method.

6.1 Shape Functions

When using a finite element software package, the derivation of the stiffness and mass matrices from the shape functions are transparent to the analyst. In many cases, the displacement and shape functions are not reported in the user's manual. You must rely on experience to recognize the element as being a higher order element. It is certainly possible to perform vibration analysis using finite element analysis techniques without understanding the importance of (or being able to) derive admissible displacement functions. However, the more the analyst knows about the basics of the finite element analysis method, the greater the probability that an accurate analysis will be performed. Therefore, a brief discussion of shape functions are included herein to provide a basic understanding of the development of a finite element.

Consider a simple uniaxial truss element defined by two nodes, as shown in Figure 158.

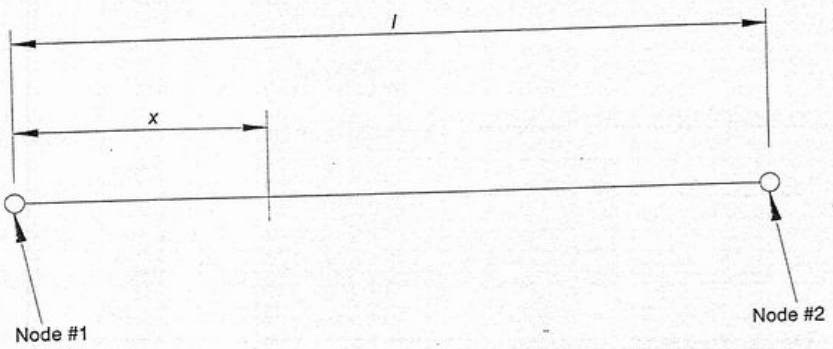


Figure 158 - *a simple uniaxial truss element*

Assuming a linear displacement function, the axial deformation anywhere along the element can be defined as:

$$u(x) = a + bx$$

or in matrix form as:

$$\{u(x)\} = \{1\ x\}\{a\ b\}^T$$

The following conditions must hold at the nodal points in order for the displacement function to be admissible:

$$u(0) = x_1 = a$$

$$u(L) = x_2 = a + bL$$

Such that the following matrix form of the displacement function can be written as:

$$\{x_1\ x_2\}^T = \begin{bmatrix} 1 & 1 \\ 0 & L \end{bmatrix} \{a\ b\}^T$$

Matrix inversion and multiplication yields:

$$\{a\ b\}^T = \frac{1}{L} \begin{bmatrix} L & 1 \\ 0 & 1 \end{bmatrix} \{x_1\ x_2\}^T$$

Substituting the above expression into the original displacement function equation yields:

$$u(x) = \left[\left(1 - \frac{x}{L}\right) \frac{x}{L}\right] \{x_1\ x_2\}^T$$

or:

$$u(x) = [H]\{x_1\ x_2\}^T$$

The matrix $[H]$ is called the **shape function matrix**. It contains the following shape functions:

$$H_1 = 1 - \frac{x}{L}$$

$$H_2 = \frac{x}{L}$$

The significance of this equation is that you can calculate the displacement anywhere within the element from the nodal displacement, if an admissible shape function matrix is available.

The stiffness matrix for the element can also be derived from the shape function. The internal strain energy for the axial deformation of the element is:

$$U = \frac{1}{2} \int \sigma \varepsilon dV$$

where

σ = Stress
ε = Strain

Substituting Hooke's material law yields:

$$U = \frac{1}{2} \int E\, \varepsilon^2 dV$$

where

E = Modulus of Elasticity
$\sigma = E\varepsilon$

Since ε (strain) = du/dx and $dV = A\,dx$, then

$$U = \frac{1}{2}\int EA\left(\frac{du}{dx}\right)^2 dx$$

Since the relationship between internal strain energy and stiffness can be expressed in matrix form as $\frac{1}{2}\{x\}^T[K]\{x\}$:

$$U = \frac{1}{2}\{x\}^T[K]\{x\} = \frac{1}{2}\int EA\left(\frac{du}{dx}\right)^2 dx$$

Substituting $u(x) = [H]\{x\}^T$ into the above expression yields the following, after some rigorous calculus:

$$[K]\{x\} = \left(\frac{AE}{L}\right)\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\{x_1\ x_2\}^T$$

So that the stiffness matrix is defined as:

$$[K] = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Similarly, the consistent mass matrix can be formulated from the kinetic energy of the element. This yields:

$$[M] = \rho A\int [H]^T[H]\,dx$$

Substituting the shape functions previously determined into the above expression yields the following consistent mass matrix:

$$[M] = \frac{\rho AL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

The above development of the stiffness and mass matrices for a simple uniaxial truss element using a linear displacement function is elementary compared to the development of a three dimensional element containing higher-order displacement functions. The utilization of the finite element technique would not be very effective if the analyst would be required to develop a finite element for each problem. Fortunately, complete element formulations are available in all finite element analysis software codes and the development of the stiffness and mass matrices is done by the program, transparent to the analyst.

7.0 ASPECT RATIO AND SKEWNESS

Two other factors which are extremely important to the accuracy of the finite element solution are the aspect ratio and the skewness of the individual elements which comprise the mesh. The **aspect ratio** is the ratio of the length of one side of the element to the length of any other side of the element. You can measure the degree of skewness by measuring the angle that any two faces of an element makes at a node point.

Many finite element software codes contain isoparametric element formulations, wherein a parent element contained in the finite element analysis software is mapped onto the actual element defined by the analyst, using the shape functions as the coordinate transformation. By using this technique, only one element formulation has to be included in the element library for each element type. These so-called "parent" elements are usually perfectly square or cubic, depending on whether the element is two or three dimensional. The accuracy of this coordinate transformation and the subsequent development of the stiffness and mass matrices is affected by the amount by which the general shape of the actual element deviates from the parent element. For linear displacement functions, elements containing equal length sides, with right angles between all faces, will provide the best results.

Figure 159 illustrates three different solid elements which can be obtained from the same parent element. The element of Figure 159(a) is a perfect cube. In this case there is a direct 1:1 transformation in the mapping process from the parent element to the actual element. The aspect ratios for the element of Figure 159(b) deviates from unity, which will have a slight effect on the accuracy of the solution.

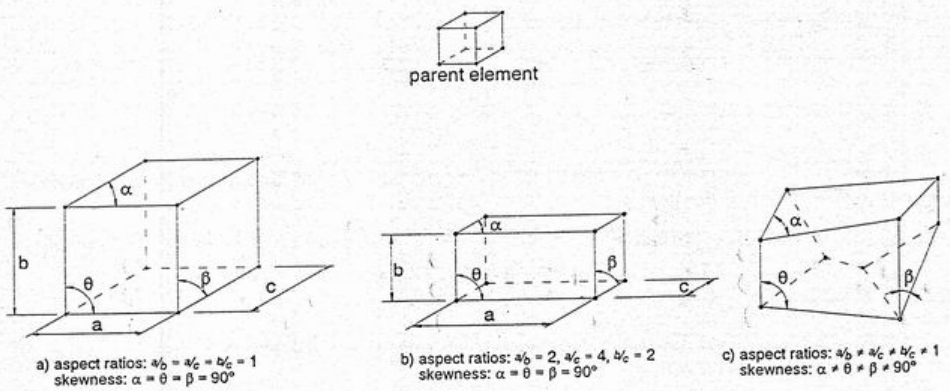


Figure 159 - *solid elements obtained from a parent element*

The element shown in Figure 159(c) is badly skewed and, if many of these elements are used in the mesh, will have a significant adverse effect on the accuracy of the solution.

You should limit the aspect ratio and skewness of all elements in the mesh. Geometric discontinuities will exist in almost all structures and machines which will make it impossible to model the entire structure with perfectly square, cubic, or triangular elements. Therefore, in most cases, finite element meshes will contain elements with aspect ratios other than unity and some skewed angles between faces.

It is important to note that, if a finite element analysis software package is used by the vibration analyst, the shape functions and parent element formulations as well as coordinate transformation and mapping procedures are part of the software. You have no control over these features. However, you have complete control in the definition of the mesh. In most cases, the accuracy of the solution is affected more by your decisions in preparing the finite element model than by the mapping and solution techniques used by the software developer.

8.0 ELEMENT TYPES

The element libraries of most commercially available general purpose finite element software packages are filled with hundreds of types of elements. Each element has been developed to address a particular set of problems. Special thermal elements are available for heat transfer analysis; plastic flow elements have been developed for nonlinear, strain rate sensitive deformation of solids; and special elements are even available for magneto-statics.

The element types most commonly used in vibration analysis include:

1. Spar Elements
2. Beam Elements
3. Thin Shell Elements
4. 2D Solid Elements
5. 3D Solid Elements
6. Mass Elements
7. Springs

8.1 Spar Elements

The spar elements are defined by two nodes, one located at each of the terminal points. It is sometimes referred to as a line element since it is represented graphically as a line between the two nodes. The spar element is used to describe structural elements that undergo uniaxial deformation, either compression on tension. Bending or rotational deformation are not considered.

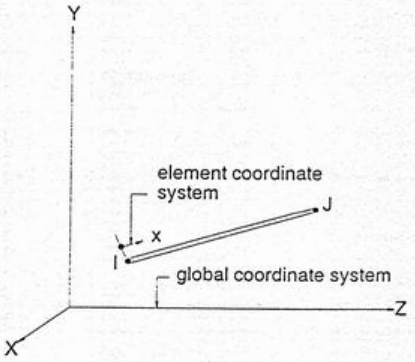


Figure 160 - *a spar element*

The spar elements can be either two- or three-dimensional. For the two-dimensional version, the element contains two degrees of freedom at each node (translation in the X and Y directions). The three dimensional version contains three degrees of freedom per node (translation in the X,Y & Z directions).

In order to perform a dynamic analysis using a spar type element, the analyst must provide the finite element software with the cross section area of the element and material properties, including the modulus of elasticity and mass density, required to formulate the stiffness and mass matrices.

Spar elements are typically used to model the structural elements of a truss. These structural members usually are designed to resist only uniaxial forces.

8.2 Beam Elements

Beam elements are similar to the spar type elements. These elements are defined by two nodes and are graphically displayed as a line between the nodes. They are available in both 2D and 3D

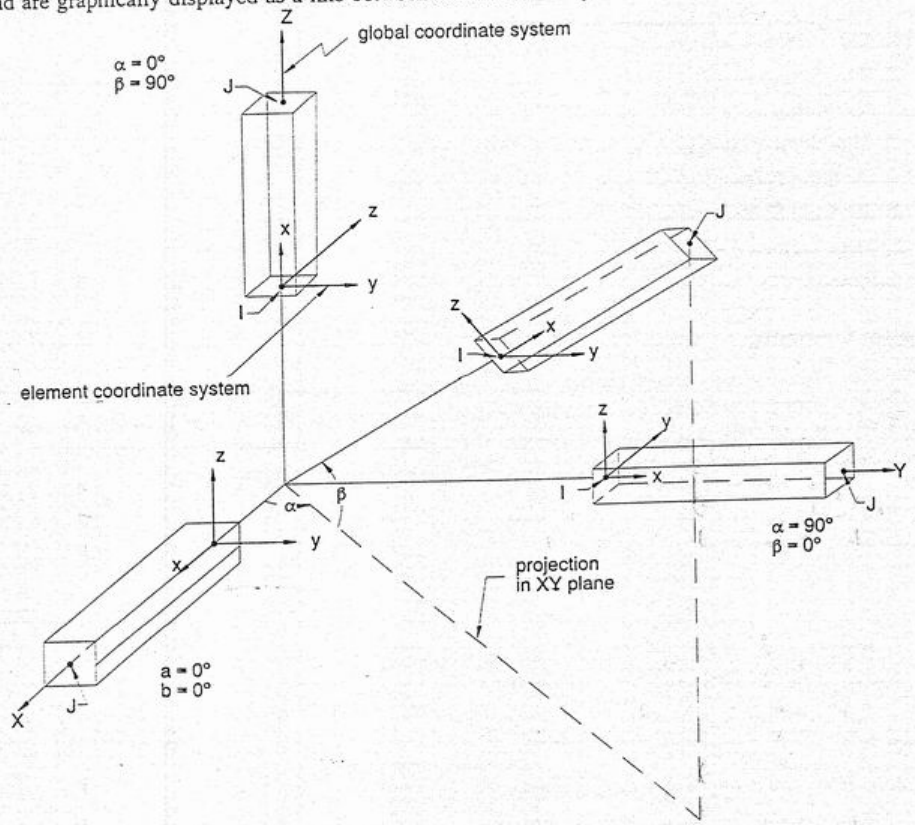


Figure 161 - *a beam element*

versions. Unlike the spar type, the beam elements contain rotational stiffness in additional to translational stiffness. Therefore, beam elements are used to model structural members subjected to bending.

The two-dimensional version of the beam element contains three degrees of freedom per node (two translational and one rotational). The three dimensional version contains six degrees of freedom per node. Translation and rotation about all three axes are included.

You must provide information relative to the physical and material properties of the actual structural member which is modeled by the beam element. Required physical properties include the cross section area and moment of inertia about each axis. The material properties required to formulate the stiffness and mass matrices include modulus of elasticity, Poisson ratio, and mass density.

Special formulations of beam type elements are available that include features such as tapered unsymmetrical properties along the length of the element.

As the name implies, this element type is used to model beams in framed structures.

8.3 Thin Shell Elements

Thin shell elements are three dimensional and generally contain both bending and membrane stiffness. Several versions of this element type are available. A thin shell element formulated from a linear displacement function will be defined by four nodes, one located at each corner of the element. Higher order shell elements formulated using quadratic or cubic displacement functions would contain four or eight additional midside nodes, respectively.

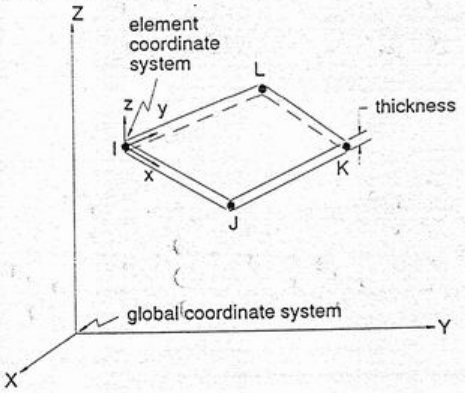


Figure 162 - *a thin shell element*

The three dimensional thin shell elements contain six degrees of freedom at each node. These include translation and rotation about each axis. An element based on a linear displacement function contains a total of 24 degrees of freedom, while an element based on a cubic displacement function contains 72 degrees of freedom. The increased accuracy obtained by the higher order elements requires a significant increase in computational resources. The additional degrees of freedom result in larger stiffness and mass matrices which must be inverted.

The only physical property which must be specified is the thickness. Some shell element formulations accept variable thickness.

The material properties which must be supplied by the analyst include the modulus of elasticity, Poisson ratio, shear modulus and mass density. Many element formulations accept orthotropic material properties.

You can specify the nodal locations so that the element is skewed. This is usually necessary when modelling complex structures. However, the skewness affects the accuracy of the solution obtained by the element. Therefore, develop the finite element mesh so that the skewing of elements is minimized.

8.4 Two Dimensional Solid Element

This element type is used for the two dimensional modelling of solid structures. Formulations are available for biaxial plane elements (plane stress and plane strain) and for axisymmetric ring elements.

Solid elements based upon a linear displacement function are defined by four nodes, one located at each corner of the element. Higher order 2D solid elements are available. An element formulated from a quadratic displacement function would contain a total of eight nodes, four corner and four midside nodes.

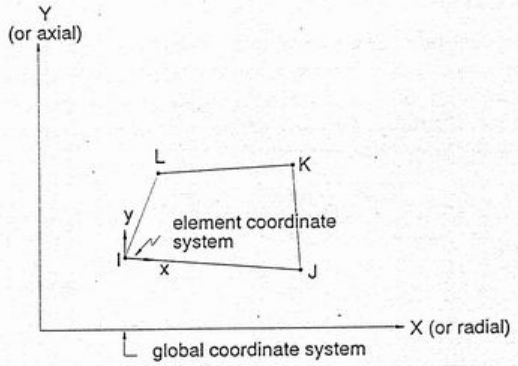


Figure 163 - *a two-dimensional solid element*

Solid elements contain only translational degrees of freedom. The 2D solid elements have two translational degrees of freedom per node.

The solid elements generally do not require the specification of any additional physical properties by the analyst. There are certain special formulations which require the specification of thickness for plane stress conditions.

The material properties required to perform a vibration analysis include the modulus of elasticity, Poisson ratio, shear modulus and mass density. Orthotropic material property specifications are available in many finite element codes.

8.5 Three Dimensional Solid Element

This element type is used for the three dimensional modelling of solid structures. Elements based upon linear displacement functions are defined by eight nodes located at each of the corners. Each node has three translational degrees of freedom. This element type is sometimes referred to as a "brick" element.

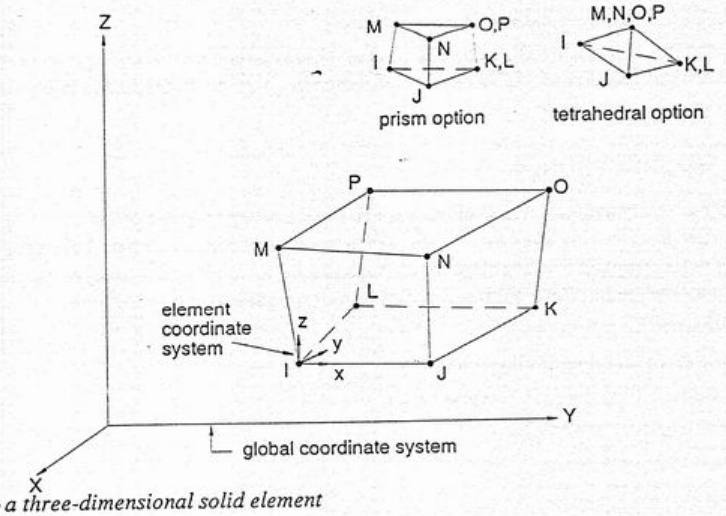


Figure 164 - *a three-dimensional solid element*

Special prism and tetrahedral formulations of the 3D solid element are available. These elements contain five and four sides instead of six. The prism elements are defined by five nodes, and the tetrahedral elements are defined by four nodes.

Higher order solid elements are also available in many finite element analysis codes. A brick type solid element based on a quadratic displacement function will contain twenty nodes, eight corner and twelve midside nodes. Since each node contains three degrees of freedom, the quadratic solid element contains a total of sixty degrees of freedom. Higher order tetrahedral elements are also available.

The three dimensional solid elements do not require the specification of any physical properties. All geometric properties are defined by the location and connectivity of the nodes.

The material properties required to perform a vibration analysis include the modulus of elasticity, Poisson ratio, shear modulus, and mass density. These can be input as orthotropic material properties for many elements. You can specify a different modulus of elasticity for each axis, and define a different Poisson ratio and shear modulus for each plane.

Models developed using three dimensional solid elements are generally very computer resource intensive. These type of elements are used to model complex non axisymmetric thick solid structures which cannot be accurately simulated with thin shell elements. Some examples are large concrete machine foundations, equipment fabricated from thick castings, and other complex thick machine parts.

8.6 Mass Elements

Unlike the other elements, the generalized mass elements do not contain any degrees of freedom. These elements are defined as a concentrated amount of mass located at a particular node in the finite element mesh. The mass elements do not increase the size of either the global stiffness or mass matrix. They are used to add lumped mass to the finite element model to account for the weight of massive nonstructural items.

8.7 Spring Elements

Spring elements are usually defined by two nodes. These elements contain one degree of freedom. Both translation and rotation are valid degrees of freedom. You must define the stiffness or spring constant.

9.0 ANALYSIS PROCEDURES

Many commercially available general purpose finite element computer programs with extensive element libraries are currently available. If the vibration analyst utilizes one of these programs, it is not necessary to develop the displacement functions and solution techniques to solve a problem. The procedures an analyst using finite element analysis software generally must follow are:

1. Define node locations
2. Choose element types & connectivity
3. Define material and geometric properties
4. Define boundary conditions
5. Define forcing functions
6. Submit file for analysis
7. Interpret results

9.1 Nodes

The finite element model is discretized by the definition of nodal points. The location of nodes define many of the geometric features of the model. The number of nodes used to describe the finite element model is an important factor in the accuracy of the solution. Unless an automatic mesh generator is used, the location of the nodes is the option of the analyst.

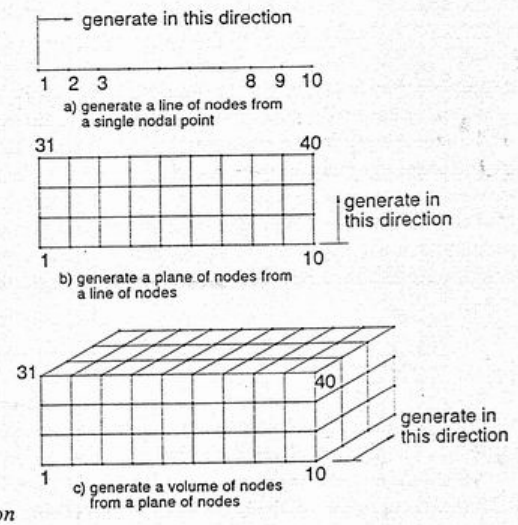


Figure 165 - *node generation*

Many of the available finite element analysis software packages contain nodal generation techniques to simplify the modelling of complex geometries. As illustrated in Figure 165, a single node can be generated into a line of nodes. Next, the line of nodes is generated into a plane of nodes. Finally, the plane of nodes is generated into a volume of nodes.

Nodes can be defined and generated using several coordinate systems, including cartesian, cylindrical, spherical, and toroidal to suit the particular shape of the structure or component to be modelled. Use local coordinate systems to describe local geometric features such as fillets and holes.

Use symmetry reflections of any given node pattern, in any coordinate system, to simplify development of the finite element model. Automatic mesh generators are available in some finite element analysis software packages. You define the boundaries of the model; all other nodes are then automatically generated by the computer.

9.2 Elements

You must choose the type of element which is best suited for the particular analysis to be performed. Use spar elements to model trusses, beam elements for framed structures, and so on. You must define all geometric (thickness, area, moment of inertia, etc.) and material (modulus of elasticity, density) properties required to form the global stiffness and mass matrices.

The shape of an element is determined by a statement that defines which nodal points are connected to form the boundary of the elements. Element formulations containing simple linear displacement functions usually require node definition only at the corner of the element. Elements with higher order displacement functions require additional midside nodes.

You can generate element connectivities to simplify the modelling procedure. Similar to nodal generation, a single element can be generated into a line of elements, assuming the required nodes exist along the line of generation. The resultant line of elements can then be generated into a plane of elements. Finally, the plane of elements can be generated into a volume of elements.

9.3 Boundary Conditions

The constraint against deformation that the surrounding environment provides to a structure or component must be approximated. The realistic modelling of the constraints is imperative if an accurate estimate of the dynamic characteristics is to be obtained. It is usually not economically feasible to explicitly model everything that affects the solution. A boundary to the finite element model must be defined to reflect the relative stiffness of everything not considered by the model.

For instance, appropriate translational and rotational degrees of freedom can be fixed at the base of a finite element model of a framed structure to account for the fixidity of the foundation without explicitly modelling the foundation. If the elastic characteristics of the foundation-soil interface are considered to provide a significant contribution to the dynamic characteristics of the framed structure, then the boundary can be defined by a set of appropriate spring constants or elements.

You can simplify the finite element modelling and analysis of symmetrical structures, in some cases, by defining boundary conditions along a plane of symmetry so that only a portion of the structure has to be modelled to provide an estimate of the dynamic characteristics of the entire structure. These boundary conditions along the plane of symmetry can be defined such that either a symmetrical or asymmetrical reflection of the mode shape is achieved.

Take extreme caution when simplifying the analysis using symmetry constraints to ensure that all pertinent mode shapes will be calculated by the analysis. For instance, if only symmetrical boundary conditions are imposed along the line of symmetry, the subsequent analysis will not include any

contribution due the excitation of asymmetric modes of natural frequency. This could result in a gross underestimation of the magnitude of dynamic response of the structure to a specified vibrational forcing function.

9.4 Dynamic Loading

If your objective of the finite element analysis is only to define the magnitude of the natural frequencies and the mode shapes, then you do not need to define a dynamic loading condition. As discussed previously, the modal analysis step in finite element analysis consists of the solution to the classical eigenvalue-eigenvector problem.

If the dynamic response of a structure subjected to a time or frequency dependent load is the objective of the analysis, then a two step procedure must be employed. First, the natural frequencies are obtained by solving the eigenvalue problem. Next, the dynamic response is obtained by defining the dynamic forcing function and calculating the response using mode-superposition or other appropriate techniques to evaluate the contribution of each mode shape to the dynamic magnification of the response of the structure to the specified load.

Dynamic response analysis is divided into the following categories:

1. Harmonic response
2. Linear transient response
3. Non-linear transient response

The above list is in order of the degree of difficulty and the required computer resources for each type of analysis.

In order to perform a harmonic response analysis for a structure, apply all dynamic forcing functions to the structure harmonically. The definition of the load must include the magnitude, frequency and phase relationship. Damping for the structure is defined as a percentage of critical damping.

Linear transient dynamic analysis is used to determine the response of a linear elastic structure or component subjected to time dependent aperiodic loads. Many dynamic loads cannot be approximated by a harmonic function or a Fourier series of harmonic functions. Traffic over a highway bridge and intermittent impact due to turbulent flow in ductwork are examples of transient dynamic loads. For this type of analysis, you must define the time-history of the forcing function. There are many numerical integration techniques available to obtain the time dependent response of the structure to the specified transient load. You must be aware of the effect of the size of the integration time step on the accuracy of the solution. Generally, decreasing the time step increases the level of accuracy as well as computer resource requirements. You must determine the optimum balance between cost and accuracy.

The specification of the forcing function for a nonlinear transient dynamic analysis is similar to the linear case. This type of analysis is used to determine the dynamic response of non-linear structures subjected to transient loading. Non linearities can be associated with material properties such as inelastic action, or geometric properties such as friction surfaces and gaps.

9.5 Matrix Condensation

Many finite element software codes contain a condensation routine whereby the size of the stiffness and mass matrices are reduced for the eigenvalue extraction. The stiffness matrix is reduced through static condensation. The reduced mass matrix is obtained by a kinematic condensation technique,

first introduced by Guyan [6]. This matrix condensation technique is sometimes referred to as Guyan Reduction. This technique preserves the potential energy of the structure but modifies the kinetic energy. Therefore the subsequent reduced stiffness matrix is exact, but the reduced mass matrix is approximate.

The computer resources required to perform eigenvalue extraction and modal superposition is dependent upon the size of the reduced matrices. The accuracy of the solution is also dependent upon the accuracy of the approximate reduced mass matrix. Therefore, an understanding of kinematic condensation is imperative if the vibration analyst hopes to provide an accurate solution using an optimal amount of computational resources.

Guyan mass matrix reduction involves the declaration of master degrees of freedom, which dictate the size and accuracy of the final reduced matrix. The accuracy of the reduced matrix not only depends upon the number of master degrees of freedom, but also upon the location of the masters. If all degrees of freedom in the finite element model are chosen as masters, then there is no approximation. However, for extremely large models, declaring all degrees of freedom as masters may not be either advisable or even feasible.

Use the following guidelines when selecting master degrees of freedom to be used for the purpose of kinematic condensation:

- The total number of master degrees of freedom should be at least equal to twice the number of modes of natural frequency desired from the analysis.
- Declare nodes located at or near areas of concentrated mass as masters.
- If a motion of a structure in one direction induces a motion in another direction, then master degrees of freedom should be declared for both directions.

10.0 CASE HISTORY #1

The first case history involves the finite element analysis of an existing structure which had been experiencing excessive vibrations during normal operating conditions.

10.1 Problem Description

A section of ductwork on the outlet side of a fan of an emissions control system for a rolling mill was experiencing excessive vibrations. The vibrations led to fatigue cracking of the structural elements resulting in leakage of the system.

The section of ductwork spanned from the fan impeller housing to a clarifier vessel. The ductwork was approximately 4 feet x 6 feet in cross section and 15 feet in length. It was fabricated of rather thin gage plate with several stiffener bars. The fan operated at 1200 rpm (20 Hz).

Due to the fact that flexible rectangular ductwork of this type typically contains numerous closely coupled resonant conditions, an analysis utilizing experimental modal modification techniques was considered to be impractical. Therefore, a finite element analysis of the ductwork, correlated to experimental field data, was used to solve the problem.

The experimental testing portion of the vibration study consisted of two separate field tests: impact excitation of the ductwork to obtain resonant frequencies (used to correlated the finite element model), and the collection of vibrational data during normal operating conditions which was used to identity the cause of vibration.

Figure 166 contains a typical frequency response measured during the impact test of the ductwork. Many closely spaced modes of natural frequency existed. Fourteen independent mode shapes were

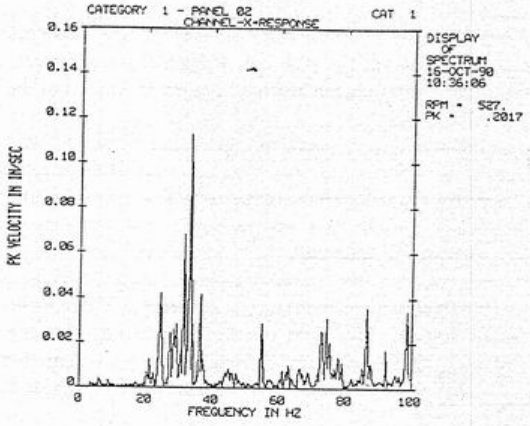


Figure 166 - *impact data from ductwork*

identified between 0 - 60 Hz which is the third order of the operating speed of the fan. Many of these modes of resonance occurred around both the first & second orders of the operating speed.

Figure 167 contains the frequency spectrum obtained at the fan bearing during normal operating conditions. This data supported a conclusion that the excessive vibrations were a result of natural frequency excitation caused by pressure pulse excitation emitted by the fan impeller (1x, 2x, 3x, ...RPM). Therefore, the objective of the finite element analysis was to provide a design modification to increase the resonant conditions beyond the frequency of the principal dynamic forcing functions.

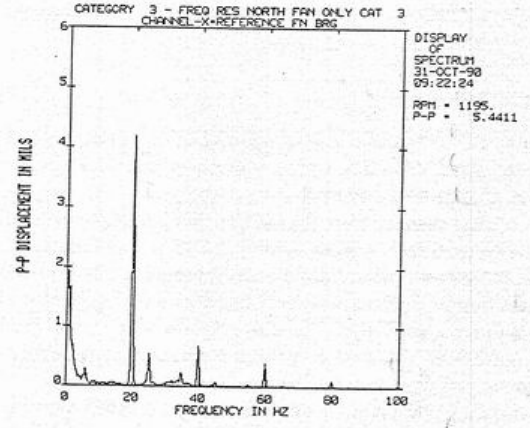


Figure 167 - *frequency spectra at fan bearing*

10.2 Finite Element Solution

The finite element model which was developed to analyze the existing ductwork is illustrated in Figure 168. Over 1400 thin shell elements were required to accurately model the side plates and stiffeners which comprise the section of ductwork under consideration.

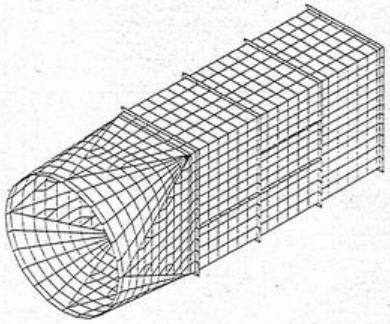


Figure 168 - *finite element model for existing ductwork*

Translational and rotational degrees of freedom associated with the end nodes which form the boundaries between the ductwork and fan housing and the ductwork and purifier vessel were assumed to be completely rigid and were held fixed. Since the actual boundary conditions were not completely rigid, this assumption introduced a slight error in the calculation due to the overestimation of the stiffness at the boundaries. However, it was determined by comparing experimental test results with the finite element analysis results that the effect of the boundary conditions were not a significant factor in the determination of the magnitude of the natural frequencies and associated mode shapes. This indicated that, although not completely rigid, the stiffness of the boundary was much greater than that of the ductwork.

The results of the finite element modal analysis predicted that the principal mode of natural frequency occurred at 21.63 Hz and that twenty closely spaced resonant conditions existed between 21.63 Hz and 60 Hz. These results correlated well with the results of the experimental impact test. Since the major dynamic forcing functions occurred at the first three orders of the operating speed of the fan (20 Hz, 40 Hz & 60 Hz), it was obvious that the existing ductwork must be modified to increase the many resonant conditions outside the frequency range of the dynamic forces.

The finite element model was revised to study the effect of proposed structural modifications on the dynamic characteristics of the ductwork. It was determined that a design containing thicker plate along with additional, larger stiffeners would increase the natural frequency of the ductwork such that the principal resonant condition would be at a frequency much greater than the operating speed of the fan. Therefore, dynamic magnification of the force would be minimized.

The finite element model of the new design is illustrated in Figure 169.

The first two mode shapes which were predicted to occur at 70.71 Hz and 73.16 Hz are illustrated in Figure 170.

The mode shape associated with the 12th mode of natural frequency is shown in Figure 171, illustrating the complex deformation pattern of the higher energy mode shapes.

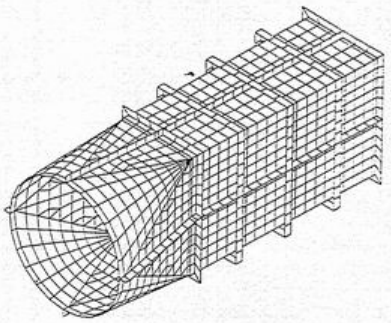


Figure 169 - *finite element model of new duct design*

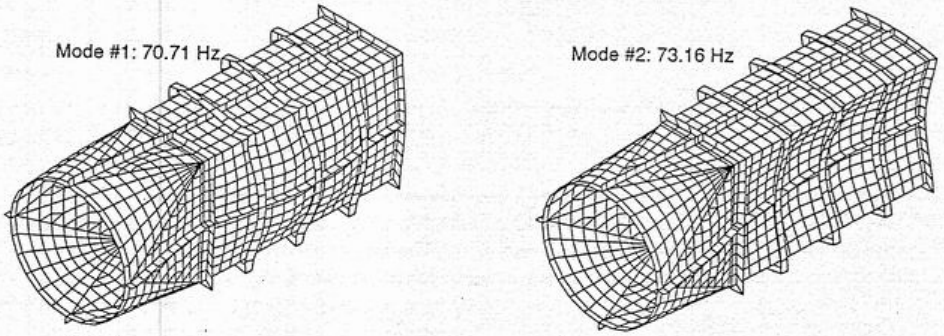


Figure 170 - *first two predicted mode shapes of new ductwork design*

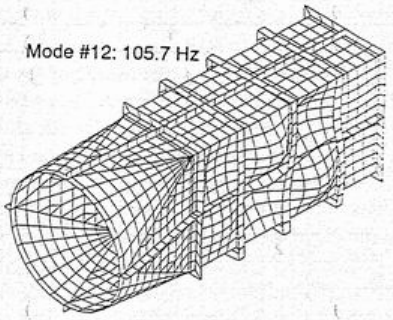


Figure 171 - *12th mode shape of new ductwork design*

Since the predicted natural frequency of the proposed design concept was greater than the frequency of the forcing functions which had been the major contributing factors to the excessive vibration level in the existing ductwork, this design concept was accepted as the solution to the problem.

11.0 CASE HISTORY #2

The vibration analysis of the ductwork presented in the previous example was for an existing flexible steel structure containing numerous closely-spaced resonant conditions. Although the cause of the dynamic forcing function was know, its magnitude was not well defined. The objective was to modify the structure such that all modes of natural frequency fell outside the range where dynamic magnification of the forcing function would be a concern.

The second case history involves a rigid structure with relatively few resonant conditions subjected to a well defined harmonic forcing function. The main difference is that the structure in the second case history was not yet built at the time of the analysis. Finite element techniques were employed in this case as a design tool, evaluating the dynamic response of conceptual engineering designs prior to developing the final detailed design.

11.1 Problem Description

A machine that cuts sheets of paper at a high speed was to be installed in a new process line at a major paper plant. The machine incorporated a concept wherein a single blade attached to a rotating cylinder makes a precision cut in a continuous strip of paper as the strip of paper travels past the rotating cutter. The relationship between the rotational speed of the cutting blade and the translational speed of the paper establishes the final length of each sheet of paper. The rotational speed of the cutting blade is variable over a certain operating range (0 - 7 Hz) to provide the necessary adjustment required to produce a large variety of paper sheet sizes.

Since most of the paper produced at this plant is used by printers, the precision of the cut is critical to reduce fraying of edges and subsequent dusting that adversely affects printing operations. Therefore, it is imperative to provide a machine foundation which minimizes vibrations. If the cutter vibrates relative to the paper, the quality of the edge of the paper would not be acceptable.

A finite element model of the conceptual machine foundation design was prepared to evaluate the dynamic characteristics thereof. Given the machine manufacturers dynamic loading specifications, the dynamic response of the foundation could be calculated and compared to the maximum vibration criteria established by the manufacturer to insure that the foundation would provide an acceptable base for the machine. This would increase the probability that the machine would provide a precise and acceptable quality cut.

11.2 Finite Element Analysis

The finite element model of the foundation is illustrated in Figure 172. Over 600 solid elements were used to define the reinforced concrete blocks and piers which form the foundation. Over 40 thin shell elements were used to define the machine supported on top of the foundation.

The elements which form the massive block sections of the foundation are large since this portion of the foundation was not expected to deflect much. The aspect ratios were kept close to unity. The elements that form the piers which support the equipment were spaced closer to provide more definition in this area. This was done because it was expected that the principal mode of vibration would be associated with flexure of the piers.

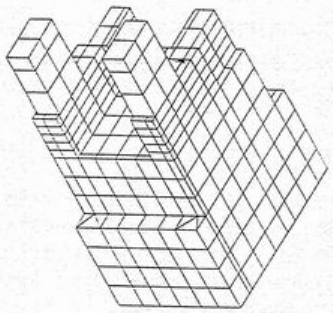


Figure 172 - *finite element model of machine foundation*

The conceptual design of the foundation specified that the base would be dowelled to bedrock using anchors. Therefore, boundary conditions were specified such that all translational degrees of freedom at the foundation/rock interface were fixed against movement.

The analysis proceeded in two steps. First, the magnitude of natural frequencies and associated mode shapes were determined by solving the eigenvalue/eigenvector problem defined by the finite element formulation. Next, the predicted dynamic response was obtained using the dynamic loading condition specified by the manufacturer.

The results of the finite element analysis for the first two resonant conditions are shown in Figure 173. The first two modes of natural frequency were predicted to occur at 18.8 Hz and 19.8 Hz respectively.

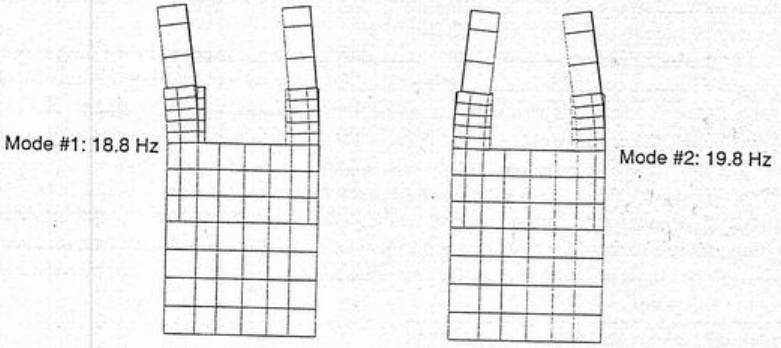


Figure 173 - *first two predicted mode shapes of machine foundation*

These first two mode shapes are similar in that they are both characterized by the "flagpoling" of the machine frames on top of the concrete piers. The concrete piers are bending similar to a cantilever beam. The machine frames are vibrating in phase in the first mode shape and are 180 degrees out-of-phase in the second mode.

Due to the manner in which the cutter blade operated, harmonic forces of first, second and third order of the operating speed could be developed. Since the operating speed could reach 7 Hz, the third order of the forcing function could excite the resonant conditions of the first two modes.

A forced harmonic response analysis was performed using the finite element model. This analysis showed that, although the third order harmonic of the applied dynamic load could be close to the principal resonance of the structure, the resultant vibration would be tolerable. This was due to the fact that the magnitude of the third order harmonic was very small, the foundation was extremely stiff and sufficiently damped.

12.0 CASE HISTORY #3

The next case history addresses the analysis of the dynamic response of a structure when subjected to an explosion or blast emanating from an unconfined vapor cloud. This example involves the analysis of the dynamic response of a structure which could not be accomplished by experimental modal analysis for two reasons:

1. The structure had not yet been constructed. Therefore, it was not available for field testing.
2. It would be impractical to simulate the impact force associated with a major explosion.

The analysis and design of structures to resist explosions can be separated into two basic categories: blast resistant and blast proof. A structure is considered to be blast resistant if it can survive the explosion without failure. The structure can yield, but must not collapse. A structure is considered to be blast proof if it does not yield during the explosion. A blast resistant structure can withstand only one explosive event of the rated magnitude and must be reconstructed afterward since it will realize a significant amount of structural damage. A blast proof structure could withstand multiple explosions of the rated magnitude without experiencing significant structural damage.

Blast resistant and blast proof analysis is an important consideration in the design of control structures at petroleum and petrochemical plants where there is a probability of an unconfined explosive vapor cloud forming due to a leak in the process. Other type of structures which must include an evaluation of the dynamic response to blast loading include military structures of all kind and nuclear containment vessels.

An in depth discussion of the development of the pressures developed during a blast and the mechanics involved in the evaluation of the dynamic response and structural resistance of a building is beyond the scope of this chapter. The reader is referred to the literature [9], [10], [11], for a more detailed discussion on the subject. The following is included only as an introduction.

When an explosion occurs in air, the sudden release of a large amount of energy produces a shock front which generates extremely high temperatures and pressures. This high intensity shock front expands outward from the point of detonation into the surrounding air at supersonic speed.

As the shock front continues to expand, it decays in strength. This is due to the rapid increase in volume of compressed air enclosed by the shock front which results in a corresponding decrease in pressure. Eventually, a partial vacuum is produced, since a large mass of air has been forced away from the point of detonation. The shock wave collapses inward, resulting in a negative pressure phase of the blast.

Figure 174 contains an illustration of a time dependent pressure function which is typical for an explosion.

The force of the air blast can be divided into the following three phases:

1. Initially, the structure experiences an increase in static air pressure above the normal atmospheric pressure. This increase in pressure is generally referred to as **overpressure.**
2. As the initial shock front passes the structure, dynamic drag pressures are developed due to the high velocity of winds directly behind the shock front.

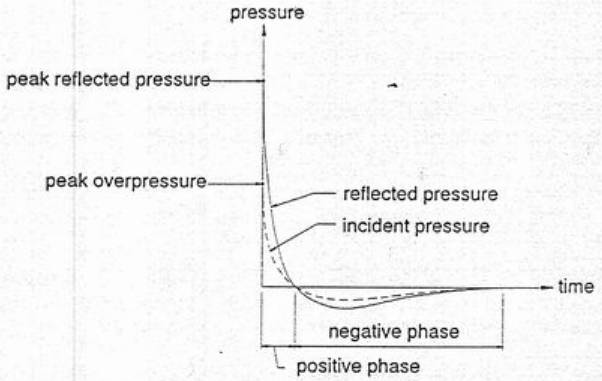


Figure 174 - *pressure-time variation for an air burst*

3. Finally, negative pressures are developed as the shock front collapses.

The resultant pressure on structural elements which are perpendicular to the direction of the shock wave will be greater than the overpressure associated with the blast. This is due to the sudden change in momentum of the shock front. This increased pressure is referred to as the reflected pressure. Vertical walls of blast resistant buildings are usually designed to resist the peak reflected pressure, whereas roofs are generally designed to resist the peak overpressure.

The duration of the positive pressure phase of the loading can be measured in milliseconds. The transient dynamic loading developed during an air blast is similar to an impact. It will result in the excitation of all resonant conditions as the damped response of the structure "rings down" toward equilibrium. This is probably the ultimate in impact tests!

The finite element method provides a tool by which the dynamic response and structural integrity of a building can be evaluated for any blast scenario. The following case history summarizes the methods and procedures of performing a finite element analysis for blast proof design.

12.1 Problem Description

The objective of this study was to evaluate the dynamic response of a reinforced concrete control house subjected to a specified blast loading. The control house was to be designed as a blast proof structure as opposed to a blast resistant structure. This requirement actually simplified the analysis. For the case of blast resistant analysis, the nonlinear structural behavior associated with the development of failure mechanisms has to be evaluated. The blast proof analysis can proceed assuming that the structural response of the control house remains linear and elastic throughout the entire event. A blast proof structure will be significantly larger than if it had been designed using the blast resistant philosophy.

The blast loading specified for the analysis consisted of a peak reflected pressure of 45 psi to be applied to the exterior walls of the control house and a peak overpressure of 20 psi applied to the roof. The duration of the positive pressure phase of the blast was specified to be 3 milliseconds (.03 sec). The pressure was assumed to decrease linearly from the specified magnitude to zero at the end of the positive pressure phase. The negative pressure phase of a blast is generally disregarded since the

magnitude of pressures developed during this phase are insignificant relative to the positive pressures associated with a blast.

12.2 Finite Element Analysis

The finite element model of the control house is illustrated in Figure 175. A two dimensional analysis of the control house was employed since the width of the building was much larger than the depth. Therefore, it was determined that this simplification could be made without sacrificing much in accuracy. Even with this simplifying assumption, the finite element method provides a more realistic evaluation than the approximate manual methods described in the references which treat the structure as a single degree of freedom vibrating mass and damper.

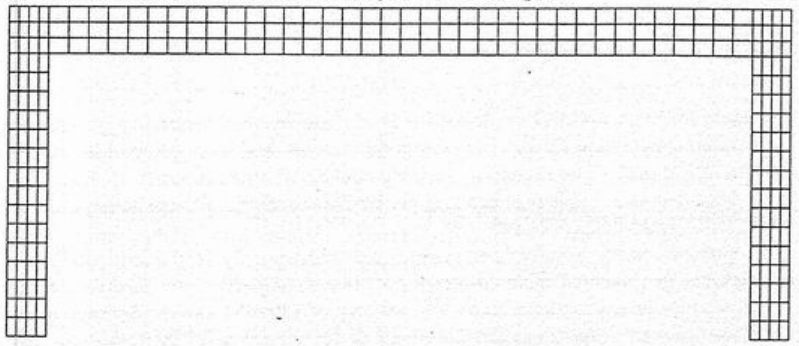


Figure 175 - *finite element model of the control house*

A sufficient number of two dimensional, solid plane strain elements were used to describe the mode shapes which were expected. Several elements were used through the thickness of the wall and roof since a significant amount of flexure was expected. Elements containing extra displacement shape functions were used to increase the accuracy of the solution.

The first step of the solution technique is to obtain the natural frequencies and associated mode shapes of the structure. A modal analysis of the finite element model was performed to determine the resonant conditions. Figures 176 and 177 contain the first two mode shapes as predicted by the finite element analysis.

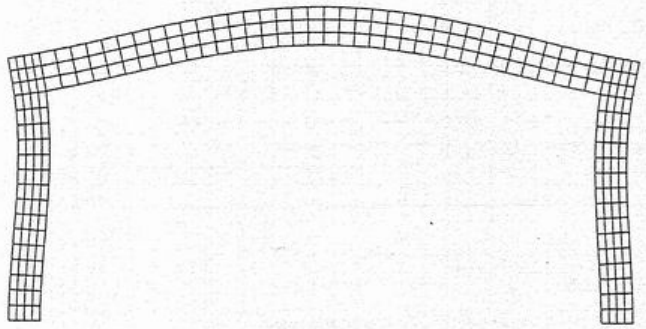


Figure 176 - *second mode shape of control house*

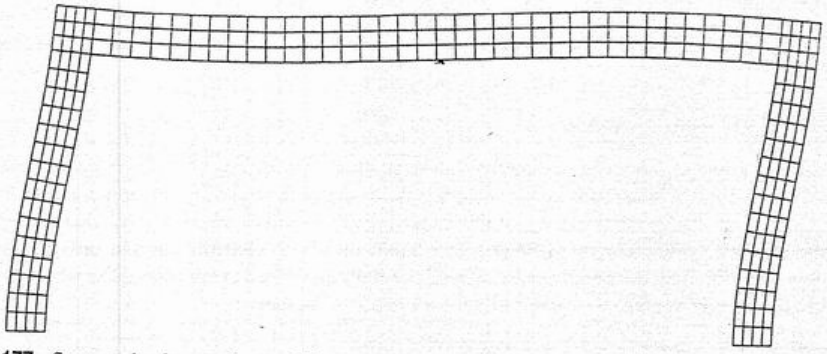


Figure 177 - *first mode shape of control house*

The first mode shape occurs at 11 hz. It is a "sidesway" mode characterized by flexure of the walls as the roof oscillates back and forth. The maximum stresses will be developed in the wall at the foundation and at the roof. The second mode shape occurs at approximately 17.9 Hz. This mode shape can be described as a simple roof bending mode. Due to the rigid frame action of the building, the side walls also flex as the roof bends.

Once the resonant conditions have been identified by modal analysis, the structure's response to the blast loading can be predicted using the technique of modal superposition described earlier in this chapter. The response analysis comes under the category of a linear transient dynamic analysis.

The predicted transient response of the front wall of the control building during the first second after the blast is shown in Figure 178. The point at which the response was obtained is located half

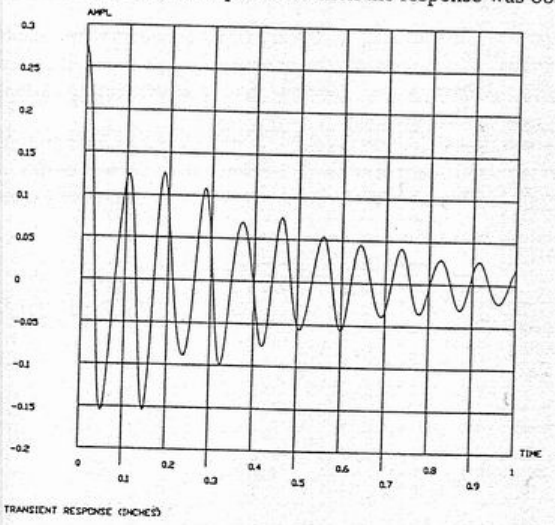


Figure 178 - *predicted response of control house front wall*

way between the foundation and the roof. Since the first mode shape of the building is predominant, there are eleven cycles in the first second, which corresponds to the principle resonance of 11 Hz. However, the response does not decay smoothly. This indicates the presence of other modes.

The final step of the analysis would be a stress pass to determine the stresses due to the blast. The magnitude of the stresses are then compared to the allowable values as per ACI code 318.[12]

13.0 CONCLUSION

The finite element method is a numerical technique which provides an *approximate* solution to complex engineering problems. The accuracy of the finite element solution is dependent upon many factors.

The most important of these are:

- the number of elements used to define the finite element mesh
- the shape of the elements relative to aspect ratio and skewness
- the type of elements used in the mesh
- the interpolation or shape function used to formulate the element stiffness and mass matrices
- the definition of boundary conditions
- the number and location of master degrees of freedom

The finite element technique can be used in vibration engineering to estimate the natural frequency of structures and determine the associated mode shapes. It can also be used to calculate the response of a structure to a dynamic loading condition.

14.0 REFERENCES

[1] "Introduction to the Finite Element Methods", C.S. Desai & J.F. Abel, Van Nostrand Company, N.Y., 1972.

[2] "The Finite Element Method in Engineering Science", O.C. Zienkiewicz, McGraw-Hill Company, London, 1977.

[3] "Finite Element Procedures in Engineering Analysis", K.J. Bathe, Prentice-Hall, Inc., New Jersey, 1982.

[4] "Variational Methods for the Solution of Problems of Equilibrium and Vibrations", R. Courant, Bulletin Amer. Math. Soc., Volume 49, 1943.

[5] "The Finite Element Method in Plane Stress Analysis", R.W. Clough, Journal of Structural Division, ASCE, Proceedings 2nd Conference on Electronic Computation, 1960.

[6] "Reduction of Stiffness and Mass Matrices", R.J. Guyan, AIAA Journal, Volume 3, No. 2, February 1965.

[7] "A Frontal Solution Program for Finite Element Analysis", B.M. Irons, International Journal for Numerical Methods in Engineering, Volume 2, No. 1, January, 1970.

[8] "A Non-Conforming Element for Stress Analysis", R.L. Taylor, P.J. Beresford & E. L. Wilson, International Journal for Numerical Methods in Engineering, Volume 10, 1976.

[9] "Structures to Resist the Effects of Accidental Explosions, Volume I, Introduction", U.S. Army Armament Research, Development and Engineering Center, Dover, New Jersey, December 1987.

[10] "Design of Structures to Resist Nuclear Weapons Effects", American Society of Civil Engineers, Manual of Engineering Practice Number 42, New York, N.Y., 1963

[11] "Protective Construction: Nuclear Blast Resistant Design", Federal Emergency Management Agency, TR-20, Volume 4, March 1985.

[12] "Building Code Requirements for Reinforced Concrete", American Concrete Institute, ACI-318, September 1983.

B APPENDIX B - EXAMPLE ANALYSES

INTRODUCTION

This Appendix contains descriptions of actual vibration analysis procedures performed by the authors. Each of these analyses provides an example of how the problem-solving techniques described in this volume are applied in the field.

ANALYSIS #1 - MULTIPLE RESONANT FREQUENCIES

A hospital had installed a large air handler to provide cool air for an operating room. The fan impeller in the air handler was driven by a 200 HP motor, connected to the impeller by five D-sized V-belts. Hospital maintenance personnel asked the authors to verify the air handler's design before its initial startup

The authors performed impact tests on the air handler base and on the belt strands, and discovered that the belt strand resonance was exactly equal to the motor mount resonance (both were 28 Hz). This was a very bad situation, since the snapping of the belt strand as it left the sheave would provide a forcing function at 28 Hz, which could excite the motor mount into resonance.

However, there was a bit of uncertainty in the analysis: the 28 Hz vibration detected by impacting the motor mount could have been the belt strand resonance instead of the motor mount resonance. The best way to tell for sure if the motor mount was resonant at 28 Hz would have been to remove the belts. If the 28 Hz vibration disappeared when the mount was impacted without belts, the authors would have been sure that the belt strand was the source. If the 28 Hz vibration was still present after the belts had been removed, however, the authors would have been certain that the motor mount was also resonant at that frequency.

Unfortunately, it was not practical to disassemble the air handler to remove the belts. So, having been warned of the potential danger, the hospital personnel started the air handler. And the handler literally destroyed itself! The belts flew off and flew through the filter. It was shaking so badly that the authors and the hospital personnel ran from the structure.

The top plot in Figure 179 is a time domain plot of the air handler's vibration during startup. Notice the total displacement around 70 seconds, where the air handler was moving over 16 inches per second (over 1 foot!).

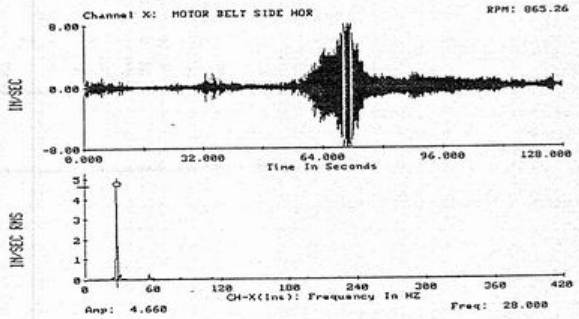


Figure 179 - *time and frequency domain plots of air handler vibration*

The bottom plot in Figure 179 is a frequency domain spectra of the area of maximum displacement. Notice the large spike at 28 Hz, which is both the belt strand resonance and the motor mount resonance.

This example illustrates the importance of ensuring that a structure's resonance does not coincide with a forcing function within the system.

ANALYSIS #2 - BEAT FREQUENCY BETWEEN SPINDLES

The authors were called to analyze a vibration problem in a multi-spindle machine used to manufacture pistons for V8 engines. Each spindle performs a different cut, so that it only takes about 8 seconds to machine a piston. However, each spindle runs at a different speed from the others. This creates beat frequencies which are present as vibrations in the machine.

Figure 180 shows a time-domain plot of the beat frequencies between spindles 3 and 4. Notice that there are 15 peaks in the 16-second plot (slightly less than one peak per second).

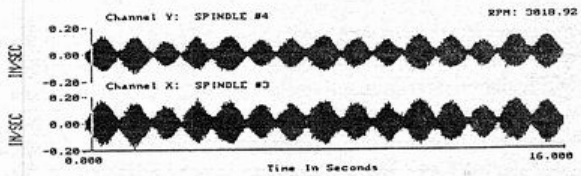


Figure 180 - *beat frequency between two boring spindles*

Since one piston is machined in about eight seconds, this beat frequency occurs during the machining of one piston. This causes the pattern shown in the plot to occur on the surface of the piston, which is unacceptable.

Ideally, these two spindles should run at the same speed; this would completely eliminate the beat. This was not a pragmatic solution, since it would be difficult to maintain the exact speed all the time. The authors attempted to bring the speeds of the two spindles closer together, which would spread the beat farther apart. Once the beat was minimized, the authors balanced the spindles.

Figure 181 shows the effect of the speed change and balancing on the beat frequencies between the spindles.

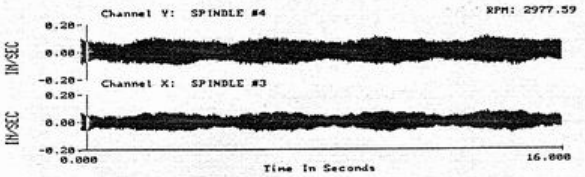


Figure 181 - *beat frequency plot after balance*

Now there are only 4 beats in 16 seconds, and the peaks are much lower in amplitude than in Figure 180. Now only two peaks occur in an 8-second machining pass, which creates an acceptable surface tolerance for the piston.

ANALYSIS #3 - HARMONIC GROUPS OF BELT FREQUENCY

A large manufacturer uses several belt-driven fans to provide air for a coating process. The plant personnel asked the authors to bring all vibration levels below .08 in/sec, since fan vibration was affecting the uniformity of the coating.

The authors began their analysis by calculating the belt frequencies in the fans (refer to Formula 15-1 in the Power Transmission Components chapter). These calculations showed that the first and second orders of the primary belt frequency were the source of vibration in the system.

Figure 182 shows a frequency domain spectra of the fan vibration. The source of each vibration peak is identified.

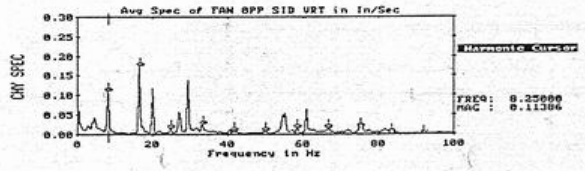


Figure 182 - *frequency domain spectra of fan vibration*

The authors aligned the fan sheaves and installed new belts, and were able to reduce the vibration to below .06 in/sec.

ANALYSIS #4 - RUBBING GIBS

An engine manufacturer called the authors to analyze a table used to machine engine blocks. This was a large, circular table, capable of holding several blocks. The table indexes around to pre-defined locations, where motors are suspended from the ceiling. Each motor drives a cutter; when the block stops below the cutter, the motor/cutter assembly moves in and machines the block. When the cut is complete, the motor/cutter assembly moves away and the table rotates to the next position. When a block has made the complete circuit around the table, it is fully machined and is removed from the table.

The cutter at Station 2 was driven by an 80 HP motor suspended vertically. Excessive chatter at this station was causing unacceptable machining on the block. The authors took a time domain plot of the Station 2 cutter, which is shown in Figure 183.

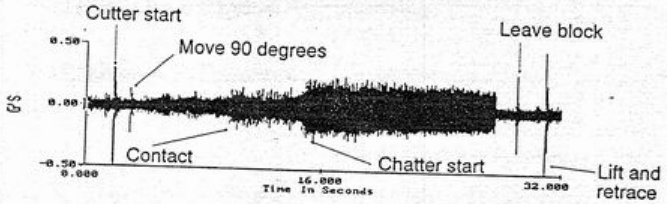


Figure 183 - *time domain plot of Station 2 cutter*

Figure 183 shows vibration at six steps in the cutting operation:

1. **Cutter start** - this is when the cutter first begins its motion into the table.
2. **Move 90 degrees** - the cutter moves 90 degrees to its initial path, heading toward the block.
3. **Contact** - the cutter first comes in contact with the block and begins cutting.
4. **Chatter start** - the chatter begins approximately 30% of the way into the machining operation, causing excessive vibration.
5. **Leave block** - the point at which the cutter has completed its path and is no longer in contact with the block.
6. **Lift and retrace** - the motor/cutter assembly raises up and returns to its starting place.

The customer assumed that the chatter was due to another spindle on machine. He timed how long it took for the chatter at Station 2 to begin, and discovered that another spindle on the table changed speed at the exact same moment that the chatter started. The authors suggested that the other spindle should be shut off to verify this hypothesis.

However, it wasn't possible to shut off other spindle. The machine operator suggested that, instead, they should remove all the other blocks from the table. That way, if the interaction between the other spindle and its block was causing the chatter at Station 2, the chatter should disappear when all the other stations were empty. Figure 184 shows the time domain plot at Station 2 when all the other stations were empty.

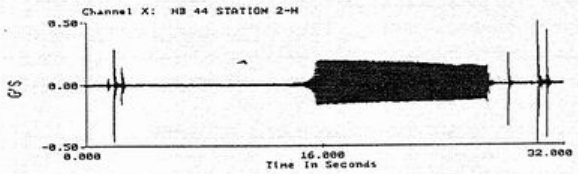


Figure 184 - *vibration at Station 2 only*

Figure 184 shows the vibration at Station 2 when the table was empty, and the cutter was just moving in air. Notice that, although the vibrations associated with cutter movement and block contact are gone, the chatter is still present.

The authors suggested that, since the chatter was still present, it must be caused by the gibs squealing. A *gib* is a wearing material (such as brass) that supports a motor yet is free to slide in a channel. The gibs on Station 2 were rubbing in the channel too tight, and began to vibrate at that point in the machining process where the chatter began. The authors readjusted the gibs and the chatter ended. Figure 185 shows the time domain plot taken after the gibs were readjusted.

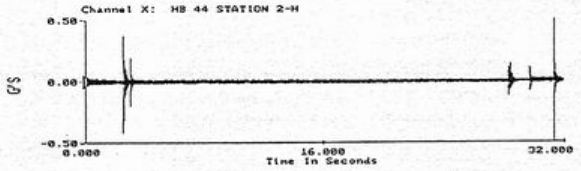


Figure 185 - *time domain plot after readjusting gibs*

As it turns out, it was purely coincidental that the other spindle began its speed change at the same time that the chatter began.

ANALYSIS #5 - USING COHERENCE

The authors were called to diagnose another multi-spindle machine that was experiencing excessive vibration at one spindle. This vibration was causing a high reject rate since the part machined by that spindle was affected by the vibration. The authors mounted a transducer on the spindle housing (a casting) and performed an impact test to determine the casting's resonant frequency. Figure 186 shows the coherence plot obtained from the impact.

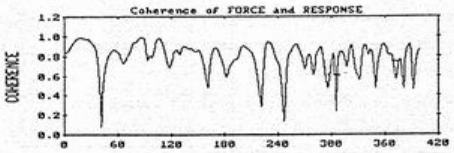


Figure 186 - *coherence plot obtained from impacting spindle*

This coherence was very poor, suggesting some non-linearity in the housing. On castings such as this, a crack is a very common non-linearity. The authors cleaned the casting and discovered a 7-inch crack around its base. Plant maintenance personnel replaced the housing with a new casting, and the authors performed another impact test. The coherence for the new casting is shown in Figure 187.

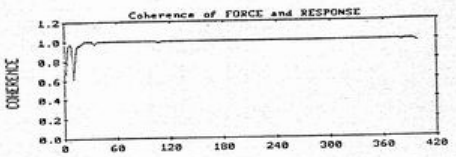


Figure 187 - *coherence after repairing spindle casing*

This analysis demonstrates the value of coherence. The impact test is easy to set up and perform, and saves a lot of money by quickly locating non-linearities that a balancing procedure would be unable to correct.

ANALYSIS #6 - BEARING FAULT

The authors were called to investigate high noise levels in a two-spindle boring machine at a large engine manufacturer. This boring machine is used to manufacture six-cylinder engine blocks. The machine bores two cylinders, redraws and indexes down the block, and bores the next two cylinders. It rebores a total of three times to complete a six-cylinder block.

Each cutting tool mounted in the spindles has two cutting heads. The first head cuts a hole the entire length of the cylinder, to accommodate the connecting rod for the piston. The second head is mounted above, and is slightly larger than, the first head. This second head cuts partway down the cylinder, and creates the smooth surface that the piston rings slide against. Figure 188 shows the time domain plot of the boring machine vibration.

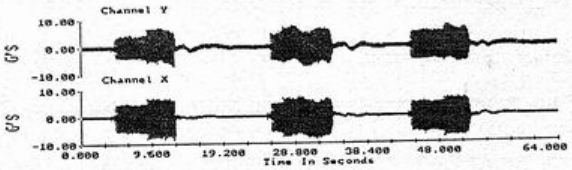


Figure 188 - *time domain plot of boring machine vibration*

This plot has three noticeable areas of high vibration, where the cutters were vibrating excessively as they cut the cylinder. Consequently, the fine cut for the piston volume was unacceptably rough. Also, the machine was very loud during the cut.

A frequency domain analysis of these areas of high vibration showed a definite spike at 320 Hz. After performing an analysis of possible forcing functions, the authors discovered that 320 Hz was the outer race fault frequency for the spindle bearings.

The spindle bearings were replaced, and the authors took the time domain plot in Figure 189.

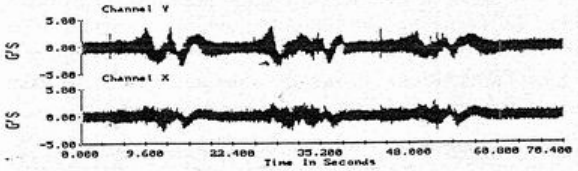


Figure 189 - *time domain plot after replacing spindle bearings*

ANALYSIS #7 - PRESSURE PULSATIONS

A large surface grinder was installed at a factory on a massive isolation pad. The isolation pad is a reinforced concrete slab 6 feet wide, 5 feet high, and 42 feet long. Parts being ground had unacceptable variations in the surface that were caused by vibration. The authors took vibration readings all over the machine, and found that this vibration was at 296 Hz (refer to Figure 190). This frequency was all over the machine, even on the pad!

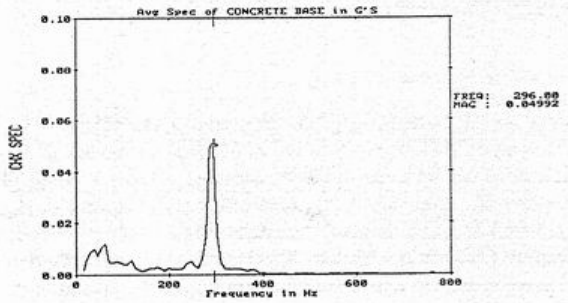


Figure 190 - *vibration reading at 296 Hz*

The authors first assumed that the vibration was coming from another machine through the floor, since nothing in the grinder was rotating at 296 Hz (the grinder motor was only 1,200 RPM - 20 Hz). They mounted a transducer on the plant floor, only two inches from the isolation pad, and recorded the frequency domain spectra shown in Figure 191.

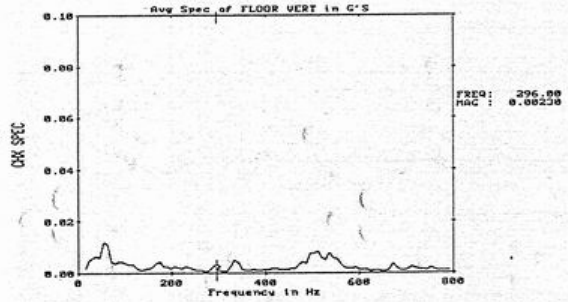


Figure 191 - *vibration on plant floor, adjacent to pad*

The 296 Hz vibration is there, but is extremely low. This indicates that the vibration is obviously not coming from the floor. So, the authors looked further, and discovered a 1/3 HP motor driving a lubrication pump on the grinder. It seemed very unlikely that a tiny 1/3 HP motor would be shaking a multi-ton concrete pad, but the authors had to be sure. They shut off the lubrication pump and took another vibration reading on the pad, which is shown in Figure 192.

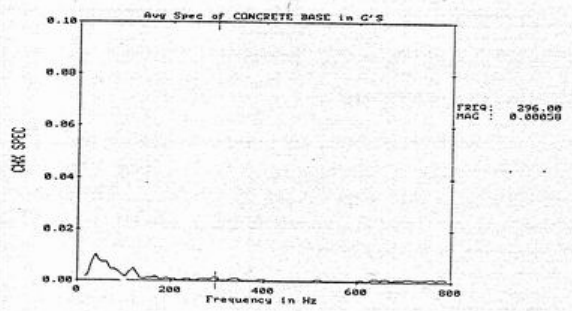


Figure 192 - *vibration on machine with lube pump off*

Figure 192 indicated that the lubrication pump was the source of the vibration (even in the pad!).

Solving this problem presented a challenge. There *must* be lubrication on the part being run through the grinder. One solution is to use an **accumulator**, which is a large volume placed in-line to accumulate fluid and damp out vibration. However, accumulators don't work on high frequencies.

Instead, the authors created a dynamic damper. They placed two tees in the lubrication line that allowed part of the lubricant to flow off in a branch. The branch was reconnected to the line farther on. The length of the branch determined where the lubricant was in the pulsation cycle when it was re-introduced into the main line. After a few experiments, the branch was set to a length that re-introduced the pulsating lubricant 180 degrees out of phase with the pulsations in the main line. This canceled out the pulsations and reduced the vibration, as shown in Figure 193.

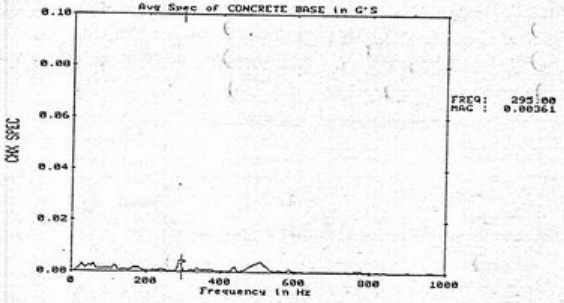


Figure 193 - *vibration after inserting tee in lube pump line*

Figure 193 shows that the dynamic damper reduced the amplitude at 296 Hz down to .0036 g's; a better than 10 to 1 improvement.

C - APPENDIX C - RESONANT FREQUENCY CALCULATIONS

INTRODUCTION

This Appendix contains 10 formulas for calculating approximate resonant frequencies. Five common cases of shaft, wheel, and bearing support configuration are shown, as well as five common cases of loaded beams. The formula for calculating the approximate resonant frequency of that shaft (or beam) is given with a brief explanation of its use.

Definition of Terms

Several terms, common in the field of physics, are used in this Appendix:

- **Lumped mass** - the assumption that all the mass of a particular machine component is concentrated at one point. In reality, machine components are large, and their mass is spread out over a large area. However, assuming that a component is a lumped mass greatly simplifies the equations. The lumped mass in each configuration is denoted in the equations as *W*.
- **Moment of inertia** - a measure of a body's resistance to angular acceleration, equal to the sum of the products of each mass element multiplied by the square of its distance from the rotational axis.

- **Shear Modulus** - a measure of the rigidity of a body (its resistance to deflection), equal to the shearing stress (a non-longitudinal force) divided by the shearing strain (the deflection angle resulting from the non-longitudinal force).
- **Young's Modulus** - the longitudinal elasticity of a structural member, equal to the longitudinal stress (the force stretching or compressing the object) divided by the longitudinal strain (the change in length as a result of the stress). Within a material's elastic limits, elastic deformation (strain) is proportional to the applied force (stress).

Assumptions

Several assumptions have been made in the derivation of these formulas. These assumptions do not, by themselves, add significant error to the frequency calculations. Even when taken together, these assumptions still allow you to obtain a very reasonable approximation of the shaft's resonant frequency.

- The shaft or beam is massless. You can add in the effect of the shaft or beam if you desire by using the procedure described in section 11 of this Appendix.
- Cases 1 through 5 only include translational inertia, since the center of gravity of the masses lies along the centerline of the shaft. Cases 6 through 10 include both rotational and translational inertia, since the center of gravity of the masses does not lie along the beam's centerline.
- Cases 6 through 10 assume that the masses exert a load at a single point on the beam (that is, that they are lumped masses).
- The bearings provide no moment to the shaft.
- The first mode shape at resonance is nearly approximated by the static deflection of the shaft or beam. This assumption introduces very little error since the static deflection is a close approximation of the first mode shape. Errors in the calculated resonant frequency will be roughly 10% of the error in the assumed mode shape.
- The system has no damping; all energy exists as either kinetic (undeflected) energy or potential (deflected or strain) energy. Although all systems have damping, this assumption does not introduce much error in the calculation: engine/compressor and rotor systems as described here are very lightly damped.

1.0 SHAFT CASE ONE

This case shows a shaft with a single wheel supported by two bearings. The wheel is inboard of both bearings.

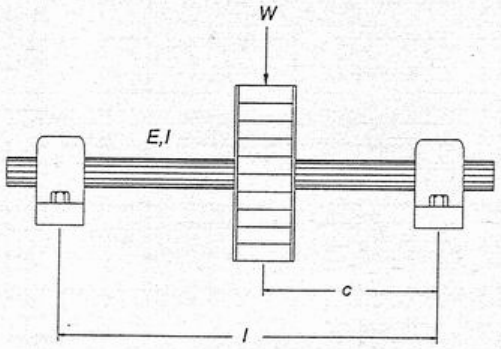


Figure 194 - *Shaft Case One: single wheel*

(Formula C-1)

$$\omega_n = \sqrt{\frac{3EIgl}{Wc^4}\left(\frac{1}{\left(\frac{l}{c}-1\right)^2}\right)}$$

where

ω_n = natural frequency (radians/sec)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
g = acceleration due to gravity (in./sec^2)
l = distance between bearing centers (inches)
W = weight of lumped mass (pounds)
c = distance to lumped mass (inches)

Even though Figure 194 shows the lumped mass (the wheel) mounted off-center, Formula C-1 can also be used in cases where the wheel is mounted in the center of the shaft. In this case, $c = 1/2l$.

Refer to section 11 for information on adding the mass of the shaft to the frequency calculation.

2.0 SHAFT CASE TWO

This case shows a shaft with a single cantilevered wheel supported by two bearings. The wheel is outboard of both bearings.

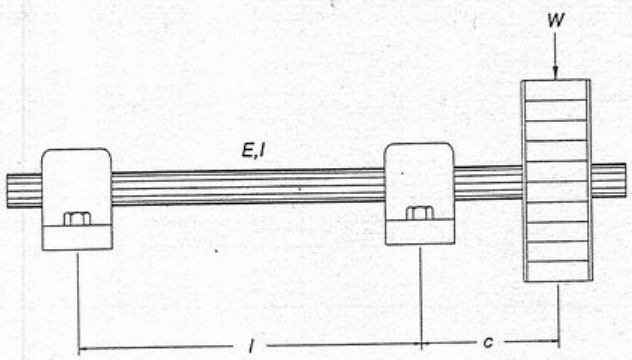


Figure 195 - *Shaft Case Two: outboard mass*

$$\omega_n = \sqrt{\frac{3EIg}{Wc^3}\left(\frac{1}{\frac{l}{c}+1}\right)} \qquad \text{(Formula C-2)}$$

where

ω_n = natural frequency (radians/sec)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
g = acceleration due to gravity (in./sec^2)
W = weight of lumped mass (pounds)
c = distance to lumped mass (inches)
l = distance between bearing centers (inches)

3.0 SHAFT CASE THREE

Case three shows a shaft with two cantilevered wheels supported by two bearings. Both wheels are outboard of the bearings, one on each end of the shaft.

To calculate the resonant frequency of this configuration, you must first calculate the static deflection at each lumped mass (how much each mass deflects the shaft due to its own weight).

$$y_1 = \frac{W_1 c_1^3}{3EI}\left(\frac{l}{c_1}+1\right) + \frac{W_2 l c_2^2}{6EI} \qquad \text{(Formula C-3)}$$

$$y_2 = \frac{W_1 l c_2^2}{6EI} + \frac{W_2 c_2^3}{3EI}\left(\frac{l}{c_2}+1\right) \qquad \text{(Formula C-4)}$$

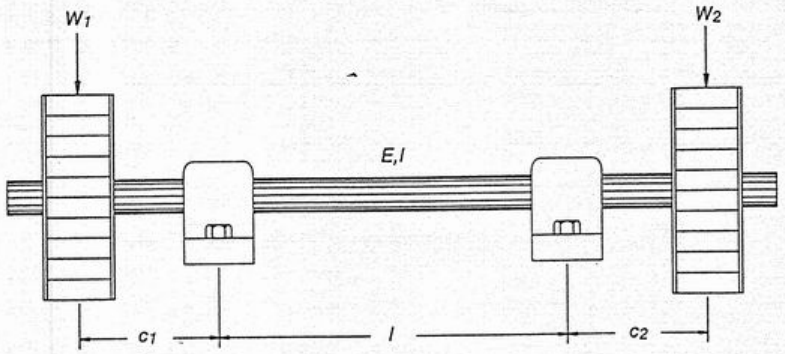


Figure 196 - *Shaft Case Three: two outboard masses*

where

y_1 = static deflection of lumped mass W_1 (inches)
y_2 = static deflection of lumped mass W_2 (inches)
W_1 = weight of lumped mass 1 (pounds)
c_1 = distance from W_1 to closest bearing (inches)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
l = distance between bearing centers (inches)
W_2 = weight of lumped mass 2 (pounds)
c_2 = distance from W_2 to closest bearing (inches)

Now calculate the approximate resonant frequency using the following formula:

$$\omega_n = \sqrt{g \frac{W_1 y_1 + W_2 y_2}{W_1 y_1^2 + W_2 y_2^2}} \qquad \text{(Formula C-5)}$$

where

ω_n = natural frequency (radians/sec)
g = acceleration due to gravity (in./sec^2)
W_1 = weight of lumped mass 1 (pounds)
y_1 = static deflection of lumped mass W_1 (inches)
W_2 = weight of lumped mass 2 (pounds)
y_2 = static deflection of lumped mass W_2

4.0 SHAFT CASE FOUR

Case four shows two wheels on a single shaft supported by two bearings. One wheel is cantilevered (outboard of both bearings), while the other wheel is between the two bearings.

To approximate the resonant frequency of this configuration, you must first calculate the static deflection at each lumped mass (how much does each mass deflect the shaft due to its own weight).

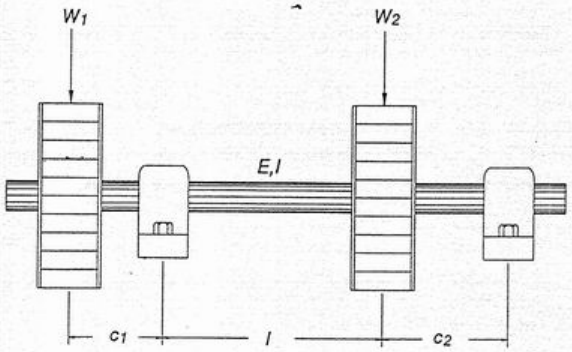


Figure 197 - *Shaft Case Four: one mass inboard, one mass outboard*

$$y_1 = \left| \frac{W_1 c_1^{\,3}}{3EI} \left(\frac{l}{c_1} + 1 \right) + \frac{W_2 c_1 c_2}{6EIl} \left(l^2 - c_2^{\,2} \right) \right| \qquad \text{(Formula C-6)}$$

$$y_2 = \left| \frac{W_2 c_2^{\,4}}{3EIl} \left(\frac{l}{c_2} - 1 \right) + \frac{W_1 c_1 c_2}{6EIl} \left(l^2 - c_2^{\,2} \right) \right| \qquad \text{(Formula C-7)}$$

where

y_1 = static deflection of lumped mass W_1 (inches)
y_2 = static deflection of lumped mass W_2 (inches)
W_1 = weight of lumped mass 1 (pounds)
c_1 = distance from W_1 to closest bearing (inches)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
l = distance between bearing centers (inches)
W_2 = weight of lumped mass 2 (pounds)
c_2 = distance from W_2 to closest bearing (inches)

Unlike Shaft Case Three, where the terms were added, the two terms are subtracted here. Therefore, it is possible that y_1 or y_2 could come out negative, depending on the values of the two terms. A negative static deflection is meaningless, so the absolute value is used to ensure that the static deflections are a positive amount.

Calculate the approximate resonant frequency using the following formula:

$$\omega_n = \sqrt{g \frac{W_1 y_1 + W_2 y_2}{W_1 y_1^{\,2} + W_2 y_2^{\,2}}} \qquad \text{(Formula C-8)}$$

where

ω_n = natural frequency (radians/sec)
g = acceleration due to gravity (in./sec^2)
W_1 = weight of lumped mass 1 (pounds)
y_1 = static deflection of lumped mass W_1 (inches)
W_2 = weight of lumped mass 2 (pounds)
y_2 = static deflection of lumped mass W_2

5.0 SHAFT CASE FIVE

This case shows three wheels on a shaft supported by two bearings. Two of the wheels are outboard of the bearings, while one is between the bearings.

First, you must calculate the static deflection of each of the wheels using the following formulas:

(Formula C-9)

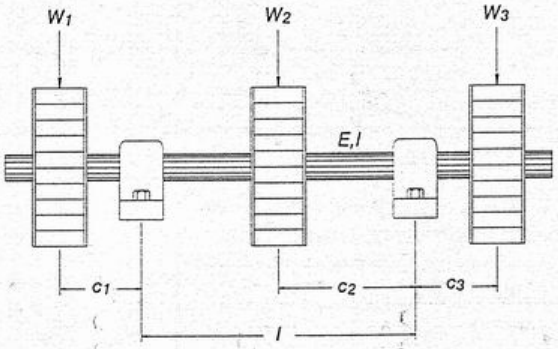


Figure 198 - *Shaft Case Five: two outboard wheels, one inboard wheel*

$$y_1 = \frac{W_1 c_1^3}{3EI}\left(\frac{l}{c_1}+1\right) + \frac{W_2}{6EI}\left(\frac{c_1 c_2}{l}\right)(l^2 - c_2^2) + \frac{W_3 l c_1^2}{6EI}$$

(Formula C-10)

$$y_2 = \frac{W_1 c_1 c_2}{6EIl}(l^2 - c_2^2) + \frac{W_2 c_2^4}{3EIl}\left(\frac{l}{c_2}-1\right)^2 + \frac{W_3 c_2 c_3}{6EIl}\left(l - c_2\right)\left(2l - c_2\right)$$

(Formula C-11)

$$y_3 = \frac{W_1 c_3^2 l}{6EI} + \frac{W_2 c_2 c_3}{6EIl}\left(l - c_2\right)\left(2l - c_2\right) + \frac{W_3 c_3^3}{3EI}\left(\frac{l}{c_3}+1\right)$$

where

y_1 = static deflection of lumped mass W_1 (inches)
y_2 = static deflection of lumped mass W_2 (inches)
y_3 = static deflection of lumped mass W_3 (inches)
W_1 = weight of lumped mass 1 (pounds)
c_1 = distance from W_1 to closest bearing (inches)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
l = distance between bearing centers (inches)
W_2 = weight of lumped mass 2 (pounds)
c_2 = distance from W_2 to closest bearing (inches)
W_3 = weight of lumped mass 3 (pounds)
c_3 = distance from W_3 to closest bearing (inches)

Calculate the approximate resonant frequency using the following formula:

$$\omega_n = \sqrt{g \frac{W_1 y_1 + W_2 y_2 + W_3 y_3}{W_1 y_1^2 + W_2 y_2^2 + W_3 y_3^2}} \qquad \text{(Formula C-12)}$$

where

ω_n = natural frequency (radians/sec)
g = acceleration due to gravity (in./sec^2)
W_1 = weight of lumped mass 1 (pounds)
y_1 = static deflection of lumped mass W_1 (inches)
W_2 = weight of lumped mass 2 (pounds)
y_2 = static deflection of lumped mass W_2
W_3 = weight of lumped mass 3 (pounds)
y_3 = static deflection of lumped mass W_3

6.0 BEAM CASE ONE

Beam case one shows a single lumped mass at the end of a single fixed beam.

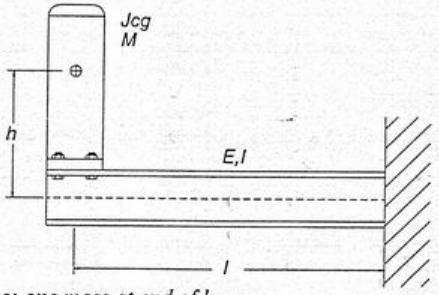


Figure 199 - *Beam Case One: one mass at end of beam*

The center of gravity of the mass is displaced from the center line of the beam. Therefore, this is a two degree of freedom system, where both vertical deflection and angular rotation can contribute to the motion of the mass.

First, we must define the quantity J, which is the effective mass moment of inertia of mass m with respect to the center line of the beam.

$$J = J_{cg} + mh^2$$

where

J = effective mass moment of inertia (slug-ft^2)
J_{cg} = mass moment of inertia at center of gravity of m (slug-ft^2)
M = mass (slugs)
h height of center of gravity of m (feet)

Also, to simplify the formulas in cases one through three, we will define a term B to equal a constant of several terms:

$$B = \sqrt{9J^2 + 3mJl^2 + m^2 l^4}$$

The formulas for calculating first- and second-order resonances are:

$$\omega_1 = \sqrt{\frac{2EI}{mJl^3}\left(3J + ml^2 - B\right)} \qquad \text{(Formula C-13)}$$

$$\omega_2 = \sqrt{\frac{2EI}{mJl^3}\left(3J + ml^2 + B\right)} \qquad \text{(Formula C-14)}$$

where

ω_1 = first-order natural frequency (radians/sec)
ω_2 = second-order natural frequency (radians/sec)
E = Young's Modulus (lb/ft^2)
I = moment of inertia (feet^4)
m = mass (slugs)
J = effective mass moment of inertia (slug-ft^2)
l = distance from fixed end of beam to center of gravity of m (feet)

7.0 BEAM CASE TWO

Beam case two consists of a single lumped mass at some distance l from the end of a single fixed beam. Unlike beam case one, the mass is not at the end of the beam.

First calculate the quantity J, which is the effective mass moment of inertia of mass m with respect to the center line of the beam.

$$J = J_{cg} + mh^2$$

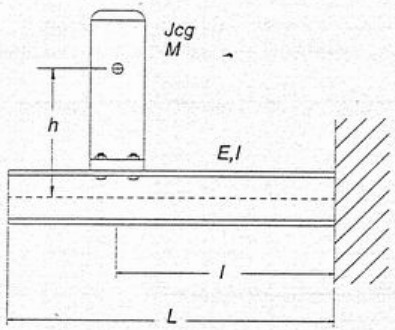


Figure 200 - *Beam Case Two: mass not at end of beam*

where

J = effective mass moment of inertia (slug-ft^2)
J_{cg} = mass moment of inertia at center of gravity of m
m = mass (slugs)
h = height of center of gravity of m (feet)

Also, to simplify the formulas in cases one through three, calculate the term B to equal a constant of several terms:

$$B = \sqrt{9J^2 + 3mJl^2 + m^2 l^4}$$

The formulas for calculating first-order resonance of this structure (assuming a massless beam) is:

$$\omega_n = \sqrt{\frac{2EI}{mJl^3}\left(3J + ml^2 - B\right)} \qquad \text{(Formula C-15)}$$

where

ω_1 = first-order natural frequency (radians/sec)
ω_2 = second-order natural frequency (radians/sec)
E = Young's Modulus (lb/ft^2)
I = moment of inertia (feet^4)
m = mass (slugs)
J = effective mass moment of inertia (slug-ft^2)
l = distance from fixed end of beam to center of gravity of m (feet)

Refer to Section 11.2 to see how to calculate the natural frequency including the beam mass.

8.0 BEAM CASE THREE

Beam case three shows two lumped masses supported by a single fixed beam. One of the masses is at the end of the beam, while the other mass is located closer to the fixed end of the beam.

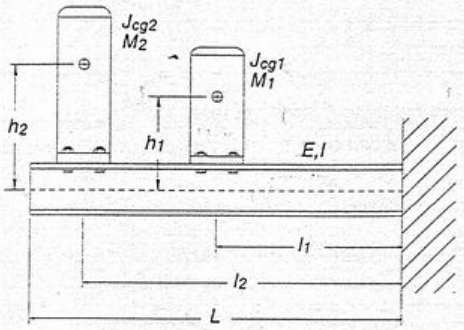


Figure 201 - *Beam Case Three: two masses on one beam*

The resonant frequency for this case is calculated in three steps:

- Calculate the resonant frequency as if mass m_1 were the only mass on the beam.
- Calculate the resonant frequency assuming mass m_2 is the only mass
- Combine the two frequencies following the procedure described in Section 11.2.

8.1 Resonant Frequency for Mass m_1

To find the structure's resonant frequency with mass m_1 only, first calculate the quantity J_1, which is the effective mass moment of inertia of mass m_1 with respect to the center line of the beam.

$$J_1 = J_{cg1} + m_1 h_1^2$$

where

J_1 = effective mass moment of inertia for mass $m1$(slug-ft^2)
J_{cg1} = mass moment of inertia at center of gravity of $m1$
M_1 = mass of m_1(slugs)
h_1 height of center of gravity of $m1$ (feet)

Then calculate the term B_1:

$$B_1 = \sqrt{9J_1^2 + 3m_1 J_1 l_1^2 + m_1^2 l_1^4}$$

The first-order resonance ω_1 of mass m_1 is:

$$\omega_1 = \sqrt{\frac{2EI}{m_1 J_1 l_1^3}\left(3J_1 + m_1 l_1^2 - B_1\right)} \qquad \text{(Formula C-16)}$$

where

ω_1 = first-order natural frequency (radians/sec)
E = Young's Modulus (lb/ft^2)
I = moment of inertia (feet^4)
m_1 = mass (slugs)
J_1 = effective mass moment of inertia of mass $m1$ (slug-ft^2)
l_1 = distance from fixed end of beam to center of gravity of $m1$ (feet)

8.2 Resonant Frequency for Mass m_2

To find the structure's resonant frequency with mass m_2 only, first calculate the quantity J_2, which is the effective mass moment of inertia of mass m_2 with respect to the center line of the beam.

$$J_2 = J_{cg2} + m_2 h_2^2$$

where

J_2 = effective mass moment of inertia for mass $m2$(slug-ft^2)
J_{cg2} = mass moment of inertia at center of gravity of $m2$
M_2 = mass of m_2(slugs)
h_2 = height of center of gravity of m_2 (feet)

Then calculate the term B_2:

$$B_2 = \sqrt{9J_2^2 + 3m_2 J_2 l_2^2 + m_2^2 l_2^4}$$

The first-order resonance ω_2 of mass m_2 is:

$$\omega_2 = \sqrt{\frac{2EI}{m_2 J_2 l_2^3}\left(3J_2 + m_2 l_2^2 - B_2\right)} \qquad \text{(Formula C-17)}$$

where

ω_2 = first-order natural frequency (radians/sec)
E = Young's Modulus (lb/ft^2)
I = moment of inertia (feet^4)
m_2 = mass (slugs)
J_2 = effective mass moment of inertia of mass $m2$ (slug-ft^2)
l_2 = distance from fixed end of beam to center of gravity of $m2$ (feet)

8.3 The Combined Natural Frequency

The natural frequency ω_b for the beam without the masses is found by

$$\omega_b = 3.52\sqrt{\frac{EI}{m_b L^3}} \qquad \text{(Formula C-18)}$$

where

ω_b = natural frequency for a massless beam (radians/sec)
E = Young's Modulus (lb/ft^2)
I = moment of inertia (feet^4)
m_b = total of masses $m1$ and $m2$ (slugs)
L = total length of beam (feet)

The natural frequency ω_n for the structure including the mass of the beam is found by

$$\omega_n = \sqrt{\frac{\omega_1\omega_2\omega_b}{(\omega_1\omega_2)^2 + (\omega_1\omega_b)^2 + (\omega_2\omega_b)^2}} \qquad \text{(Formula C-19)}$$

9.0 BEAM CASE FOUR

Beam case four shows a single mass supported by two fixed beams. Both beams are the same length, and the mass is located near the end of both beams.

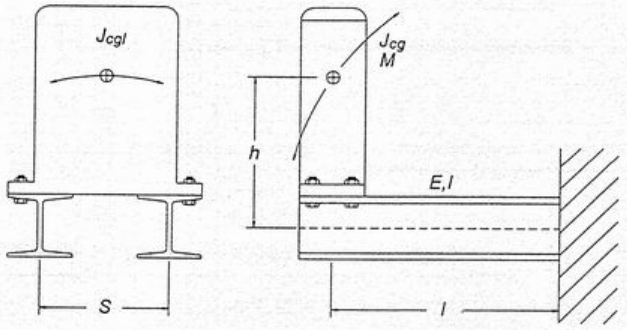


Figure 202 - *Beam Case Four: one mass supported by two beams*

This system has both a vertical (up and down) resonance and a torsional resonance. The vertical resonance is calculated exactly as the procedure shown in 7.0, Beam Case 2. The formula for calculating the torsional resonance ω_t is

$$\omega_t = \sqrt{\frac{6EI\,S^2 + 2GJl}{l^3(J_{cgl} + mh^2)}}^{\,2} \qquad \text{(Formula C-20)}$$

where

ω_t = torsional resonance (radians/sec)
E = Young's Modulus (lb/ft^2)
I = moment of inertia (feet^4)
S = distance between beam centers (feet)
G = shear modulus (1.6 * 10^9 lb/ft^2)

J = polar area moment of the I-beam section
l = length from fixed end of beam to center of gravity of m (slug-ft^2)
J_{cgl} = polar mass moment of inertia about longitudinal axis through center of gravity of m
m = mass (slugs)
h = height from beam center to center of gravity of m (feet)

10.0 BEAM CASE FIVE

Beam case five shows a skid (a platform that is not attached to the floor or any fixed structure) supporting four lumped masses.

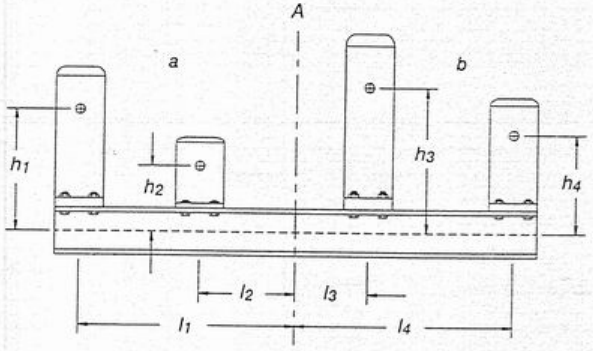


Figure 203 - *Beam Case Five: skid resonance*

First, assume that the first mode shape has a node (a point of zero displacement) somewhere along the length of the skid. We will assume that this node is at point A, somewhere between the second and third masses. If this assumption is true for the skid, then each half of the skid can be considered as a system similar to the one shown in Section 8.

Having made this assumption, calculating the skid's natural frequency is an iterative procedure:

1. Measure length l_1, the distance from the end of the skid to your assumed node point A. From l_1, find lengths l_2 through l_4 as shown in Figure 203.
2. Calculate ω_a, the natural frequency for that portion of the skid to the left of node point A. Also calculate ω_b, the natural frequency for the right portion of the skid. Use the method described in Section 8 to calculate ω_a and ω_b
3. If $\omega_a > \omega_b$, then move point A to the right a small amount. Recalculate lengths l_1 through l_4 and repeat step 2.
4. If $\omega_a < \omega_b$, then move point A to the left a small amount. Recalculate lengths l_1 through l_4 and repeat step 2.
5. If $\omega_a \approx \omega_b$, then the iteration has converged and the natural frequency ω_n of the skid is

$$\omega_n = \omega_a = \omega_b \qquad \text{(Formula C-21)}$$

11.0 ADDING SHAFT OR BEAM MASS

The method for including the mass of the shaft (or the beam) is very simple:

- Calculate the natural frequency for the system, assuming that the shaft or beam has no mass.
- Calculate the natural frequency of the shaft or beam by itself.
- Combine the two calculations.

11.1 Adding Shaft Mass

Follow these steps to include the mass of the shaft in your resonant frequency calculations for shaft cases one through five:

- First use formulas C-1 through C-5 as shown here to calculate the natural frequency for the massless shaft. Call this answer ω_{nw}.
- Next, calculate the natural frequency for the shaft by itself, with no lumped masses. Call this answer ω_{ns}. The shaft's natural frequency can be approximated by

$$\omega_{ns} = 9.87 \sqrt{\frac{EI}{m_s L^3}} \qquad \text{(Formula C-22)}$$

where

ω_{ns} = natural frequency of shaft (radians/sec)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
m_s = total mass of shaft
L = length of shaft (inches)

- Finally, combine ω_{nw} and ω_{ns} using the following formula:

$$\omega_n \approx \frac{\omega_{nw}\omega_{ns}}{\sqrt{\omega_{nw}^2 + \omega_{ns}^2}} \qquad \text{(Formula C-23)}$$

where

ω_n = natural frequency of system including shaft mass (radians/sec)

11.2 Adding Beam Mass

Follow these steps to include the mass of the beam in your resonant frequency calculations for beam cases one through five:

- First use formulas C-6 through C-10 as shown here to calculate the natural frequency for the massless beam. Call this answer ω_{nw}.
- Next, calculate the natural frequency for the beam by itself, with no lumped masses. Call this answer ω_{nb}. The beam's natural frequency can be approximated by

$$\omega_{nb} = 3.52\sqrt{\frac{EI}{m_b L^3}} \qquad \text{(Formula C-24)}$$

where

ω_{nb} = natural frequency of beam (radians/sec)
E = Young's Modulus (lb/in^2)
I = moment of inertia (inches^4)
m_b = total mass of beam
L = length of beam (inches)

- Finally, combine ω_{nw} and ω_{nb} using the following formula:

$$\omega_n = \sqrt{\frac{\omega_{nw}^2 \omega_{nb}^2}{\omega_{nw}^2 + \omega_{nb}^2}} \qquad \text{(Formula C-25)}$$

where

ω_n = natural frequency of system including beam mass (radians/sec)

GLOSSARY OF TERMS

acceleration - Change in velocity with respect to time.

accelerometer - A transducer that measures acceleration (also see **transducer**).

aliasing - A false signal equal to the difference between the input frequency and the sampling rate of a digital analyzer.

amplification range - The range of operating frequencies where the vibratory motion due to the forcing function is amplified by the system.

amplitude - The distance of a mass from its at rest position. When vibratory motion is plotted as a sine wave, the peak amplitude is the maximum distance of the wave from the zero line.

analog analyzer - An electronic vibration analyzer whose output is proportional (or ANALOGous) to its input. Analog analyzers display the amount of vibration on an analog meter.

attenuation - The decrease in strength of an electrical signal, usually as a result of the distance the signal travels. As electrical signals and vibratory waves travel, they decrease in strength the farther they travel.

axial plane - The plane in which the axis of a shaft or bearing lies.

bandwidth - The range of frequencies that a transducer or analyzer can accommodate. For filters, bandwidth specifies the smallest frequency difference that can be detected (this is also called **resolution**). Also see **constant percentage bandwidth filter**.

baseband - The maximum frequency of interest in a vibration signal.

bearing misalignment - A misalignment condition that results when the bearings supporting a shaft are not aligned with each other. The bearings may be mounted in different planes, cocked relative to the shaft, or distorted due to foundation settling or thermal growth.

beat frequency - A frequency that results when two or more primary frequencies are "beat" together (are present at the same time). The sum and difference of these primary frequencies create new frequencies called beat frequencies.

belt strand - That portion of a belt that is not in direct contact with one of the sheaves.

blade pass frequency - For fans, a frequency equal to the number of blades times the RPM of the fan.

cavitation - A condition that occurs when a local pressure drop inside a pump allows cavities filled with fluid vapor to form. These cavities collapse when they contact the high pressure side of the pump, causing both noise and vibration. This same condition in fans and blowers is caused **starvation**.

chord - A line segment that joins two points on a curve or circle.

circular frequency - The amount of revolution in a unit time.

coherence - The linear relationship between two signals. Coherence is calculated over several averages by multiplying individual frequency values by their corresponding output values. The coherence at the first average is always 1 or 100%, and approaches zero as the number of averages increases due to the contribution of non-linear components.

commutator - A device used to reverse the current flow in the armature coil of a DC motor. A commutator is basically a split metal ring that is attached to the ends of the armature coil. An external DC source is applied to the commutator, and the split in the ring reverses the current flow halfway through a revolution.

conjugacy - The condition where the normal to the common tangent at the point of contact between two gear teeth passes through the pitch point, and the pitch point lies on the center to center line of the two gears. Conjugacy is necessary to maintain a constant velocity ratio between the gears.

constant percentage bandwidth filter - A filter whose frequency resolution increases as the frequency to which it is tuned decreases. These filters are used in analog analyzers to tune in to specific vibration components.

contact angle - The angle at which a rolling element in an anti-friction bearing contacts the bearing races. Changes in bearing load often change the contact angle.

continuous spectra - A frequency domain display that results from an impact, since energy is distributed across the entire baseband. Compare with **line spectra**.

Coulomb damping - Another way to refer to vibratory damping (also see **damping**).

couple imbalance - Two mass imbalances that are 180 degrees apart. Vibration readings taken at both ends of a rotor with pure couple imbalance would be 180 degrees out of phase. Compare with **static imbalance**.

CPS - Cycles per second (also called **Hertz**).

crest factor - The peak value of a wave divided by the wave's RMS value. Pure sine waves have a crest factor of 1.414. Some analog analyzers only display a crest factor reading instead of a time waveform. Comparing these readings to 1.414 allows you to estimate the nature of the time waveform.

critical speed range - see **amplification range**

critically damped - A condition where a system's damped resonant frequency is equal to zero. Critically damped systems will move through one cycle when excited, but they do not vibrate (also see **damping**).

cross effect - The resultant vibration in an imbalance plane (or planes) caused by the addition of a trial weight in another plane.

cross-channel analysis - Using an analyzer with two or more input channels to calculate such functions as phase, coherence, and transfer function.

cycle - The time separation between two identical conditions of motion (also see **period**).

damping - The dissipation of vibratory energy. Damping can occur from friction, viscosity, and internal hysteresis.

degree of freedom - The number of axes along which a system is free to move.

differentiation - The mathematical process used to convert displacement into velocity, or velocity into acceleration. Differentiation is the reverse of integration.

digital analyzer - An analyzer that converts analog vibration information into a digital format suitable for computers. Digital analyzers allow precise, real-time analysis of vibration.

disk skew - A condition where an entire fan disk assembly is not mounted perpendicular to the shaft center line.

displacement - The total amount of movement of a vibrating system, equal to two times amplitude.

double differentiation - Performing differentiation twice to convert displacement into acceleration (also see **differentiation**).

double integration - Performing integration twice to convert acceleration into displacement (also see **integration**).

dynamic range - The ratio of the smallest amplitude to the largest amplitude that an analyzer can detect.

fault frequency - A frequency generated by a fault in an anti-friction bearing.

fixed bearing - An anti-friction bearing whose outer race is prevented from moving with the shaft (usually by inserting spacers on either side of the outer race). Compare with **floating bearing**.

fixed machine - The machine whose position is not changed during shaft alignment. Compare with **shim machine**.

floating bearing - An anti-friction bearing whose outer race is free to move with the shaft. Compare with **fixed bearing**.

fluid dynamics - The study of fluids in motion. Fluid dynamics is a branch of science that figures prominently in fan design, since moving air behaves like a fluid.

forced vibration - Vibration that is maintained from cycle to cycle by a forcing function (also see **forcing function**).

forcing function - A periodic force that causes a system to vibrate, and then maintains that vibration.

Fourier transform - The process of converting a time domain display into its various frequency components.

free space amplitude - The amplitude at which no amplification due to resonance occurs.

free vibration - Vibration where no energy is added after the initial force that begins the vibration.

frequency - The number of complete cycles in unit time.

frequency analysis - Machine analysis performed by examining a frequency domain display. The frequencies that are present are used to determine the vibration forcing function.

frequency domain - A display of all the frequencies of vibration that are present in a sample.

friction damping - Damping that occurs when two parts of a system slide against each other, like piston rings against a cylinder wall (also see **damping**).

harmonic - A frequency that is an integer multiple of a given frequency.

harmonic distortion - Distortion of a given wave occurring from the presence of one or more harmonics of that wave (also see **harmonic**).

head - The height of a column of fluid above a specified plane, typically expressed as inches of water. Head is another measure of pressure, since pressure is what pushes the water up the tube.

hydraulic instability - A vibratory condition where the lubricant in a sleeve bearing no longer flows as a fluid film. Caused primarily when the shaft moves off-center due to load, eccentricity, or unbalance, hydraulic instability leads to oil whirl and oil whip.

hysteresis damping - Damping that occurs due to internal friction created when the molecules of the material slide against each other (also see **damping**).

Hz (Hertz) - the number of cycles per second of a wave.

induction motor - An AC motor whose rotor is an electromagnet, but is not energized from an external source. Stator current induces a current flow in the rotor, turning the rotor into a magnet, which rotates as it tries to keep up with the rotating magnetic current.

influence coefficients - A set of numbers that define how changes made in one imbalance plane affect other imbalance planes in the system.

integration - The mathematical process used to convert acceleration into velocity, or velocity into displacement. Integration is the reverse of differentiation.

leakage components - Energy appearing in the first cell (and higher harmonics) of a frequency display, resulting from sudden changes in signal level at the beginning and end of the time window.

least squares balancing - A balancing method used to find a compromise balance solution. This method is used in systems where a satisfactory correction in one plane inevitably results in high vibration in other planes. By using least squares balancing, acceptable vibration readings are found in all planes.

line - One data point in a frequency domain signal. A line is sometimes called a **cell** or a **bin**.

line spectra - A frequency domain display that results when the vibration energy is at definite frequencies that transform into discrete frequencies (lines) in the spectra. Compare with **continuous spectra**.

mass - A body of matter that occupies a certain volume of three-dimensional space.

misalignment - A condition where the center lines of two coupled shafts are not exactly in line with each other. The two center lines may be displaced parallel to each other (called **parallel misalignment**) or they may intersect at an angle (called **angular misalignment**).

motor action - The motion that results from the interference of two magnetic fields. Motor action is the operating principle behind DC motors.

multiple plane balancing - A balancing method where a rotor is "divided up" into several imbalance planes. Correction weights are calculated and added in each plane.

natural frequency - The frequency at which a system will vibrate when excited by a single input force. The frequency of a system's free vibration.

non-synchronous components - Vibration frequencies that are not an integer multiple of the reference signal.

Nyquist frequency - A sampling rate equal to twice the input frequency, allowing the analyst to confidently reconstruct the input frequency from the discrete digital values.

offset misalignment - A misalignment condition where the shaft center lines are displaced parallel to each other and do not intersect. This condition is also called **parallel misalignment**.

oil whirl - A condition of hydraulic instability where the shaft inside a sleeve bearing rides on the unstable vortex created by the shaft being moved off center. Oil whirl is proportional to shaft frequency.

oil whip - A resonant condition that occurs when the oil whirl frequency coincides with the first critical of the shaft (also see **oil whirl**).

overdamped - A condition where a system's damped resonant frequency is imaginary. This condition arises when g/l in Formula 2-3 (see System Response chapter in Volume 1) is less than $(\omega_{2m})^2$. Overdamped systems exhibit non-oscillatory motion (also see **damping**).

parallel misalignment - See **offset misalignment**.

peak - The value of a wave's maximum amplitude.

period - The time distance between two identical conditions of motion of a wave.

phase - The time separation between two events.

phase angle - The time separation between two events when time is measured in degrees (also see **phase**).

phase shift - Any change in phase that occurs as a result of the system response.

pitch - For fans, the angle between the fan blade and an imaginary line perpendicular to the center line of the shaft.

pitch point - The point on a gear where the normal to the common tangent between two gear teeth intersects a line drawn between the two gear centers.

Power Spectral Density (PSD) - The unit of measure for a continuous spectrum (see). Power spectral density is determined by squaring the RMS voltage in a frequency band and dividing by the center frequency of the band: $\frac{V^2}{Hz}$.

pressure - Force per unit area, such as pounds per square inch or Newtons per square meter. Fan pressure is typically measured in Pa (for Pascal); 1 Pa = 1 N/m^2.

pump head - The mechanical energy that is imparted to a fluid (in foot-pounds per pound of fluid) by a pump. Pump head is independent of the fluid's density.

radial plane - The plane in which the radius of a shaft or bearing lies.

radian - An angular measurement where the length of a subtended arc segment is equal to the radius of the circle on which it lies.

resonance - The condition where the forcing function frequency coincides with a system's resonant frequency.

resonant frequency - The frequency at which a system will amplify the energy input from the forcing function, causing a dramatic (and often disastrous) rise in vibration amplitude.

restoring force - the backwards pull (or push) of a spring against the deflecting force (also see **spring constant**).

resultant - The vector obtained when two or more vectors are added or subtracted. When as-is and trial weight imbalance vectors are added, the vibration that results is the resultant vibration.

RMS - A measure of the power contained in a wave. The RMS of a pure sine wave is equal to .707 times the peak value of the wave.

sampling - In digital signal analysis, the process of cutting an analog (time) signal into a number of individual values.

sampling frequency - In digital signal analysis, the rate at which the input signal is sampled. Sampling frequency is also called the "sample rate".

shim machine - The machine whose position is changed during shaft alignment. Compare with **fixed machine**.

signal-to-noise ratio - The ratio of a wave's peak amplitude to that of the wave's noise floor (the **noise floor** is the level of noise that is present even if no signal is present; this level is the quietest that the wave can be).

sine wave - A wave whose periodic behavior exactly matches the plot of the sine function.

single plane balancing - A balancing method that assumes that a rotor's imbalance is located in a single plane. Correction weights are added in this plane. Also see **multiple plane balancing**.

sinusoidal - Having the shape of a sine wave.

slip cycle - The number of stator magnetic field rotations required for an induction motor's slip frequency to rotate once around the stator.

slip frequency - The difference between the frequency of the rotating magnetic field of an induction motor stator and the rotational frequency of the rotor.

soft foot - A poorly shimmed foot on a machine that, if tightened, deforms the machine enough to cause a bow or eccentricity.

spalling - Pitting and removal of metal caused by lubrication breakdown.

spectra - A display of the various frequencies present in a vibration signal.

spring - Any part of a system that supports mass and behaves like a spring (that is, the part pushes or pulls against a force).

spring constant - the number of pounds necessary to extend a spring (or deflect a structural member) one inch. The spring constant is expressed in lbs/in, and is written mathematically by the letter *k*.

spring-mass system - A mechanical system containing a spring, a mass, and damping. Mechanical systems treated for vibration are typical spring-mass systems.

starvation - A condition that occurs when a local pressure drop inside a fan allows small vacuum cavities to form. These cavities collapse when they contact the high pressure side of the fan, causing both noise and vibration. This same condition in pumps is caused **cavitation**.

static imbalance - A single mass imbalance acting through the shaft centerline in a single direction. Vibration readings taken at both ends of a rotor with static imbalance would be in phase, since the imbalance acts in the same direction at both ends. Compare with **couple imbalance**.

strobe light - A bright, rapidly flashing light used to "freeze" a rotating shaft, thus allowing you to read phase from the apparent still image.

sub-synchronous components - Vibration frequencies that are less than 1*RPM of the shaft.

synchronous components - Vibration frequencies that occur at integer multiples of shaft RPM. Synchronous components are also called **phase locked components**.

synchronous motor - An AC motor where the rotor rotates at exactly the same frequency as that of the stator field. The motor runs at a fixed speed regardless of the load on the motor.

system - A mechanical assembly that consists of a mass, a spring, and damping.

thermal growth - A change in length of a system component caused by expansion due to heating.

thermal vector - A vibratory force brought about as a result of uneven heating in a mechanical system.

time domain - A display of a system's vibratory behavior during a specific period of time.

time window - The time required for a digital analyzer to take the number of samples required to accurately reconstruct the input signal.

transducer - A device used to convert vibratory motion into an electrical signal proportional to that motion. The signal may be proportional to the displacement, the velocity, or the acceleration.

transfer function - A frequency domain description of how the system acts on one signal to produce another signal.

trial weight - A weight used in single- and multi-plane balancing to determine the system's response characteristics.

tribo effect - The induction of noise in a shielded cable as a result of the cable being bent.

underdamped - A condition where a system's damped resonant frequency is greater than zero, arising when s/l in Formula 2-3 (see System Response chapter in Volume 1) is greater than $(c/2m)^2$. Underdamped systems vibrate through several cycles before coming to rest (also see **damping**).

vector - A mathematical quantity having a magnitude (length) and a direction. Vibration forces are represented as vectors.

velocity - The change in displacement with respect to time.

vibration - The complex, periodic motion of a rotating or reciprocating system in response to one or more forcing functions.

vibration signature - The measure of all the frequencies comprising a system's vibratory movement.

viscosity - The measure of the internal friction of a fluid (its "thickness" or resistance to flow). Gases are also viscous, but fluids have much higher viscosities than gases.

viscous damping - Damping caused by the resistive force of a viscous liquid against a moving body. Most processes in industry are viscous processes (also see **damping**).

weight - The acceleration of a mass by gravity.

wiping - The rubbing of a shaft against the bearing material in a sleeve bearing.

Index

A

Acceleration
  defined, 329
Accelerometer
  defined, 329
Aliasing, 202
  avoiding, 204
  defined, 329
Amplification
  amplification range defined, 329
Amplitude
  defined, 329
Analog analyzers
  basics, 193
  calculating filter response time, 195
  defined, 329
  reading vibration amount, 193
  reading vibration frequency, 194
  reading vibration phase, 194
  strobe lights, 195
  tunable filters, 194
  using a strobe as a tachometer, 196
Analyzers, analog
  See Analog analyzers
Anti-friction bearings
  cautions for use, 12, 48
  fixed and floating, 11, 48
Attenuation
  defined, 329
Averaging
  exponential, 213
  linear, 213
  peak hold, 214
  synchronous, 214
  synchronous time, 214
Axial flow fans, 8
  cooling towers, 9
  pressure pulses, 8
Axial plane
  defined, 329
Axial pumps
  scroll, 44

B

Balance
  multiple plane balancing defined, 333
  single plane balancing defined, 335
Bandwidth
  defined, 330
Baseband, 202
  defined, 330
Bearings
  anti-friction, 11
  fault frequency defined, 331
  misalignment, 24, 108
  oil whirl, 25, 109
  reciprocating loads from engines, 124
Beat frequency
  defined, 330
Belt frequency
  fans, 18
Belt strand
  defined, 330
Belts
  calculating belt length, 167
  drive ratios, 163
  noise in drives, 165
  primary belt frequency, 166
  strand resonance, 168
  V-belts, 158

vibration from misalignment, 24
vibration in belt drives, 166
wear, 165
Bernoulli's equation, 35
Bin
See line
Blade pass frequency
defined, 330
fans, 18
Blowers
compared to fans, 3

C

Catastrophic maintenance, 223
Cavitation
compressors, 108
defined, 330
fans, 24
pumps, 45
Cell
See line
Centrifugal fans, 4
blade angle, 4
blade types, 5
Chains
calculating chain length, 176
major types, 171
sprocket teeth, 174
vibration in chain drives, 178
Chord
defined, 330
Coherence, 215
defined, 330
Commutator
defined, 330
See also motors
Compressors
axial, 98
centrifugal, 97
compared to fans, 3
head (vs. pressure), 94
high blade pass frequency, 109
reciprocating, 95
rotating, 95
screw compressors, 96
Conjugacy
defined, 330
See also gears
Contact angle
defined, 330
Continuous spectra
defined, 330
Cooling towers, 9
Coulomb damping
defined, 330
Couple imbalance
defined, 331
Couplings, 187
accommodating misalignment, 187
gear couplings, 188
lock up, 108
torsional resonance, 188
types, 187
Crest factor
defined, 331
Cross channel analysis
defined, 331
Cross effect
defined, 331
Cycle
defined, 331

D

Damping
critical damping defined, 331
defined, 331
friction damping defined, 332
hysteresis damping defined, 332
overdamped defined, 333
underdamped defined, 336
viscous damping defined, 336
Data collection
broadband or spectral, 226
Degrees of freedom
defined, 331
Differentiation
defined, 331
double differentiation defined, 331
Digital analysis
aliasing, 202

avoiding aliasing, 204
bandwidth, 204
baseband frequency, 202
coherence, 215
constant percentage bandwidth, 199
continuous spectra, 206
exponential averaging, 213
frequency spectra, 204
Hanning window, 212
instantaneous spectra, 212
leakage, 211
line spectra, 205
linear averaging, 213
lines, 204
Nyquist frequency, 202
peak hold, 214
PSD (Power Spectral Density), 206
real-time analyzer rate, 212
sampling, 201
skirts, 206
synchronous averaging, 214
synchronous time averaging, 214
time window, 204
transfer function, 217
triggering, 213
Digital analyzers
defined, 331
first practical, 200
Disk skew
defined, 331
Displacement
defined, 331
Dynamic range
defined, 331
Dynamic vibration absorbers
defined, 51
problems with, 51

E

Engines
compression ratio, 115
diesel cycle, 114
external combustion, 112
internal combustion, 112
otto cycle, 113
PV (pressure volume) curves, 113
rotating and reciprocating forces, 116
slider crank mechanism, 115
vibration from accessories, 124
vibration sources, 115
Experimental modal analysis
apparent mass, 246
apparent stiffness, 246
basic steps, 247
compared with finite element analysis, 238
disadvantages, 238
frequency response functions, 246
imaginary frequency response curve, 242
magnification factor, 241
multiple degree of freedom systems, 243
operating deflection shape, 263
orthogonality, 244
overview, 237
single degree of freedom systems, 239
structural modification, 255

F

Fans
axial flow, 8
background vibration, 20
belt frequency, 18
belt strand frequency, 19
blade angle (centrifugal), 4
blade pass frequency, 18
cantilevered, 5
center hung, 4
centrifugal, 4
cooling towers, 9
fan curves, 3
head, 3
high blade pass frequency, 25
interpreting time domain data, 22
mounting, 9
operating principles, 2
operational sources of vibration, 14
pressure, 2
rotating stall, 14
stall, 14, 23
structural sources of vibration, 9
surge, 14

taking static data, 18
variable pitch, 26
vibration analysis, 15
Fast Fourier Transform
See FFT
FFT (Fast Fourier Transform), 200
direct Fourier transform, 201
discrete Fourier transform, 201
inverse Fourier transform, 201
Filters
anti-aliasing, 204
constant percentage bandwidth, 199
Finite element analysis
analysis procedure, 290
dynamic analysis, 274
element types, 285
increasing accuracy, 271
modal superposition, 274
principles, 270
static analysis, 272
two categories, 269
Fixed bearing
defined, 331
Fixed machine
defined, 332
Floating bearing
defined, 332
Fluid dynamics
Bernoulli's equation, 35
defined, 332
equation of continuity, 35
overview, 35
relating to fans, 2
Reynolds number, 37
viscosity, 36
Forcing function
defined, 332
Fourier transform
defined, 332
Free space amplitude
defined, 332
Frequency
circular frequency defined, 330
defined, 332
frequency analysis defined, 332
natural frequency defined, 333
Frequency analysis
using an analog analyzer, 199
Frequency domain
continuous spectra, 206
defined, 332
display, 201
line spectra, 205
PSD of continuous spectra, 206

G

Gear couplings, 188
Gears
basic gear theory, 145
bevel, 150
conjugacy, 146
crossed helical, 149
helical, 149
herring bone, 149
hypoid, 151
involute tooth profile, 147
load pulses, 151
mesh frequencies, 151
pitch point, 145
spur, 148
straight bevel, 151
torsional vibration in gear boxes, 153
transmission error, 151
worm, 150

H

Hanning window, 212
Harmonic
defined, 332
Harmonic distortion
defined, 332
Head
defined for compressors, 94
defined for fans, 3
general definition, 332
Hertz
defined, 332
Hydraulic instability
defined, 332

I

Induction motor
defined, 332
Influence coefficients
defined, 332
Integration
defined, 333
double integration defined, 331

L

Leakage components
defined, 333
Least squares balancing
defined, 333
Line
defined, 333
Line spectra
defined, 333

M

Machinery maintenance
catastrophic (hysterical), 223
predictive (histogram), 224
preventive (historical), 224
Mass
defined, 333
Misalignment
angular, 46
bearing misalignment defined, 330
defined, 333
offset misalignment defined, 333
Modal analysis
See Experimental modal analysis
Motor action
defined, 333
Motors
AC motor principles, 62
commutator, 68
DC motor principles, 66
electrical sources of vibration, 76
electricity and magnetism, 60
electromagnets, 60
induction, 65
interpoles (DC), 74
left hand rule, 60
magnetic flux lines, 59
mechanical sources of vibration, 79
motor action (DC motors), 67
rotor (AC), 64
series, 75
shunt, 75
stator (AC), 62
synchronous, 64

N

Non-synchronous components
defined, 333
Nyquist frequency, 202
defined, 333

O

Oil whip/oil whirl
oil whip defined, 333
oil whirl defined, 333
Oil whirl, 25, 109
Operating deflection shape, 263
"pseudo" frequency response function, 265
test setup, 264
Overhung fan
See Fans, cantilevered

P

Pa (Pascal)
measure of fan pressure, 2
Peak
defined, 334
Period
defined, 334
Phase
defined, 334
phase angle defined, 334
phase shift defined, 334
reading at operating frequency, 23
Pitch

defined, 334
Pitch point
defined, 334
See also gears
Power Spectral Density (PSD)
defined, 334
Power transmission
chain drives, 171
flexible coupling types, 187
flexible couplings, 187
torsional resonance in couplings, 188
U joints, 178
V-belt types, 160
V-belts, 158
vibration in belt drives, 166
vibration in chain drives, 178
Predictive maintenance, 224
Pressure
defined, 334
Preventive maintenance, 224
PSD (Power Spectral Density), 206
Pump head
defined, 334
Pumps
axial, 43
cavitation, 45
centrifugal, 38
diffusion vanes, 41, 44
head, 32
head (vs. pressure), 32
NPSH (Net Positive Suction Head), 34
operating principles, 32
operational sources of vibration, 44
resonance, 50
structural sources of vibration, 47
system head, 33
volute, 39

R

Radial plane
defined, 334
Radian
defined, 334
Resonance
acoustic resonance (pumps), 46
belt strand excitation, 168
defined, 334
formulas for approximating, 313
pumps, 50
torsional resonance in couplings, 188
Resonant frequency
defined, 334
fans, 18
Restoring force
defined, 334
Resultant
defined, 334
Reynolds number, 37
RMS (Root mean squared)
defined, 334

S

S/N (signal to noise ratio)
defined, 335
Sampling
defined, 334
Sampling frequency
defined, 334
Shim machine
defined, 335
Sine wave
defined, 335
sinusoidal defined, 335
Slip cycle
defined, 335
See also motors
Slip frequency
defined, 335
See also motors
Soft foot
defined, 335
Spalling
defined, 335
Spectra
defined, 335
Spring
defined, 335
spring constant defined, 335
spring mass system defined, 335
Starvation

defined, 335
Static imbalance
defined, 335
Strobe light
defined, 335
Strobe lights
uses of, 195
using as a tachometer, 196
using with analog analyzers, 195
Sub-synchronous components
defined, 335
Synchronous components
defined, 335
Synchronous motor
defined, 335
System
defined, 336

T

Thermal growth
defined, 336
Thermal vector
defined, 336
Time domain
defined, 336
described, 201
display, 201
Time domain data
for fans, 22
impacts, 107
non-periodic components, 23, 107
Time window
defined, 336
Torsional vibration
gear boxes, 153
Transducer
defined, 336
Transfer function, 217
defined, 336
output/input ratios, 218
plotting options, 218
possible types, 218
Transmission error
in gears, 151
Trial weight
defined, 336
Tribo effect
defined, 336
Triggering, 213
Turbines
and generator rotors, 142
balancing, 132
couple imbalance, 139
intermediate solution balancing, 140
mechanical imbalance, 129
misalignment, 130
rotor bow, 130
static couple balancing, 135
static imbalance, 138
steam imbalance, 129

U

Universal joints, 178
vibration, 185

V

V-belts, 158
drive ratios, 163
types, 160
Vectors
defined, 336
Velocity
defined, 336
Vibration
belt drives, 166
chain drives, 178
collection techniques, 225
defined, 336
fans (structural sources), 9
forced vibration defined, 332
free vibration defined, 332
universal joints, 185
vibration signature defined, 336
Vibration analysis
fans, 15
static data (fans), 18
Viscosity
defined, 336

W

Weight
    defined, 336
Windows
    exponential, 212
    force-exponential, 212
    Hanning, 212
    uniform, 212
Wiping
    defined, 336

The *Simplified Handbook of Vibration Analysis* series explains how to determine the health of rotating/reciprocating machinery by analyzing vibration signatures. Written in a non-technical, easy-to-follow manner, these books emphasize practical experience and real-world problem solving.

Although some mathematical equations are included in the text, a background in physics or mathematics is not necessary. The emphasis is on *how* to measure and resolve vibration problems, not on *why* the vibrations behave the way they do. These texts will be an invaluable resource for any maintenance personnel responsible for machinery vibration analysis.

Volume 2 focuses on applied vibration analysis. There are chapters focusing on vibration sources and analysis techniques for specific machinery: fans, pumps, motors, compressors, turbines, and engines. Other chapters discuss vibration considerations for power transmission components such as gears, belts, chains, universal joints, and flexible couplings. Two chapters discuss vibration analysis techniques using analog and digital analyzers. These are followed by a chapter on data collection techniques, which provides detailed information on planning and implementing a vibration data collection program. Finally, two chapters discuss the theory and application of Experimental Modal Analysis and Finite Element Analysis.

Computational Systems Incorporated